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Quantum Statistics of a Laser Cooled Ideal Gas

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We study the dynamics of a system of bosonic or fermionic atoms in a microscopic trap undergoing laser cooling. We show that the stationary state can be described by a Bose-Einstein or Fermi-Dirac distribution, respectively. Fluorescence from the system reflects quantum statistical properties.

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Recently there has been a growing interest in studies of quantum statistical properties of cold atoms. In part, this is motivated by progress in laser cooling and trapping of neutral atoms [1]. In addition, and parallel to these efforts, the experimental realization of the Bose-Einstein condensation (BEC) in magnetic traps has become a challenging task of atomic physics [2].

In laser cooling, an atomic transition is excited by near-resonant laser light so that the momentum transfer associated with the induced and spontaneous emission cycles provides a dissipative mechanism for the atomic center–of–mass motion. Recent achievements in laser cooling are exemplified by new cooling mechanisms which have led to temperatures close to or below the one–photon recoil limit [1,3], and the observation of quantized atomic motion in optical molasses [4].

With increasing atomic densities and decreasing temperatures one expects the quantum statistics related to the bosonic or fermionic properties of the cold atoms to become observable. In addition, we expect collective radiative atomic interactions and the complex coupling of the laser to the *electronic* atomic degrees of freedom to alter significantly the dynamics of the cooling process. Thus, on the theoretical side the challenge is to generalize the single atom master equation of laser cooling to include many particle and quantum statistics effects. Such a master equation will provide a complete description of the dynamics of the cooling and predict the steady state distribution of the atoms. Of particular interest is the relationship between these stationary solutions of the master equation for a finite number of atoms N and the thermodynamic canonical ensemble as given by the Bose-Einstein (BED) or Fermi-Dirac distributions (FDD). We emphasize that in the case of laser cooling the laser provides the coupling to an effective *external* reservoir which establishes the equilibirum distribution. This allows us to study an *ideal* Bose or Fermi gas to the extent that collisions are negligible. This is in contrast to the standard approach of, for example, BEC in a *dilute* Bose gas, where collisions play a fundamental role in establishing a thermodynamic equilibrium and provide the thermalization necessary for an evaporative cooling mechanism [2].

In this Letter we study the dynamics of a one–dimensional system of noninteracting atoms in a microscopic trap undergoing laser cooling. We show that in the present model the stationary state of such a system can be described by the BED or FDD. The control parameter of the system is the detuning of the driving laser which plays a role analogous to the temperature. In particular we consider a system of N identical two–level atoms with (electronic) ground state $|g\rangle$ and excited state $|e\rangle$ in a trap. The atoms interact with a laser field of wavelength λ. We assume that the trapping potential is harmonic, so that the energy level scheme of an atom in the trap consists of a double ladder of equidistant states. For simplicity we locate the trap at the node of a standing wave laser [5]. When the ground state function of the trap has a size $a_0 \ll \lambda$ we are in the Lamb–Dicke limit (LDL), so that the Lamb–Dicke parameter $\eta = 2\pi a_0/\lambda$ is small and the model becomes solvable.

The second quantized Hamiltonian describing such a

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system of atoms can be constructed along the lines described in Refs. [6]. Using standard methods [7] we can then eliminate the photonic degrees of freedom and obtain a second quantized master equation for the atomic density operator ρ ($\hbar = 1$),

$$\dot{\rho} = -i[H_0 + H_L, \rho] + \mathcal{L}\rho . \quad (1)$$

The free atomic Hamiltonian is

$$H_0 = \sum_{k=0}^{\infty} \left[E_k g_k^\dagger g_k + (E_k + \omega_0) e_k^\dagger e_k\right] , \quad (2)$$

where $E_k = k\nu$ are energies of levels in the harmonic trap potential, ν is the trap frequency, and ω_0 is the frequency of the electronic transition. The operators $g_k^\dagger$, g_k, $e_k^\dagger$, and e_k describe creation and annihilation of atoms in the ground and excited electronic states, respectively. These operators fulfill the canonical bosonic commutation or fermionic anticommutation relations, depending on the spin of the atoms. In the LDL ($\eta < 1$) the Hamiltonian H_L describing interactions of atoms with the laser takes on the simple form

$$H_L = \frac{\eta\Omega}{2} e^{-i\omega_L t} \sum_{k=1}^{\infty} \sqrt{k} \left[e_{k-1}^\dagger g_k + e_k^\dagger g_{k-1}\right] + \text{H.c.},$$

with Ω the Rabi frequency of the atomic transition, and ω_L the laser frequency. We denote the detuning by $\delta = \omega_L - \omega_0$. The Hamiltonian H_L describes the transitions from the kth ground electronic state to the excited electronic states with indices $k \pm 1$ [8]. The absence of excitations diagonal in the vibrational quantum number k is due to the fact that the trap is located at the node.

Finally, the last part of the master equation describes spontaneous emission processes. Since all atoms are located within a distance smaller than λ, the spontaneous emission has here a purely super-radiative character [9]. In the second quantized theory of moving atoms the spontaneous emission causes transitions from the excited to ground states with the same k so that

$$\mathcal{L}\rho = \frac{\Gamma}{2} \sum_{k,k'}^{\infty} \left[2 g_k^\dagger e_k \rho e_{k'}^\dagger g_{k'} - e_{k'}^\dagger g_{k'} g_k^\dagger e_k \rho - \rho e_{k'}^\dagger g_{k'} g_k^\dagger e_k\right] ,$$

where $\Gamma = 2\gamma$ denotes the spontaneous emission rate for a single atom. Note that the extra diffusion terms give higher order contributions in η, and can therefore be neglected. We stress that the characteristic dissipative time scale of the system is associated with the inverse of the cooperative spontaneous emission rate $\Gamma_N = N\Gamma$.

Note that in the absence of the laser ($\Omega = 0$) the master equation (1) describes the dynamics of super-radiance [9]. In that case, every state of the system for which all atoms are in the ground electronic state, is stationary. Turning on the laser introduces the possibility of transitions to the excited electronic states. The rates of those transitions in the limit of weak field [$\eta\Omega \ll \max(\Gamma_N, \delta)$] will be of the order of $\eta^2\Omega^2/\max(\Gamma_N, \delta)$, i.e., much smaller than Γ_N. Since only a few atoms will be in the electronic excited state, we can project the master equation onto the subspace of N-atom ground states and single atom excitations. Using adiabatic elimination of the excited electronic state we systematically derive a master equation for the diagonal part of the reduced density operator of the ground state atoms (trap populations),

$$\dot{\rho} = \frac{\Gamma_-}{2} \sum_{k=0}^{\infty} (k+1)(2A_k \rho A_k^\dagger - A_k^\dagger A_k \rho - \rho A_k^\dagger A_k) + \frac{\Gamma_+}{2} \sum_{k=0}^{\infty} (k+1)(2A_k^\dagger \rho A_k - A_k A_k^\dagger \rho - \rho A_k A_k^\dagger), \quad (3)$$

where $A_k = g_k^\dagger g_{k+1}$, and $\Gamma_\pm = \Gamma(\eta\Omega/2)^2/[(\Gamma_N/2)^2 + (\nu \mp \delta)^2]$. Physically, Eq. (3) describes the situation when all of the atoms are in the ground electronic states, but they redistribute among different kth trap states with rates of the order of $\eta^2\Omega^2/\Gamma_N$. Only the rates of transitions from the kth to $k \pm 1$ level and back are nonzero, and interestingly they depend explicitly on the populations of the levels in question. In particular, for negative detuning the system reaches a steady state of the canonical form

$$\rho = \frac{1}{Z} e^{-\beta \sum_k k\nu g_k^\dagger g_k} . \quad (4)$$

with Z being the partition function, and

$$\beta = 1/k_B T = \ln(\Gamma_-/\Gamma_+)/\nu . \quad (5)$$

For $N = 1$, the state (4) reduces to the result for the single atom case [8]. Note that the temperature actually depends on the number of atoms and the atomic and laser parameters. The detuning provides a simple control parameter since the effective temperature is a monotonically increasing function of $\delta (< 0)$. On the other hand, we find a dependence of the temperature on the number of particles, which is a consequence of super-radiance. Thus, for a given detuning, the temperature grows as one increases the number of atoms in the trap.

The physical meaning of the master equation (3) is readily discussed in the decorrelation appproximation, valid in the thermodynamic limit ($N \to \infty$). We introduce mean level occupations $n_k(t) = \langle g_k^\dagger(t) g_k(t)\rangle = \text{Tr}[g_k^\dagger g_k \rho(t)]$ and neglect their fluctuations, obtaining the following equation:

$$\dot{n}_k = (k+1)[\Gamma_- n_{k+1}(1 \pm n_k) - \Gamma_+ n_k (1 \pm n_{k+1})] - k[\Gamma_- n_k (1 \pm n_{k-1}) - \Gamma_+ n_{k-1}(1 \pm n_k)], \quad (6)$$

where the upper and lower signs refer to bosons and fermions, respectively. Equation (6) describes the change of population of level k due to transitions to and from levels $k \pm 1$. This equation resembles the rate equation of a single particle trapped in a harmonic potential [8]; however, in the present case the transition rates between the levels of the harmonic oscillator are *collective* cooling and heating rates which depend on the number of particles in the particular level. For finite N the master equation (3) can be interpreted as a jump process with transition rates between the trap levels. This allows us to simulate the time–dependent dynamics of the system. In contrast to the familiar Monte Carlo simulations in statistical mechanics for the lattice models in thermodynamic equilibrium [10], our simulation has an immediate physical meaning since each of the jumps is associated with an optical pumping transition and thus the emission of a fluorescence photon. The simulation gives us not only the trap level distribution but also the photon statistics of the emitted light.

Let us first consider the case of bosons. The properties of the stationary distribution (4) for finite N are illustrated in Fig. 1. In Fig. 1(a) we plot the relative occupation of the ground state N_0/N as a function of the detuning for $N = 1, 10$, and 100 bosons. The curves were obtained directly from Eq. (6), i.e., in the decorrelation approximation, whereas the crosses denote the results of numerical simulations. As we see, the decorrelation approximation is valid for $N \gtrsim 10$. When $\Gamma_N \lesssim \nu$ and for a given detuning, the relative occupation of the ground state increases as one adds more particles to the trap. This can also be seen in Fig. 1(b), which displays the total number of atoms in the excited states versus N for several values of the laser detuning. For a given detuning the number of atoms in the excited states remains practically constant as N grows, provided $\Gamma_N \lesssim \nu$. This indicates a macroscopic occupation of the ground state ($\simeq N$). Note that in the one-dimensional model studied here, there is no well defined transition point for BEC [11]. This occurs for any value of the detuning. However, when $\Gamma_N \gtrsim \nu$, occupation of the excited levels starts to increase dramatically due to an effective increase in the temperature as a function of N via the rates $\Gamma_\pm$ as expressed by Eq. (5).

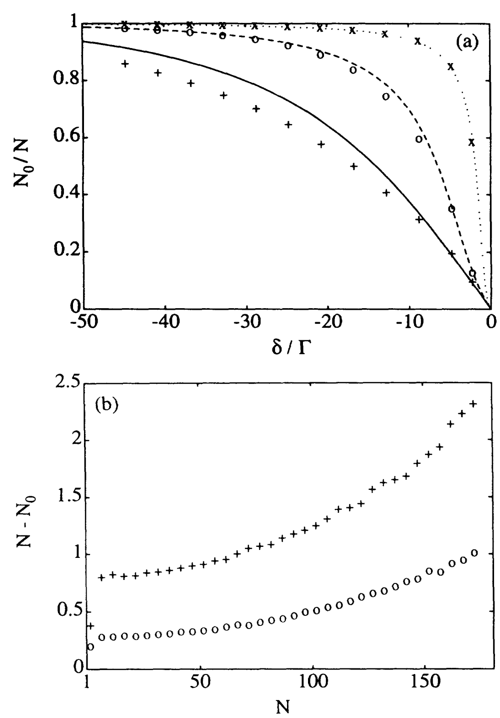

FIG. 1. (a) Relative occupation of the ground state as a function of the detuning for $N = 1, 10$, and 100 bosons, and $\nu = 100\Gamma$. Solid lines correspond to the solution of the master equation under the decorrelation approximation. Crosses denote results obtained from Monte Carlo simulations. (b) Occupation of the excited trap states as a function of the number of atoms N for $\nu = 100\Gamma$, and $\delta = -25\Gamma$ (crosses) and $\delta = -40\Gamma$ (circles). The results were obtained from Monte Carlo simulations.

For fermions, the steady state is a Fermi–Dirac distribution with the temperature as discussed above for the case of bosons. For low temperatures (large negative detunings), atoms occupy the Fermi sea up to the Fermi level (in 1D given by the number of atoms). As the detuning increases, higher and higher levels become populated.

In view of the difference between the stationary states for bosons and fermions, one expects to detect these differences when performing appropriate measurements on the system. The most obvious of such measurements is the detection of fluorescence photons emitted by the atoms. Interestingly, the mean fluorescence intensity does not depend on the statistical character of the atoms, $I = \Gamma_N \eta^2 \Omega^2 / 8\nu|\delta|$. This is readily understood for low and high temperatures. For low temperatures the N bosons are essentially in the trap ground state and the transition rate to the first excited state will be proportional to $N\Gamma_+$; for fermions we have transitions of the atom at the edge of the Fermi sea between level $N-1$ and N which, according to Eq. (6), is again proportional to $N\Gamma_+$. On the other hand, for high temperatures the occupations of the trap levels are much less than one, and the difference between bosons and fermions disappears. However, the quantum statistics of the atoms is reflected in the quantum fluctuation properties of the fluorescent light. In Fig. 2 we show this behavior for $\nu = 100\Gamma$, $N = 50$ (a), and $\nu = 5\Gamma$, $N = 10$ (b). For low and high temperatures (large and small detunings), bosons and fermions behave in the same way. Contrarily, there are significant differences both in the waiting time (Fig. 2) and the photon counting distributions (Fig. 3). For bosons, the emitted photons tend to be more bunched, so that the photon counting distribution

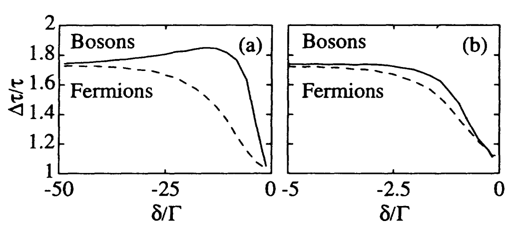

FIG. 2. Mean square variance of the delay time τ between emission of two successive photons as a function of the detuning for $N = 50$ and $\nu = 100\Gamma$ (a), for $N = 10$ and $\nu = 5\Gamma$ (b), for bosons (solid lines) and fermions (dashed lines).

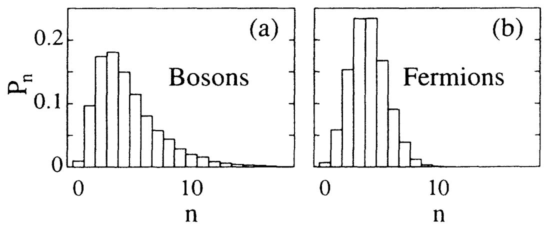

FIG. 3. (a) Photon-counting distributions for $N = 50$ bosons, $\nu = 100\Gamma$, and $\delta = -10\Gamma$; (b) same as (a) but for fermions. The results were obtained from Monte Carlo simulations. The sampling time was taken as $4/I$, where I is defined in the text.

is wider than that for fermions.

The observation of this behavior within the validity of our model requires confinement of $N \approx 10$ to 100 atoms in a volume λ^3. The tight binding of atoms in a particle trap required for the LDL can be realized with microscopic magneto-optical (rectified force) or magnetostatic traps with large magnetic field gradients, or with a dipole trap [12]. In addition, the present calculations are based on an independent particle model which assumes that the interaction energies between the atoms are small compared with the excitation energy in the trap. For $N < 10^2$ atoms and a trap ground state of size λ we find that the trapping potential is not modified by interatomic interactions for typical atomic parameters [13]. It is straightforward to extend the model to include these interactions within an "effective" (renormalized) single particle trap potential. Our theory, which is one dimensional, can be easily extended to the case of 2D and 3D. Furthermore, one can study other laser configurations with the same formalism.

To summarize, we have studied the relation between quantum statistics and laser cooling dynamics of atoms in a small trap. The model we present is exactly solvable, and predicts a stationary state described by a Bose–Einstein or a Fermi–Dirac distribution. The steady state distribution has such a canonical form for any finite number of atoms, and does not require us to evoke the thermodynamical limit. The laser detuning is the control parameter of the system, and plays a role analogous to the temperature. In contrast to the standard canonical partition function, the temperature depends also on the number of atoms, as a consquence of super-radiance. The intensity of the fluorescence photons emitted by bosons and fermions is the same for any detuning (i.e., temperature). On the other hand, correlations between emitted photons are different and characterize the quantum statistical nature of the particles in the trap. Finally, we have investigated other cooling schemes and atomic ground–ground and excited–ground state interactions. We have found, for sufficiently high densities, deviations of the equilibrium distributions from the standard BED and FDD which display novel phase transitions [14].

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