Novel Edge Excitations of Two-Dimensional Electron Liquid in a Magnetic Field

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We investigate the low-energy spectrum of excitations of a compressible electron liquid in a strong magnetic field. These excitations are localized at the periphery of the system. The analysis of a realistic model of a smooth edge yields new branches of acoustic excitation spectrum in addition to the well known edge magnetoplasmon mode. The velocities are found and the observability conditions are established for the new modes.

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The dispersion relation for plasmons in a nonrestricted two-dimensional (2D) electron liquid is well known to have a form $\omega \propto k^{1/2}$ [1–3]. If the liquid has a boundary, an edge mode appears in addition to these bulk excitations. The spectra of the edge and bulk modes differ from each other only by a numerical factor [4]. A magnetic field applied perpendicularly to the plane of the liquid changes the plasmon spectrum drastically. The spectrum of the bulk mode acquires a gap of the width equal to the cyclotron frequency ω_c. The only known gapless mode existing in the presence of the magnetic field propagates along the boundary [4–7]. The "chirality" of this edge magnetoplasmon determined by the direction of the magnetic field (i.e., by the sign of the Hall conductivity σ_{xy}) was demonstrated explicitly in the time-domain experiments [8].

The solved theoretical models of the edge modes assumed a sharp electron density profile at the boundary [4–7]; i.e., the width of the boundary strip was assumed to be infinitesimal. The existence of only a single branch of the edge magnetoplasmons follows directly from this assumption. For a realistic shape of a potential confining the electron liquid, the density profile is smooth at the boundary [9–11]. The results of Refs. [4–7] can be extended on this case only under the assumption that the current and charge oscillations forming the magnetoplasmon wave are homogeneous across the boundary strip. However, the latter condition is excessively restrictive. We demonstrate in this paper the existence of other soundlike modes propagating along the edge. The current for each of these modes alternates across the boundary strip, and therefore the new branches could not be predicted on the basis of a "sharp" boundary model.

Below we present an exactly solvable model correctly describing all the edge excitations in the strong magnetic field limit. We obtain also the values of the oscillator strengths and the damping of these modes. The new branches become robust and are not destroyed by a finite relaxation time in the achievable region of relatively short wavelengths.

The dynamics of the compressible electron liquid is governed by the Euler equation and the continuity equation linearized in the velocity of the liquid $\boldsymbol{v}(\boldsymbol{\rho},t)$ and in the deviation of the concentration $\delta n(\boldsymbol{\rho},t)$ from its equilibrium value $n_0(\boldsymbol{\rho})$:

$$\dot{\boldsymbol{v}} + \omega_c(\hat{\boldsymbol{z}}\times\boldsymbol{v}) - \frac{e^2}{\varepsilon m}\boldsymbol{\nabla}_\rho \int d^2\rho_1 \frac{\delta n(\boldsymbol{\rho}_1)}{|\boldsymbol{\rho}-\boldsymbol{\rho}_1|} = 0, \qquad (1)$$

$$\delta\dot{n} + \boldsymbol{\nabla}_\rho(n_0\boldsymbol{v}) = 0. \qquad (2)$$

Here $\boldsymbol{\rho}$ is radius vector in the plane X-Y of the 2D electron liquid, $\hat{\boldsymbol{z}}$ is the unit vector along the Z axis, and ε is the dielectric constant. The last term in (1) represents the Coulomb interaction [12].

In the following we assume that the electron liquid is homogeneous in the y direction and occupies half plane $x > 0$. Since the system is translationally invariant in the y direction, we will seek the solution of Eqs. (1) and (2) in the form

$$\boldsymbol{v} = \exp(iky - i\omega t)\,\boldsymbol{w}(x),$$
$$\delta n(x,y) = \exp(iky - i\omega t)\,f(x). \qquad (3)$$

Substituting Eqs. (3) into the system (1) and (2) and eliminating $\boldsymbol{w}(x)$, we find an integral equation for $f(x)$:

$$\left(\omega_c^2-\omega^2\right)f + \frac{2e^2}{\varepsilon m}\left\{k^2 n_0 - n_0\frac{d^2}{dx^2} - n_0'\frac{d}{dx} + \frac{k}{\omega}\omega_c n_0'\right\}\int_0^\infty K_0(|k||x-x_1|)\,f(x_1)dx_1 = 0, \qquad (4)$$

where $n_0' \equiv dn_0/dx$ and $K_0(x)$ is the modified Bessel function. The homogeneous equation (4) comprises the eigenvalue problem that determines the spectrum of edge excitations $\omega_j(k)$. The spectrum is controlled by the parameters of the problem, by the magnetic field determining ω_c, and by the concentration profile $n_0(x)$. The latter depends on a particular type of the confining potential [9,11]. We are interested in the low-frequency, long-wavelength modes, and this allows us to neglect the terms proportional to ω^2 and k^2 in Eq. (4).

Futher simplifications are possible in the case of a strong magnetic field. Keeping only the terms propor-

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tional to ω_c^2 and ω_c, and introducing a new function

$$g(x) = \left(\frac{dn_0}{dx}\right)^{-1/2} f(x) \qquad (5)$$

instead of $f(x)$, we find from (4) the following reduced equation:

$$g(x) = \lambda \int_0^\infty K_0(|k||x-x_1|) \frac{1}{\bar{n}} \sqrt{\frac{dn_0}{dx}\frac{dn_0}{dx_1}} g(x_1) dx_1, \qquad (6)$$

$$\omega = -\frac{2}{\lambda}\frac{\bar{n}e^2}{\varepsilon m \omega_c} k. \qquad (7)$$

Here $\bar{n} \equiv n_0(x \to \infty)$ is the density of the homogeneous electron liquid far from the boundary. One can estimate from Eqs. (6) and (7) the typical value of $|\omega/k|$ to be of the order of $\bar{n}e^2/m\omega_c$. It follows also from (5) and (6) that the charge distribution $f(x)$ in the wave is localized mainly within the region where dn_0/dx is large, i.e., within the boundary strip of width a. Now we can establish the validity criterion of the strong magnetic field approximation. The neglected terms $\propto \bar{n}/a$ in Eq. (4) are smaller than the main terms $\propto m\omega_c^2/e^2$ if the condition

$$\omega_c^2 \gg \frac{\bar{n}e^2}{\varepsilon m a} \qquad (8)$$

is satisfied. For realistic parameters of the 2D electron system formed in a GaAs heterostructure [10], $\bar{n} \sim 1/a_B^2$ and $a \sim 10a_B$, the latter condition is equivalent to a rather weak restriction on the filling factor, $\nu \lesssim 10$ (here a_B is the Bohr radius for GaAs). The opposite case of a weak magnetic field was considered by Nazin and Shikin [13] for the electron system above helium.

Equation (6) is the integral equation of Fredholm type. Its kernel is symmetric and positively defined; hence, all the eigenvalues λ are real and positive. If one makes an approximation $K_0(|k||x-x_1|) \approx \ln(1/|ka|)$ leading to the degeneracy of the kernel, then only a single finite eigenvalue λ exists. This eigenvalue corresponds to the known magnetoplasmon mode [4]. The actual kernel in (6) is nondegenerate, however, and thus there are many edge modes.

The eigenvalue problem (6) cannot be solved analytically for an arbitrary distribution $n_0(x)$. Below we present a model for the density profile,

$$n_0(x) = \frac{2}{\pi}\bar{n} \arctan\sqrt{\frac{x}{a}}, \quad x \geq 0, \qquad (9)$$

that allows a complete analytical solution of the problem. This model describes correctly the asymptotic behavior of the density formed by an electrostatic confinement, reproducing the characteristic [9–11] $\sqrt{x}$ singularity at $x \to 0$.

The proposed model allows us to solve the eigenvalue problem (6) using an expansion of $g(x)$ in a form

$$g(x) = \frac{1}{(x)^{1/4}(x+a)^{1/2}} \sum_{j=0}^{\infty} g_j T_{2j}\left(\sqrt{\frac{a}{x+a}}\right), \qquad (10)$$

where $T_n(\xi) \equiv \cos(n \arccos \xi)$ are the Chebyshev polynomials [14]. Substitution of (10) into (6) leads to the following system of equations for coefficients g_j of the expansion:

$$\frac{1}{\lambda} g_0 = \ln\left(\frac{e^{-\gamma}}{2|ka|}\right) g_0 - \sum_{j=1}^{\infty} \frac{(-1)^j}{j} g_j,$$

$$\frac{1}{\lambda} g_j = \frac{1}{j} g_j - 2\frac{(-1)^j}{j} g_0, \quad j \geq 1. \qquad (11)$$

When deriving Eqs. (11), we used the approximation $K_0(kx) \approx \ln(2e^{-\gamma}/|kx|)$ which is valid in the long-wavelength limit, $|ka| \ll 1$; here $\gamma \approx 0.577...$ is the Euler constant. The system (11) leads directly to the following transcendent equation for the eigenvalues:

$$\frac{1}{\lambda} + 2\Psi(1-\lambda) = \ln\left(\frac{e^{-3\gamma}}{2|ka|}\right). \qquad (12)$$

Here $\Psi(1-\lambda)$ is the digamma function [14] that has simple poles at $\lambda = 1, 2, \ldots$. Because $|ka| \ll 1$, the solutions of Eq. (12) are close to the points $\lambda = 0, 1, 2, \ldots$ where the left-hand side of this equation is singular. The smallest root of Eq. (12) belongs to the region $\lambda \ll 1$. Expanding the left-hand side of this equation in power series in λ and retaining only the two leading terms of the expansion, we find the spectrum of the conventional [4] edge magnetoplasmon mode:

$$\omega_0(k) = -2\ln\left(\frac{e^{-\gamma}}{2|ka|}\right)\frac{\bar{n}e^2}{\varepsilon m\omega_c} k. \qquad (13)$$

Other roots $\lambda \geq 1$ are close to the poles of the digamma function in Eq. (12). It is these roots that determine the new branches of the edge excitations with the acoustic spectrum:

$$\omega_j(k) = -s_j k, \quad s_j = \frac{2\bar{n}e^2}{\varepsilon m \omega_c j}, \quad j = 1, 2, \ldots. \qquad (14)$$

The difference between the acoustic modes and the "usual" plasmon ($j = 0$) originates in the structure of charge distributions associated with these waves. In the usual plasmon wave, charge does not oscillate across the boundary strip, whereas in the acoustic mode j charge oscillates $j+1$ times in the x direction (see Fig. 1), the average density being smaller by a factor of $|j \ln(|ka|)|^{-1}$

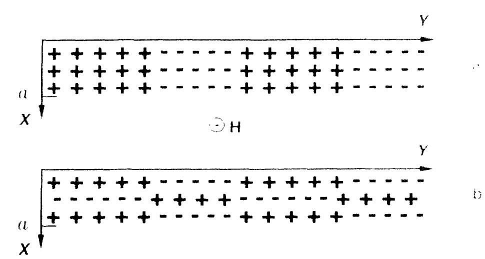

FIG. 1. Characteristic charge distributions for (a) the edge magnetoplasmon mode, $j = 0$, and (b) the edge acoustic mode with $j = 2$. Charge patterns shown in the figure move along the y axis according to Eq. (3).

than the oscillations amplitude for each mode with $j \neq 0$. The potential energies U_j produced by the charge distribution types depicted in Fig. 1 can be easily estimated. For the same characteristic amplitudes of charge density perturbation in all the waves, we find the ratio $U_0/U_j \simeq j|\ln(|ka|)|$. This explains the difference between the spectra (13) and (14), as energies U_j provide the restoring forces for the modes. For the higher harmonics $j \gg 1$ the latter considerations are obviously independent of the particular density profile (9) that allows one to expect certain universality of the spectrum (14). Indeed, for an arbitrary profile $n_0(x)$, the function

$$g(x) = \left(\frac{dn_0}{dx}\right)^{1/2} \cos\left(\pi j \frac{n_0(x)}{\bar{n}}\right) \tag{15}$$

may be used at $j \gg 1$ as an asymptotic solution of the eigenvalue problem (6). In the case of the profile derived in Ref. [9], we find the corrections to the velocities s_j [see Eq. (14)] to be of the order of $1/j^2$.

The observability of the acoustic modes requires sufficiently large oscillator strengths and rather slow decay for these modes. To begin with, we evaluate the oscillator strengths $S_j^{\alpha\beta}(k)$ for all the modes; here $\alpha, \beta = x, y$ denote the polarization of the applied ac electric field, $E_\alpha(y,t) = E_\alpha \exp(iky - i\omega t)$, and $j = 0, 1, 2, \ldots$ is the mode number. The power P absorbed from the ac field within the unit length of the boundary is related to the oscillator strengths of the different modes by

$$P \equiv \frac{e}{2L}\mathrm{Re}\int n_0 \boldsymbol{v}\boldsymbol{E}^*\, d^2\boldsymbol{\rho} = \frac{1}{2}\sum_{j=0}^{\infty} S_j^{\alpha\beta}(k) E_\alpha E_\beta^* \delta(\omega - \omega_j(k)). \tag{16}$$

Here L is the length of the boundary, and velocity $\boldsymbol{v}$ is the linear response to the external electric field $\boldsymbol{E}$. To calculate $\boldsymbol{v}$, one has to add the term $e\boldsymbol{E}/m$ to the right-hand side of the equation of motion (1). The use of Eqs. (1), (5)–(7), and (16) allows one to express $S_j^{\alpha\beta}(k)$ in terms of the eigenfunctions $g_j(x)$. We present here the results for the diagonal components of the oscillator strengths:

$$S_j^{\alpha\alpha}(k) = \frac{\pi\omega_j(k)\bar{n}e^2}{m\omega_c k} F_j^\alpha(k), \tag{17}$$

$$F_j^x = \left(k\int dx \frac{n_0}{\bar{n}}\sqrt{\frac{\bar{n}}{n_0'}}g_j\right)^2, \quad F_j^y = \left(\int dx \sqrt{\frac{n_0'}{\bar{n}}}g_j\right)^2,$$

functions $g_j(x)$ here being normalized by the condition

$$\int dx\, g_j^2(x) = 1. \tag{18}$$

It is obvious from Eq. (5) that F_j^y is proportional to the square of the average charge mode j bears. As was mentioned already, this charge in the acoustic modes is parametrically smaller than the one in the usual edge magnetoplasmon mode. Therefore, the interaction of the ac field polarized along the boundary with the acoustic modes is much weaker than the interaction with the magnetoplasmon. An explicit calculation in the framework of our model gives

$$S_j^{yy} = \frac{1}{\varepsilon}\left(\frac{\bar{n}e^2}{m\omega_c}\right)^2 \times \begin{cases} 2\pi|\ln(|ka|)|, & j = 0, \\ 4\pi|\ln(|ka|)|^{-2}j^{-3}, & j \geq 1. \end{cases} \tag{19}$$

The ac electric field applied perpendicular to the boundary interacts with the x component of the dipolar moment of the modes. These moments, and correspondingly factors F_j^x, are of the same order of magnitude for all the modes. Therefore, the difference in absorption for this polarization is due only to the difference in the mode frequencies:

$$S_j^{xx} = \frac{1}{\varepsilon}\left(\frac{\bar{n}e^2}{m\omega_c}\right)^2 \times \begin{cases} 2\pi^3|\ln(|ka|)|^{-1}, & j = 0, \\ 4\pi^3|\ln(|ka|)|^{-2}j^{-1}, & j \geq 1. \end{cases} \tag{20}$$

It is interesting to notice that the absorption anisotropies $S_j^{xx}/S_j^{yy} - 1$ are of the opposite signs for the usual plasmon and for the acoustic modes, respectively.

To estimate the decay rates of different edge modes, we include a phenomenological relaxation time τ into the equation of motion (1) by the substitution $\dot{\boldsymbol{v}} \to \dot{\boldsymbol{v}} + \boldsymbol{v}/\tau$. After such a modification the energy of the mode ϵ_j becomes time dependent, $\epsilon_j(t) \propto \exp(-2t/\tau_j)$. The latter relation allows one to define the relaxation rates $1/\tau_j$. For small dissipation the result can be expressed in terms of the unperturbed eigenmodes $g_j(x)$:

$$\frac{1}{\tau_j} = \frac{\omega_j(k)}{\omega_c \tau k}\int dx\, n_0(x)\left(\frac{d}{dx}\frac{g_j}{\sqrt{n_0'}}\right)^2, \tag{21}$$

$g_j(x)$ being normalized by the condition (18). As one can easily see from Eq. (21), the relaxation rate increases with the mode number because of oscillatory behavior of the eigenfunctions. We find for the plasmon and acoustic modes

$$\omega_j\tau_j = |ka|\omega_c\tau \times \begin{cases} 2(\ln|ka|)^2, & j = 0, \\ \beta_j/j^2, & j \geq 1, \end{cases} \tag{22}$$

where $\beta_1 = 6/5$, $\beta_2 = 60/53, \ldots, \beta_\infty = 1.128$ are slowly varying with j numerical factors.

Solutions of Eq. (6) and perturbative results (22) obtained above are applicable as long as dissipation is small enough so that it does not affect the charge distribution in the eigenmodes. The characteristic length of the charge spreading l_ω caused by dissipation is inversely proportional to frequency, $l_\omega = e^2\bar{n}/\varepsilon m\omega\omega_c^2\tau$. The smallness of the redistribution requires l_ω to be shorter than the characteristic length scale $\sim a/(j+1)$ for spatial variations of the eigenfunctions $g_j(x)$. The latter condition imposes different restrictions for the plasmon and acoustic modes; with the help of (13) and (14), we find $|ka|\ln(1/|ka|) \gtrsim 1/\omega_c\tau$ for $j = 0$ and $|ka| \gtrsim j^2/\omega_c\tau$ for $j \neq 1$. At smaller wave vectors, the results of Volkov and Mikhailov [4] for the spectrum and decay rate are applicable [15], whereas the acoustic modes are overdamped. The region of observability of the new branches is shown in Fig. 2.

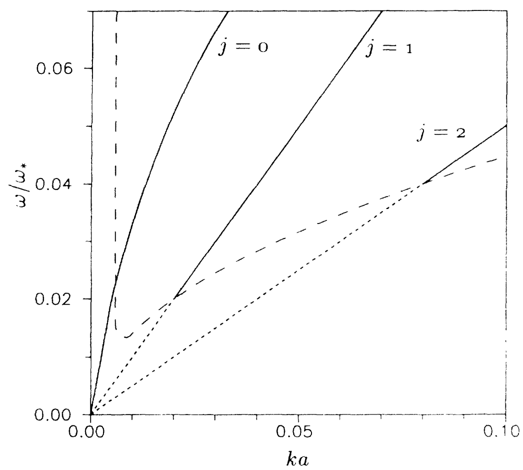

FIG. 2. The first three branches of the edge excitations; $\omega_* = 2\bar{n}e^2/\varepsilon m\omega_c a$. The dashed line separates the regions of strong damping (below the line) and weak damping (above the line). In the latter region the acoustic edge modes become observable.

The observability condition for the new modes is quite restrictive. Indeed, as follows from (22), the product $\omega_c\tau$ must be at least greater than $1/|ka|$ for the first acoustic mode to be observed. If the characteristic values of k are determined by a sample perimeter (which is typically ~ 1 cm [16]), the condition (22) cannot be satisfied even for the mobility of 10^6 $\mathrm{cm}^2/\mathrm{V\,sec}$. The technique of using a metallic grating coupler [17] appears to be more promising. In such an experiment the wave vector $k = 2\pi/d$ is determined by the grating period d that may be made [17] of the order of 1 μm. For the typical width of the boundary strip ~ 2000 Å this implies the condition $\omega_c\tau \gtrsim 1$.

We treated the 2D electron system as a classical compressible liquid. It was shown in [10] that under the conditions of the quantum Hall effect, the boundary region $x \lesssim a$ in fact is divided into alternating strips of compressible and incompressible liquids. However, the incompressible strips occupy only a small fraction $\sim (a_B/a)^{1/2} \ll 1$ of the boundary area and thus do not affect acoustic modes with a sufficiently smooth charge distribution. It means that the spectra of modes with $j < (a/a_B)^{1/2}$ are not affected by the quantum Hall effect. Furthermore, because the dissipation is suppressed within the incompressible strips, the damping rate of the novel modes is reduced below our estimate (22).

In conclusion, we found the new low-frequency excitations propagating along the edge of a 2D electron liquid in the presence of a magnetic field. These new modes have acoustic spectra with velocities inversely proportional to the mode indices. At a given wave vector k, the frequencies of acoustic modes are lower than that of a conventional edge magnetoplasmon by a factor $1/|\ln ka|$, where a is the width of the boundary strip. The oscillator strengths and decay times for the acoustic branches with low indices differ from the corresponding values for the magnetoplasmon by powers of the same small parameter. The logarithmic function appears due to the long-range nature of the Coulomb interaction. If the electron system is confined by gate-induced potentials, the differences in the mentioned parameters for the acoustic modes and the conventional magnetoplasmon become less significant: the logarithmic function should be replaced by a factor of the order of unity because of the screening of the Coulomb interaction by the metallic gates.

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