Critical Dynamics and Plastic Flow in Disordered Josephson Junction Arrays

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We present numerical simulations of Josephson junction arrays with positional disorder. We study their IV characteristics and vortex dynamics as a function of disorder. We find that above the critical current i_c there is a plastic flow of vortices and antivortices through channels, characterized by strong fluctuations of the total vorticity. For large currents there is a crossover to homogeneous flow without vortex fluctuations. We also study the dynamical critical behavior close to i_c in the gauge glass model, calculating critical exponents for the voltage onset and voltage fluctuations.

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Two dimensional Josephson junction arrays (JJA) can be designed with very specific geometries and properties through the use of modern photolithographic techniques [1]. In particular, experiments in JJA with controlled disorder have been done in the past [2], mainly directed to the study of the effects of randomness in the Kosterlitz-Thouless phase transition [3]. Also recently, the study of the JJA as dynamical systems with a large number of coupled degrees of freedom has become of great interest [4–7]. Even when the thermodynamical properties of disordered JJA have been studied as a function of temperature [2,3], their zero temperature dynamical behavior has received little attention in the past. Here we address this problem, studying for the first time the different dynamical regimes as a function of disorder and driving current.

The nonlinear dynamics of disordered media driven by an external force leads to critical phenomena in a great variety of systems. Examples of this are sliding charge density waves [8,9], fluid invasion into porous media [10], critical current phenomena in type II superconductors [8,11,12], elastic strings in a random potential [13], transport in arrays of metallic dots [14], and fluid flow down a rough inclined plane [15]. Basically, the interplay between the elastic interactions of a large number of degrees of freedom and a random static potential originates a threshold value for the driving force F. Above the threshold force F_T, a collective motion of the system takes place. It has been proposed by Fisher [8] that the onset of motion with velocity v should follow a critical behavior as $v \sim (F - F_T)^\zeta$, with diverging characteristic lengths and times. Two types of behaviors can be distinguished: "elastic flow," where the elastic medium distorts but does not break, and the time averaged flow is homogeneous in space [8–10]; and "plastic flow," where the motion breaks up into channels and therefore the flow is intrinsically inhomogeneous [8,10,14–16]. In this paper, we propose the disordered JJA as a new example of these phenomena, where controllable experiments at low temperatures can be done.

A related problem is the onset of flux flow in type II superconductors [8,11,12,16]. It has been shown that for all but very weak pinning, close to the critical current, the dissipation starts through channels of plastic vortex flow [16]. Recently, Bhattacharya and Higgins [12] have done experiments in layered superconductors, determining a nonequilibrium phase diagram as a function of effective pinning strength and current.

We consider current driven two dimensional JJA described by the Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{r},\hat{\mu}} E_J \left\{1 - \cos[\theta(\mathbf{r}+\hat{\mu}) - \theta(\mathbf{r}) - 2\pi f_{\hat{\mu}}(\mathbf{r})]\right\} + \frac{I\Phi_0}{2\pi}\sum_{r_y}[\theta(r_x=0,r_y) - \theta(r_x=Na,r_y)] , \qquad (1)$$

where $\theta(\mathbf{r})$ is the phase of the superconducting wave function at site $\mathbf{r}$; $\hat{\mu} = \hat{\mathbf{x}}, \hat{\mathbf{y}}$; $f_{\hat{\mu}}(\mathbf{r}) = \frac{1}{\Phi_0}\int_{\mathbf{r}}^{\mathbf{r}+\hat{\mu}} \mathbf{A}\cdot d\mathbf{l}$ is the line integral of the vector potential with Φ_0 the flux quanta; and I is the external current applied along the y direction. The array has $N \times N$ sites, and a is the lattice constant. The Josephson energy $E_J = I_0\Phi_0/2\pi$, with I_0 the critical current of each junction, is assumed to be the same for all the junctions regardless of the disorder. Therefore, all the effect of the randomness is on the $f_{\hat{\mu}}(\mathbf{r})$. They satisfy $f(\mathbf{R}) = \sum f_{\hat{\mu}}(\mathbf{r})$, where the sum goes around the plaquette centered at $\mathbf{R}$, and $f(\mathbf{R})$ is the magnetic flux normalized by Φ_0. We will simulate the positional disorder in two ways. (i) We take random displacements of the sites, $\mathbf{r}/a = (n_x + \delta^x, n_y + \delta^y)$, with n_x, n_y integers, and $\delta^{x,y}$ a random uniform number in $[-\Delta/2, \Delta/2]$. We use the Landau gauge for which $f(\mathbf{r},\mathbf{r}') = -\frac{H}{\Phi_0}\frac{(r'_x - r_x)(r'_y + r_y)}{2}$, with H the applied magnetic field. Noting that $\langle f(\mathbf{R})\rangle = Ha^2/\Phi_0 = f$, we will consider here only the case of integer f, which for $\Delta = 0$ maps to the problem with $H = 0$. (ii) The strong disorder limit ($f\Delta \to \infty$) can be simulated by taking each $f_{\hat{\mu}}(\mathbf{r})$ as a random variable with uniform distribution in $[-1/2, 1/2]$ [and thus $\langle f(\mathbf{R})\rangle = 0$]. This corresponds to the gauge glass model (GGM), recently studied in numerical simulations of the vortex glass transition [17], assumed to take place in disordered type II superconductors.

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The Hamiltonian of Eq. (1) can be mapped [3] to a Coulomb gas of integral charges (vortices) with a long-range logarithmic interaction, perturbed by a quenched random distribution of dipoles and driven by an external electric field,

$$\mathcal{H}_{\rm CG} = 2\pi^2 E_J \sum_{\mathbf{R},\mathbf{R}'} [n(\mathbf{R}) - f(\mathbf{R})] G(\mathbf{R},\mathbf{R}') \times [n(\mathbf{R}') - f(\mathbf{R}')] + I\Phi_0 \sum_{\mathbf{R}} V^{\rm ext}(\mathbf{R}) n(\mathbf{R}) . \quad (2)$$

Here $n(\mathbf{R})$ is the integer vorticity, $G(\mathbf{R},\mathbf{R}')$ is the two dimensional Green's function, which decays logarithmically at long distances, $V^{\rm ext}(\mathbf{R}) = \sum_{R'_y} G(R'_x = Na, R'_y, \mathbf{R}) - G(R'_x = 0, R'_y, \mathbf{R})$ can be approximated by a linear function in the middle of the array. The random $f(\mathbf{R})$ are correlated and correspond to the contribution of dipoles. Each displaced site $\mathbf{r}$ contributes with a dipolar momentum $\mathbf{p}_\mathbf{r} = -f\boldsymbol{\delta}_\mathbf{r} = -f(\delta^x_\mathbf{r}, \delta^y_\mathbf{r})$, in the lowest order in Δ; and in the GGM limit, $\mathbf{p}_\mathbf{r} = f_{\hat{\mu}}(\mathbf{r})\hat{\nu}$, with $\hat{\nu}$ the direction perpendicular to $\hat{\mu}$.

We model the dynamics of the JJA, in the classical zero temperature approximation, with the resistively shunted junction model for each junction plus current conservation in each superconducting node [4–7]. The dynamical equations for the phases $\theta(\mathbf{r})$ are

$$\frac{d\theta(\mathbf{r})}{dt} = -\frac{2\pi\mathcal{R}}{\Phi_0} \sum_{\mathbf{r}'} G(\mathbf{r},\mathbf{r}') \Big\{ I^{\rm ext}(\mathbf{r}) - \sum_{\pm\hat{\mu}} I_0 \sin[\theta(\mathbf{r}'+\hat{\mu}) - \theta(\mathbf{r}') - 2\pi f_{\hat{\mu}}(\mathbf{r}')] \Big\} , \quad (3)$$

where $\mathcal{R}$ is the shunted resistance of each junction and $G(\mathbf{r},\mathbf{r}')$ is the lattice Green's function. We take periodic boundary conditions along the x direction and open boundary conditions along the y direction. At the bottom (top) of the array the external current is injected (taken out) with $I^{\rm ext}(r_y = 0) = I$ $[I^{\rm ext}(r_y = Na) = -I]$ and $I^{\rm ext}(\mathbf{r}) = 0$ otherwise. The sum over $\pm\hat{\mu}$ runs over the four neighboring links in all the interior sites, and only over the three superconducting links in the top and bottom boundary sites. We evaluate Eq. (3) with an efficient algorithm based on fast Fourier transform and tridiagonalization techniques as in Ref. [6]. The time integration is carried out using a fixed step fourth order Runge-Kutta method. Typical integration steps are $\Delta t = 0.01 - 0.1\tau_J$ ($\tau_J = \frac{\Phi_0}{2\pi\mathcal{R}I_0}$), and the integration is carried out for time intervals of $t = 5000\tau_J$, after a transient of $2000\tau_J$.

In Fig. 1 we show the results as a function of $i = I/I_0$ for strong disorder with $f = 15$ and $\Delta = 0.05$ (we basically find the same result for the GGM). In Fig. 1(a) we show the IV characteristics, where the average voltage drop is $v = \langle\overline{v(t)}\rangle = \Big\langle\overline{\frac{1}{N_x(N_y-1)}\sum_\mathbf{r} v_y(\mathbf{r},t)}\Big\rangle$, with $\overline{v}$ the average over time, $\langle v\rangle$ the average over disorder configurations, and $v_y(\mathbf{r},t) = \tau_J(\frac{d\theta(\mathbf{r}+\hat{y})}{dt} - \frac{d\theta(\mathbf{r})}{dt})$. In Fig. 1(b),

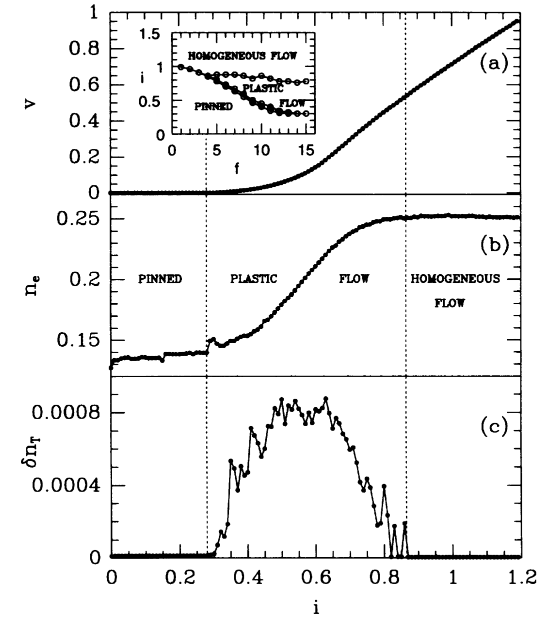

FIG. 1. (a) IV characteristics of a 32×32 disordered Josephson junction array with $f = 15$ and $\Delta = 0.05$, averaged over seven samples. (b) Number of vortex excitations n_e as a function of $i = I/I_0$. (c) Fluctuations of the total vorticity δn_T as a function of i. The inset in (a) corresponds to the dynamical phase diagram of 32×32 disordered arrays as a function of integer f, with $\Delta = 0.05$.

we show the average number of vortex excitations $n_e = \langle\overline{n_e(t)}\rangle = \Big\langle\overline{\frac{1}{(N-1)^2}\sum_\mathbf{R} |n(\mathbf{R},t) - f|}\Big\rangle$, with vorticity calculated as $n(\mathbf{R},t) = -\sum {\rm nint}[\frac{\theta(\mathbf{r}+\hat{\mu})-\theta(\mathbf{r})}{2\pi} - f_{\hat{\mu}}(\mathbf{r})]$, where the sum goes around the plaquette $\mathbf{R}$ and nint$[x]$ is the nearest integer to x. In Fig. 1(c), we show the average number of vortex temporal fluctuations, $(\delta n_T)^2 = \langle\overline{n_T(t)^2} - \overline{n_T(t)}^2\rangle$, with $n_T(t) = \frac{1}{(N-1)^2}\sum_\mathbf{R} n(\mathbf{R},t)$. The average vorticity satisfies $\langle\overline{n_T(t)}\rangle = f$, because of vortex number conservation. We find three different regimes.

(i) Below a critical current i_c there is a pinned phase where there is no dissipation ($v = 0$), and no vortex fluctuations ($\delta n_T \approx 0$). The number of vortex excitations basically counts the number of vortex-antivortex pairs (VAP) that exist around the background integral vorticity f; $n_e = 2n_{\rm VAP}$. In the pinned phase, the most defective sites (the ones with stronger $\mathbf{p}_\mathbf{r}$) nucleate VAPs. In fact, it has been shown that single defects nucleate VAPs in current driven JJA [5,6]. The external current tends to break the pinned VAPs, because of the opposite Lorentz forces acting on them [5,6]. In the pinned phase, as the current is increased, the VAPs with weaker pinning can be broken, and the freed vortices and antivortices move until they are pinned again in stronger pinning sites. Therefore, after a transient, the VAPs redistribute in the array in a new stable configuration, and n_e remains nearly the same (it increases slightly). Close to i_c there are fewer and fewer possibilities of redistributing the VAPs, and at i_c the first VAP finds a path along the array without being pinned in any other site.

(ii) Above i_c dissipation starts and v grows nonlinearly. At the same time we find that there are strong fluctuations of the vorticity ($\delta n_T \neq 0$). Also n_e grows, since once some VAPs are free to move, new ones are again nucleated at the sites. As the current is increased, the time averaged number of free VAPs increases, since they move faster and faster. In this regime we find that the VAPs move along certain channels of flow in the array, with the number of those channels increasing with increasing current. This is shown in Fig. 2 for the GGM disorder, where we plot the time averaged voltage $v_y(\mathbf{r}) = \overline{v_y(\mathbf{r},t)}$ for a given sample. We call this phase the "plastic flow" regime, in analogy to what happens in type II superconductors [11,12,16]. But, since we are considering $f = n$, there is no underlying flux lattice and therefore the plastic flow is not originated by strong deformations in a given vortex lattice as studied for thin films [16]. Instead, the plastic flow is originated by the generation of VAPs which follow an irregular motion in the disordered array. This rather inhomogeneous motion explains the fluctuations in the vorticity that we found in this case, since at a given time there is no exact cancellation of vortex and antivortex excitations, because of the lack of reflection symmetry in this regime.

(iii) At higher currents there is a crossover to a regime where again $\delta n_T \approx 0$. At the same time the voltage grows linearly with the current, and n_e saturates to a higher constant value. This dynamical regime is characterized by a homogeneous flow of vortices and antivortices [i.e., $v_y(\mathbf{r})$ is almost idependent of $\mathbf{r}$], since the number of flow channels has grown until covering all the sample.

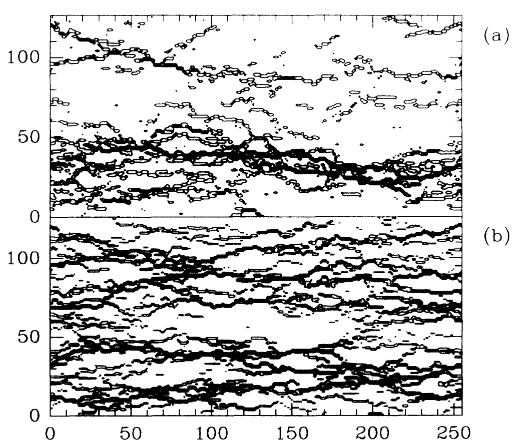

FIG. 2. Contour plot of the time averaged dissipation $v_y(\mathbf{r})$ showing the paths where vortices (antivortices) have moved. For a 256 × 128 array with gauge glass disorder. (a) For a current $i = 0.22$; (b) for $i = 0.255$, the number of channels has grown. (The critical current is $i_c = 0.215$.)

In the inset of Fig. 1(a) we show a dynamical phase diagram with the different regimes as a function of disorder. Here, we keep $\Delta = 0.05$ constant, and change f for integral values, since the effective disorder is proportional to $f\Delta$ [3]. For very weak disorder we do not see a plastic flow regime, and there is hysteresis around the onset of dissipation [18]. With increasing f the current range of the plastic regime first grows and then saturates for large f, tending to the behavior of the GGM.

Finally, we analyze the critical behavior above i_c for the GGM. Each sample has a well defined threshold current i_c, which is very sensitive to finite size effects and fluctuates from sample to sample for a given size. In the inset of Fig. 3(a) we show the sample average of i_c, and we see that $\langle i_c(N)\rangle$ decreases and seems to saturate for large N [19]. After doing a quadratic fit in $1/N$, we extrapolate for $N \to \infty$ a critical current $i_c^* = 0.205 \pm 0.010$. In Fig. 3(a) we show the voltage close to the critical current for samples of different sizes. We made a fit of the power law $v \sim (i - i_c^*)^\zeta$, as it is shown in the figure, obtaining $\zeta \approx 2.22 \pm 0.20$ [20]. Also above the critical current, there is an onset of voltage temporal fluctuations $(\delta v)^2 = \langle \overline{v(t)^2} - \overline{v(t)}^2 \rangle$, which grow nonlinearly in the plastic flow regime, and then saturate in the homogeneous flow regime. In Fig. 3(b) we show a fit of the power law $(\delta v)^2 \sim \frac{1}{N^2}(i - i_c^*)^\psi$ for samples of different sizes, obtaining $\psi \approx 2.27 \pm 0.20$. In the inset of Fig. 3(b) we show the sample to sample fluctuations of the critical current,

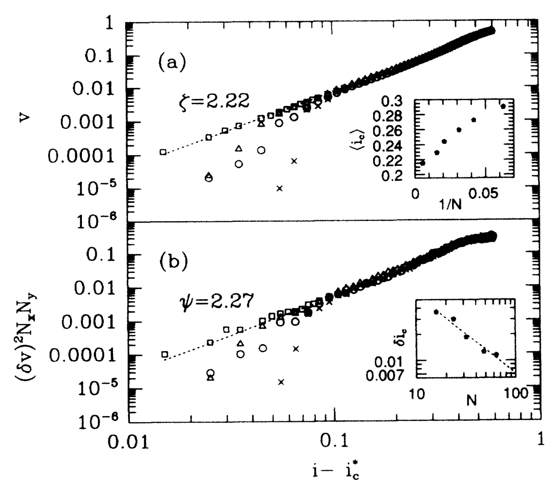

FIG. 3. Critical behavior of arrays with gauge glass disorder. (a) Average voltage v as a function of $i - i_c^*$, for different sample sizes: ×, 32 × 32; △, 48 × 48; ○, 64 × 64; and □, 256 × 128. The inset shows $\langle i_c \rangle$ as a function of N with configurational averages over n_s samples: 16 × 16, $n_s = 200$; 32 × 32, $n_s = 90$; 48 × 48, $n_s = 36$; 64 × 64, $n_s = 20$; 256 × 128, $n_s = 1$. (b) Voltage fluctuations $(\delta v)^2$ as a function of $i - i_c^*$, with the same symbols as in (a) for the sample sizes. The inset shows the fluctuations of the critical current δi_c as a function of N, averaged over the same n_s as in (a). The dotted lines correspond to nonlinear fits.

$(\delta i_c)^2 = \langle i_c^2 \rangle - \langle i_c \rangle^2$, as a function of N. They follow the power law $\delta i_c \sim N^{-1/\nu_T}$. The exponent ν_T is characteristic of a finite size scaling length $\xi_{\rm FS} \sim (i - i_c)^{-\nu_T}$, which characterizes the distribution of properties in ensembles of samples as a function of their size [9]; and $\nu_T \geq 2/d$. We find $\nu_T \approx 1.09 \pm 0.13$, which is very close to the value found in charge density wave systems [9]. If a one vortex description were valid, the voltage fluctuations would be given by $(\frac{\delta v}{v})^2 \sim \xi^{-(d-4+\eta)}(\xi/N)^d$ with the prefactor $\xi^{-(d-4+\eta)}$ coming from incoherence within a correlation volume, and $d - 4 + \eta > 0$ [8,9,13]. However, since the long range vortex-vortex interactions play a major role in this case, this may not be correct. In fact, one gets $4 - \eta = (2\zeta - \psi)/\nu_T = 1.99 \pm 0.24 \approx d$, which means that coherence within a correlation volume is relevant.

Apart from $\xi_{\rm FS}$, there are two other correlation lengths which characterize the plastic flow as it can be guessed from Fig. 2 (see also Refs. [15] and [14]). These are the typical distance between channels $\xi_{\parallel}$, and the typical distance between channel splits $\xi_{\perp}$. Both diverge when approaching i_c, leaving only one channel close to i_c when $\xi_{\parallel}, \xi_{\perp} > N$. On the other hand, the crossover to homogeneous flow occurs when $\xi_{\parallel}, \xi_{\perp} \approx a$. In the light of the present experience with charge density waves [9], a complete finite size scaling analysis would be desirable in order to get reliable critical exponents. This requires the knowledge of $\nu_{\parallel}$ and $\nu_{\perp}$, from $\xi_{\parallel,\perp} \sim (i - i_c^*)^{-\nu_{\parallel,\perp}}$. However, a detailed study of the critical behavior of these correlation lengths needs bigger lattices than the ones we are presently able to simulate.

In conclusion, we have shown that the onset of dissipation in strongly disordered JJA occurs through channels of flow of vortices and antivortices. We have given a dynamical phase diagram as a function of disorder for the current range of this plastic flow regime. Moreover, we have found some evidence that the onset of plastic flow may occur as a critical phenomenon, obtaining the critical indices for the voltage and the voltage fluctuations. As a difference with flux lattices [16], the plastic flow in this case is due to the inhomogeneous motion of vortex-antivortex excitations. Other related problems are the fluid flow down a rough inclined plane [15], and transport in arrays of metallic dots [14]. However, the long range (logarithmic) interactions present in this model should situate it in a different universality class. Controlled experiments on disordered JJA at low temperatures could be conducted in order to study these dynamical critical phenomena. In particular, recently developed vortex imaging techniques [21] would allow us to directly observe the plastic flow regime close to i_c.

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[19] However, we cannot discard that i_c could vanish logarithmically with N, as proposed in Ref. [5] for JJA with percolation disorder.

[20] Since our power-law fits are over less than two decades, we cannot assure that we are in the asymptotic critical regime yet. Simulations closer to the critical current would require very long integration times, however.

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