# Stabilization of External Modes in Tokamaks by Resistive Walls and Plasma Rotation

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It is shown that low $n$, pressure-driven, external modes in tokamaks can be fully stabilized by resistive walls when the plasma rotates at some fraction of the sound speed. The stabilization depends on toroidal coupling to sound waves and is affected by ion Landau damping. Two-dimensional stability calculations are presented to show the resulting gains in the beta limit.

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The plasma pressure, or $\beta = 2\mu_0\langle p\rangle/\langle B^2\rangle$, that can be confined by magnetic fields is limited by ideal magnetohydrodynamical (MHD) instabilities. Troyon *et al.* [1] found numerically that the beta limit of a tokamak is proportional to the plasma current, $\beta_{\max} = gI_p[MA]/a[m]B_0[T]$. This scaling has been confirmed experimentally, although rather different normalized betas, $g$, have been reached in different tokamaks. Troyon *et al.* [1] found $g \approx 2.8$, and more recent numerical studies have given higher estimates, 3.5 or 4. Experimental work on the DIII-D tokamak [2] has reached $g \approx 5$, and MHD stability analyses show that at least some of these discharges are unstable to ideal MHD modes of low toroidal mode number $n$ [3]. These are robust, global instabilities, for which kinetic corrections can be estimated as small. By convention, theoretical beta limits are computed by requiring ideal MHD stability for static equilibria without any stabilization by conducting walls. It has been pointed out that the stability of high-$g$ discharges in DIII-D could be accounted for by introducing a perfectly conducting wall at the location of the actual resistive wall [4], and this has been verified by stability analysis for certain discharges [3]. However, conventional wisdom holds that resistive walls can only slow down, but not stabilize, ideal MHD instabilities. Here, we present some first theoretical results on stabilization of toroidal, pressure-driven, external modes by resistive walls. We show that such modes can be completely stabilized in tokamaks with sonic plasma rotation and that the effect gives a significant increase of the beta limit.

It is well established that resistive walls do not change the stability boundaries of ideal MHD modes that do not have a resonant surface, where $k_\parallel \equiv (m/q - n)/R_0 = 0$, in the plasma ($m$ is the poloidal mode number, $q$ is the safety factor, and $R_0$ is the major radius); examples are the "vertical" instability and the "cylindrical" external kink mode. Resistive walls slow down the growth of these modes to the resistive time scale of the wall, $\tau_w = L/R$, but do not change the stability boundaries from their wall-at-infinity value. The growth rates are independent of plasma inertia and are insensitive to sub-Alfvénic plasma rotation [5,6]. By contrast, tearing modes can be stabilized by resistive walls, provided the rotation frequency exceeds $\tau_w^{-1}$ and a characteristic tearing growth rate [6,7]. Similar conditions apply to the toroidal, pressure-driven, external kink modes, for which $k_\parallel$ vanishes at the resonant surfaces where $q = m/n$. In a layer around each resonant surface, the rotation frequency exceeds the local Alfvén frequency. Furthermore, for pressure-driven modes, the parallel motion is important and this is influenced by inertia when the rotation is comparable to the sound speed.

Toroidal, pressure-driven modes are complicated because of the coupling between different poloidal harmonics and between the Alfvén and sound waves [8]. Therefore, we have studied the wall stabilization in toroidal geometry numerically. The spectral codes MARS [9] and NOVA [10] have been modified to include a resistive shell. The time constant of the shell, $\tau_w$, is taken to be much longer than any ideal MHD time scale. Rotation is modeled by making the shell rotate toroidally with frequency $\omega_{\rm rot}$. The equilibrium is static, and this allows us to separate wall stabilization from other effects of plasma rotation. We treat the plasma as ideally conducting, which excludes resistive modes rotating with the plasma.

When the rotation is sonic, the plasma motion excites sound waves propagating along the magnetic field lines, and in MHD, these exhibit unphysical resonant behavior. In a more complete theory, the sound waves are subject to strong ion Landau damping if $T_e \approx T_i$, and an accurate calculation must be kinetic along the field lines [11]. Nevertheless, useful approximations can be found by adding dissipative terms to the fluid equations [12,13]. We have tried three such modifications of the scalar pressure, ideal MHD equations. Two of these consist of adding a damping term for the Lagrangian pressure perturbations. The perturbed pressure is split into convective and Lagrangian parts, $p_1 = -\xi\cdot\nabla p_0 + p_{1L}$, where

$$\partial p_{1L}/\partial t = -\Gamma p_0 \nabla\cdot\mathbf{v} - \nu p_{1L}\,. \qquad (1)$$

The damping rate $\nu$ is either taken to be a fixed number or to represent a thermal conductivity following Hammett and Perkins [13], $\nu = \chi|k_\parallel v_{\rm thi}|$. As a third alternative, we add a parallel viscosity for the motion along the field lines:

$$\partial v_\parallel/\partial t = -(\mathbf{B}\cdot\nabla p)_1/B_0\rho_0 - \kappa|k_\parallel v_{\rm thi}|v_\parallel\,. \qquad (2)$$

When compared to guiding center theory [11] the formu-

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lation (2) with $\Gamma=\frac{3}{2}$, $\kappa=\sqrt{\pi}\approx 1.77$ gives a good approximation for the perturbed perpendicular pressure induced by Lagrangian perturbations of the magnetic field strength.

We now present numerical results for the modified ideal MHD eigenvalue problem including a rotating resistive shell. When the pressure exceeds the stability limit with the wall at infinity, we find two classes of modes that can be unstable: (a) one which has zero frequency in the frame of the plasma and hardly penetrates the shell, the *plasma mode*, and (b) one which penetrates the wall and rotates with respect to it at a low slip frequency equal to $O(\tau_w^{-1})\ll\omega_{\rm rot}$, the *resistive wall mode*. This mode rotates with respect to the plasma at a frequency close to the imposed rotation frequency $\omega_{\rm rot}$. Figure 1 shows an example of how the growth rates of the plasma and resistive wall modes depend on the wall radius $d$. The two modes are influenced in opposite ways by the wall distance—the plasma mode is *destabilized* as the wall is moved further from the plasma, while the resistive wall mode is *stabilized*.

The plasma mode rotates with frequency $\approx\omega_{\rm rot}\gg\tau_w^{-1}$ with respect to the wall. It does not penetrate the wall and behaves as if the wall were ideal. The plasma mode is unstable on the ideal MHD time scale when the wall radius exceeds the ideal MHD threshold, $d_{\rm ideal}$. This marginal wall position decreases with increasing pressure and goes to infinity at the conventional beta limit.

The resistive wall mode becomes increasingly stable with increasing wall radius. This counterintuitive behavior can be understood by a large aspect ratio calculation of $\Delta'$ at the resistive shell. We consider a magnetic perturbation in the vacuum, dominated by one poloidal harmonic $m$ (assumed $>0$). The perturbed magnetic flux function $\psi$ satisfies $\nabla_\perp^2\psi=0$ in the vacuum region and the poloidal harmonic $m$ is a linear combination of $r^{-m}$ and $r^m$. The growth rate of the resistive wall mode is $\gamma=d\Delta'_w/\tau_w$, where $\Delta'_w=[\psi'(d_+)-\psi'(d_-)]/\psi(d)$. If we write the logarithmic derivative of $\psi$ at the plasma edge $r=a$ as $(\psi'/\psi)_{r=a}=-(m/a)(1+x+iy)$ (with $x,y$ real and $y\neq 0$ because of the rotation) a simple calculation gives $\Delta'_w$,

$$(d\Delta'_w/2m)[1-(a/d)^{2m}]=(x+iy)/(w-x-iy)\,, \qquad (3)$$

where $w=2/[(d/a)^{2m}-1]$.

Let us first consider the case of no rotation, $y=0$, and an equilibrium that is unstable in the absence of a wall, $x>0$. Equation (3) then predicts that the resistive wall mode is unstable for $d<d_{\rm ideal}=a[1+2/x]^{1/2m}$. With increasing wall radius, $\Delta'_w\to+\infty$ when $d\to d_{\rm ideal}$, where the resistive wall mode connects to the ideal mode, which is unstable for $d>d_{\rm ideal}$. In the region of ideal instability, plasma inertia modifies $(\psi'/\psi)_{r=a}$ so as to maintain $\Delta'_w\approx+\infty$.

When the rotation frequency is finite, $y$ is nonzero. This eliminates the zero in the denominator of Eq. (3) so

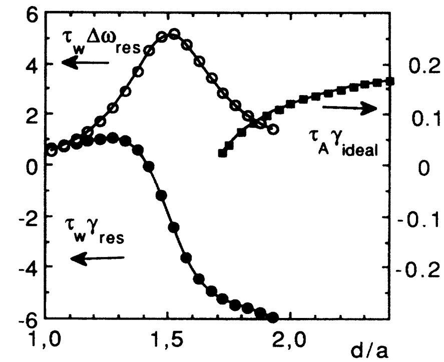


FIG. 1. Growth rate $\gamma_{\rm res}$ and slip frequency $\Delta\omega_{\rm res}=\omega_{\rm rot}-\omega_{\rm res}$ of resistive wall mode and growth rate of plasma mode $\gamma_{\rm ideal}$ versus wall radius for the $n=1$ mode with pressure about 30% above the free-boundary limit and $\omega_{\rm rot}/\omega_A=0.06$.

that $\Delta'_w$ remains finite and complex for all wall distances. Thus, rotation effectively separates the resistive wall mode from the plasma mode. The growth rate of the resistive wall mode remains $O(\tau_w^{-1})$ for all $d$, and if $\tau_w\gg\tau_A$, the plasma response can be computed neglecting the small slip frequency with respect to the wall. [Because of the damping added to the sound waves, the solution in the plasma remains well behaved as $\text{Re}(\gamma)\to 0$; i.e., the MHD continuum resonances [8] move into the stable half plane.] In Fig. 1, the resistive wall mode is stabilized when $d$ exceeds a threshold, predicted by (3) to be $d_{\rm res}=a[1+2x/(x^2+y^2)]^{1/2m}$. The present discussion is clearly oversimplified, e.g., by only considering one poloidal harmonic, but is shows that rotation separates the plasma and resistive wall modes, and that they behave in opposite ways with respect to the wall distance. It also shows that the optimum wall position is some distance away from the plasma.

We conclude that when a rotating plasma exceeds the pressure limit with the wall at infinity, there are two stability limits for the wall radius, $d_{\rm res}$ and $d_{\rm ideal}$. The plasma is stable when $d_{\rm res}<d<d_{\rm ideal}$, and this condition must apply for all $n\neq 0$. We have computed stability limits including rotation and a resistive shell for several MHD equilibria. Figure 2 shows $d_{\rm ideal}$ and $d_{\rm res}$ for $n=1$ and 2 versus normalized beta for a JET-shaped equilibrium with $\omega_{\rm rot}/\omega_A=0.06$ and a broad pressure profile, $p_0/\langle p\rangle\approx 1.7$. Wall stabilization is more powerful when the pressure profile is broad so that the beta limit is set by external modes. We have adjusted the current profile to keep $q_0=1.2$ and $q_a=2.55$ and used the parallel viscosity model (2) with $\kappa=1.77$.

In Fig. 2, $d_{\rm ideal}$ is smaller for $n=2$ than for $n=1$, so that the outer stability limit for the wall position is set by $n=2$. In fact, $n=3$ gives an even more restrictive $d_{\rm ideal}$. However, the present model is unrealistic for high $n$ modes. First, strong shaping, such as in DIII-D, can cause a transition to second stability for large and intermediate $n$. Second, the velocity profiles in experiments

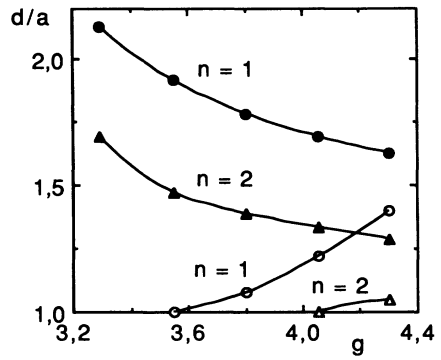


FIG. 2. Marginal wall position versus normalized beta for the plasma (filled symbols) and resistive wall modes (open symbols) with toroidal mode numbers $n=1$ and $n=2$. The plasma mode is stable for $d<d_{crit}$ and the resistive wall mode for $d>d_{crit}$. The region stable to both the $n=1$ and $n=2$ modes is bounded by the $n=1$ resistive wall mode and the $n=2$ plasma mode.

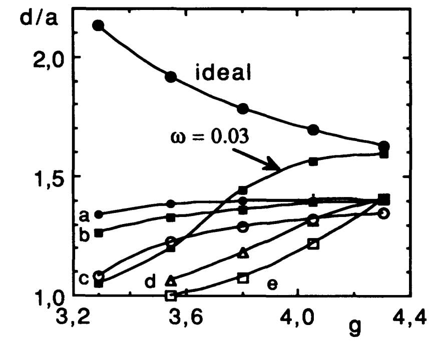


FIG. 3. Marginal wall distance versus normalized beta for the plasma mode (marked "ideal") and the resistive wall mode. Curves $a$–$e$ apply for $\omega_{rot}/\omega_A=0.06$. $a$ and $b$ give results for the pressure damping model (1) with $a$, $\nu/\omega_A=0.025$ and $b$, $\nu/\omega_A=0.0025$. $c$–$e$ give the results for the parallel viscosity model (2) with $c$, $\kappa=0.1$, $d$, $\kappa=0.885$, and $e$, $\kappa=1.77$. The curve for $\omega_{rot}/\omega_A=0.03$ was computed for $\kappa=1.77$.

are sheared, which is expected to stabilize high $n$ ballooning modes [14]. Thus, the stability boundaries of the high and intermediate $n$ modes should be more sensitive to the plasma rotation profile and to geometrical effects. Nevertheless, as exemplified by Fig. 2, the most restrictive $d_{ideal}$ can be set by toroidal mode numbers larger than 1. However, Fig. 2 indicates that the inner limit, $d_{res}$, is set by the $n=1$ resistive wall mode.

For the equilibria in Fig. 2, the highest normalized beta that is stable to both $n=1$ and $n=2$ at the prescribed rotation frequency is about 4.2, to be compared with the threshold of 3.1 in the absence of wall stabilization. For equilibria with broad pressure profiles, we find that when $q_a$ increases, $d_{res}$ for $n=1$ moves closer to the plasma boundary, and the maximum normalized beta increases.

Considerable uncertainty comes from computing the perturbed pressure from fluid rather than kinetic theory. Figure 3 shows results obtained with different fluid approximations, using the same equilibrium as in Fig. 2 and $\omega_{rot}/\omega_A=0.06$. Stability limits are shown for $n=1$, obtained with the pressure damping (1) and parallel viscosity (2) models and different dissipation coefficients. The Hammett-Perkins approximation with $\chi=2/\pi$ gives a result almost identical to the pressure damping model with $\nu/\omega_A=0.025$, while the parallel viscosity model generally gives a stronger stabilizing effect.

Also shown in Fig. 3 is a comparison case with half the rotation frequency, $\omega_{rot}/\omega_A=0.03$. The stabilization is much weaker for this case and is almost lost when $\omega_{rot}/\omega_A\leq 0.02$. Thus, there is a threshold behavior with respect to the rotation frequency. For the equilibria we have examined, $\omega_{rot}$ needs to be about $0.05\omega_A$ to give significant wall stabilization. This corresponds to about 20% of the sound frequency at the $q=2$ surface.

If the sound waves are eliminated by setting $\Gamma=0$, the wall stabilization becomes very weak at the low rotation frequencies discussed here. Thus, the stabilization by resistive walls and rotation is mainly connected to the dynamics of sound waves. By allowing $\Gamma$ to be a function of the equilibrium flux function $\psi$, we have verified that the stabilization is mainly a bulk (as opposed to singular layer) effect.

The present analysis is based on a numerical solution of the modified MHD eigenvalue problem. An analytical, large aspect ratio calculation is beyond the scope of this Letter. Such a calculation must include toroidal coupling of the different poloidal components, and terms representing inertia and fluid compression have to be added to the standard marginal stability equations. The role conventionally taken by the resonant surfaces where $k_\parallel=0$ is now taken by the continuum resonances where $\omega_{cont}=\pm\omega_{rot}$, and to complicate matters further, these involve Alfvén and sound waves coupled by the geodesic curvature in toroidal geometry [8]. In the limit of vanishing growth rate and damping coefficients, the continuum resonances show a $1/(\psi-\psi_0)$ behavior for the parallel displacement and $\log|\psi-\psi_0|$ for the normal displacement. Our numerical solutions show that realistic values of the damping coefficients broaden the singularities so that they are barely distinguishable in the normal displacement. In terms of the logarithmic derivatives of the main Fourier components at the plasma edge, $\psi'(a)/\psi(a)=-(m/a)(1+x+iy)$, the resistive wall mode generally has a smaller $x$ than the plasma mode. The imaginary part $y$ is usually larger than $x$ for a stabilized resistive wall mode and increases when the parallel viscosity is reduced. [Although the ideal MHD equations have real coefficients when $\mathrm{Re}(\gamma)$ and the dissipation coefficients vanish, the solution is complex as it must be continued around the continuum singularities.] Because of the complicated dynamics of the resistive wall modes, we feel that numerical computation is needed to establish results as unambiguously as possible, although analytic theory will undoubtedly help to clarify the physics. Finally, an accurate calculation of the sound wave dynamics requires a kinetic description along the field lines, which we plan to implement numerically using drift kinetic theory.

In summary, we have shown that low $n$ modes can be stabilized by resistive walls in combination with plasma rotation and that this wall stabilization leads to experimentally significant increases in the beta limit. The effect is more pronounced for broad pressure profiles and high $q_a$. The strongest effect occurs for $n=1$, which makes the mechanism particularly attractive for advanced tokamak operation, as ballooning modes can reach a second region of stability for large pressure and low shear, while the $n=1$ mode does not access second stability without wall stabilization [15]. The numerical example shown in Fig. 2 indicates an increase in the beta limit by about 30% by the wall stabilization. Increases of similar magnitude are observed on DIII-D, and some of these equilibria are believed to be stabilized by the vacuum vessel [3]. A certain minimum rotation frequency is needed for a significant effect. According to our numerical calculations, $\omega_{rot}/\omega_A$ needs to be at least 0.03–0.05 for typical tokamak parameters. This condition is generally satisfied in DIII-D discharges where typical values are $1\times10^6\ s^{-1} < \omega_A < 2\times10^6\ s^{-1}$ and $60\times10^3\ s^{-1} < \omega_{rot} < 200\times10^3\ s^{-1}$.

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[1] F. Troyon *et al.*, Plasma Phys. Controlled Fusion **26**, 209 (1984).

[2] T. S. Taylor *et al.*, in *Proceedings of the 13th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, 1990* (IAEA, Vienna, 1991), Vol. 1, p. 177.

[3] A. Turnbull and O. Sauter (private communication).

[4] J. Manickam and S. Jardin (private communication).

[5] S. W. Haney and J. P. Freidberg, Phys. Fluids B **1**, 1637 (1989), Sec. V.

[6] C. G. Gimblett, Nucl. Fusion **26**, 617 (1986).

[7] A. Bondeson and M. Persson, Nucl. Fusion **28**, 1887 (1988).

[8] C. Z. Cheng and M. S. Chance, Phys. Fluids **29**, 3695 (1986).

[9] A. Bondeson, G. Vlad, and H. Lütjens, Phys. Fluids B **4**, 1899 (1992).

[10] C. Z. Cheng and M. S. Chance, J. Comput. Phys. **71**, 124 (1987); D. J. Ward, S. C. Jardin, and C. Z. Cheng, J. Comput. Phys. **104**, 221 (1993).

[11] A. Bondeson and R. Iacono, Phys. Fluids B **1**, 1431 (1989).

[12] Our aim is to approximate the behavior of a collisionless plasma rather than the collisional limit described by the Braginskii equations; see C. G. Gimblett, Plasma Phys. Controlled Nucl. Fusion Res. **30**, 1853 (1988).

[13] G. W. Hammett and F. W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).

[14] F. Waelbroeck and L. Chen, Phys. Fluids B **3**, 601 (1991).

[15] J. Manickam and A. Bondeson, in Proceedings of the Sherwood Fusion Theory Conference, Newport, Rhode Island, 1993, paper 3C12 (to be published); J. Ramos, *ibid.*, paper 1C32; A. Bondeson, in *Proceedings of the 20th EPS Conference on Controlled Fusion and Plasma Physics* (European Physical Society, Geneva, 1993), Part IV, p. 1339.