Superfluidity of Excitons in a High Magnetic Field

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In a high magnetic field, such that the distance between the Landau levels exceeds the exciton Rydberg, the triplet interaction term becomes the lowest state of the system. Under these circumstances, a weak pair interaction in the triplet ground state opens a new opportunity to forming the Bose-Einstein condensate and a superfluid state of excitons at a relatively high temperature. The existence of the Bose condensate due to an essential decrease of the interaction between excitons and an increase of their binding energy in a high magnetic field are established. We show that the excitation spectrum satisfies the Landau criterion for superfluidity, and discuss the observable effects of the Bose condensate of excitons.

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The problem of the Bose condensation of excitons in a semiconductor has attracted a number of authors over the last two decades [1-10]. The idea is that the small effective mass of an exciton can make the condensation temperature quite high even at a relatively low concentration of excitons. Indeed, as is known [11] the critical temperature of an ideal Bose gas is $k_B T_c = 3.31\hbar^2 N^{2/3}/M$, where N is the concentration of the system and M is the mass of the particle. For large-radius excitons, so-called Wannier-Mott excitons (only this type of exciton will be discussed throughout the paper), the total effective mass of an excitonic atom is $M = m_e + m_h \sim 10^{-27}\text{-}10^{-28}$ g, and the condensation temperature could be quite noticeable, $T_c \sim 100$ K, at the concentration $N \sim 10^{18}$ cm^{-3}. In principle, there is no difficulty in creating the electron and hole concentration of this order by optical excitation [6,7,12], but the binding energy of the exciton, which can be evaluated in the case of Wannier-Mott excitons by the Bohr formula, is rather small, $\mathcal{R}_{\rm ex} \sim 10^{-2}\text{-}10^{-3}$ eV. On the other hand, the exciton radius is quite large, $a_0 = \varepsilon\hbar^2/me^2 \sim 10^{-6}$ cm, and becomes of the same order as the average distance between excitons. It was shown [3] that the exciton operators obey the Bose commutation relations with accuracy to terms of the order of Na_0^3. That implies the restriction from above on the concentration of the exciton gas $Na_0^3 \ll 1$. Thus, we cannot expect more or less high temperature of the transition of excitons into the Bose condensate. Another restriction comes from the possibility of formation of an excitonic bound state of the hydrogen-molecule type. Such a state arises mostly in semiconductors of the type $A^{\rm III}B^{\rm V}$ where the mass of one of the particles (holes) is much larger than the other. Because of all these reasons, the creation of a dense exciton system for observing the effects of quantum statistics still remains a very difficult problem [10].

In the previous publications [13-15] we demonstrated that a strong magnetic field B, such that the distance between the Landau levels, $e\hbar B/m_e c$, exceeds the Coulomb unit of energy, $2\mathcal{R} = m_e e^4/\hbar^2$, dramatically changes the properties of a hydrogenlike gas. Under the circumstances, the triplet (when the total electronic spin is equal to unity) evidently becomes the lowest state for the pair interaction between the atoms. In this state the pair interaction is strongly anisotropic and pretty weak [13]. The weak pair interaction and the small overlap of the atomic wave functions lead to a remarkable situation when the behavior of hydrogen and deuterium gases becomes similar to ^{4}He and ^{3}He, respectively. The transition to a superfluid state at a relatively low temperature is now possible for hydrogen and deuterium in an ultrahigh magnetic field [14,15].

A high-density gas of hydrogenlike excitons in semiconductors is the unique object for laboratory studies of extreme states of matter in an ultrahigh magnetic field. Since the excitonic binding energy is $\mathcal{R}_{\rm ex} = me^4/2\varepsilon^2\hbar^2 \sim 10^{-2}\text{-}10^{-3}$ eV, where $\varepsilon \sim 10$ is the dielectric constant, and $m = m_e m_h/(m_e + m_h)$ is the reduced effective mass of the electron and the hole, the "atomic" scale of a magnetic field for the hydrogenlike excitons $B_c = m^2 e^3 c/\varepsilon^2\hbar^3$ becomes available in a laboratory. This scale of a magnetic field may be a few orders of magnitude smaller than for hydrogen atoms. In the fields $B \gg B_c = m^2 e^3 c/\varepsilon^3\hbar^3$, i.e., $B^* \equiv B/B_c \gg 1$, the main interaction parameters of the exciton system are completely changed and it may give a real chance of observing phenomena related to the Bose condensate and the superfluidity of excitons in different types of semiconductors. In this case, as a first approximation, we can treat the motion of the electron and the hole inside a single excitonic atom as one-dimensional motion in a Coulomb potential, while considering the motion only in the magnetic field in a perpendicular plane. The expression for the ground-state wave function at a zero excitonic momentum can be written approximately as [16]

$$\Psi_0(\mathbf{r}) = \frac{1}{\sqrt{2\pi}\lambda} \exp\left(-\frac{\rho^2}{4\lambda^2}\right) \frac{1}{\sqrt{a_0\alpha}} \exp\left(-\frac{|z|}{a_0\alpha}\right), \quad (1)$$

where we chose the z axis in the direction of the magnetic field, $\lambda = \sqrt{\hbar/eB} = a_0/\sqrt{B^*}$, $\alpha = 1/\ln B^*$, and $\rho^2 = x^2 + y^2$. So the size of each exciton becomes smaller by the factor $\ln B^*$ in comparison with the Bohr radius in the direction of the magnetic field, and smaller by the factor

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$\sqrt{B^*}$ in a plane perpendicular to this direction. In the presence of a high magnetic field, a large binding energy of the exciton, $E_0 \approx \mathcal{R}_{\rm ex} \ln^2(B/B_c) \gg \mathcal{R}_{\rm ex}$, and a small characteristic size, as well as a substantial decrease of the pair interaction between hydrogenlike excitons in the triplet ground state, allow us to hope that the Bose condensation and the superfluidity will not be suppressed by the collective effects leading to an electron-hole liquid at a high density of the system. The weak pair interaction between the excitons in a high magnetic field ensures that the scattering amplitude of two excitons as a function of their energy has no poles on a real axis; i.e., there is no "molecule" of two excitons. For a semiconductor of type InSb, the reduced effective mass is 0.01–$0.02 m_e$ and the dielectric constant is $\varepsilon \approx 13$–16. As a result, a magnetic field in the range of 16–20 T corresponds to $B^* \sim 100$. Under these circumstances, the binding energy of the exciton is relatively large, ~ 10–15 meV (0.6 meV without a magnetic field), and at the concentration of excitons $N \sim 10^{18}$ cm^{-3} the average distance between excitons is approximately 10 times as large as their effective size. Therefore, the formation of the superfluid state becomes perfectly realistic in a high magnetic field even at a relatively high temperature.

The Hamiltonian of the system of electrons and holes in a high magnetic field has the form

$$\hat{H} = \sum_{\mathbf{p}} \{[\varepsilon_e(\mathbf{p}) - \mu_e] a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + [\varepsilon_h(\mathbf{p}) - \mu_h] b_{\mathbf{p}}^\dagger b_{\mathbf{p}}\} + \frac{1}{2} \sum_{\mathbf{p},\mathbf{p}',\mathbf{k}} V_{\mathbf{k}} [a_{\mathbf{p}}^\dagger a_{\mathbf{p}'}^\dagger a_{\mathbf{p}'+\mathbf{k}} a_{\mathbf{p}-\mathbf{k}} + b_{\mathbf{p}}^\dagger b_{\mathbf{p}'}^\dagger b_{\mathbf{p}'+\mathbf{k}} b_{\mathbf{p}-\mathbf{k}} - 2 a_{\mathbf{p}}^\dagger b_{\mathbf{p}'}^\dagger b_{\mathbf{p}'+\mathbf{k}} a_{\mathbf{p}-\mathbf{k}}] , \quad (2)$$

where $V_{\mathbf{k}} = 4\pi e^2 \hbar^2/\varepsilon \mathbf{k}^2$ is the Coulomb interaction, $a_{\mathbf{p}}^\dagger$ and $b_{\mathbf{p}}^\dagger$ are the Fermi operators describing creation of electrons and holes, while $a_{\mathbf{p}}$ and $b_{\mathbf{p}}$ describe electron and hole annihilation, respectively. The chemical potentials of the electrons μ_e and the holes μ_h are determined by the conditions

$$\sum_{\mathbf{p}} \langle a_{\mathbf{p}}^\dagger a_{\mathbf{p}} \rangle = \sum_{\mathbf{p}} \langle b_{\mathbf{p}}^\dagger b_{\mathbf{p}} \rangle = n , \quad (3)$$

where n is the dimensionless exciton number density, and the dependence of the electron (or hole) energy on the momentum $\mathbf{p}$ in a high magnetic field is

$$\varepsilon_{e,h}(\mathbf{p}) = \frac{1}{2m_{e,h}} \left(\mathbf{p} \pm \frac{|e|}{c} \mathbf{A} \right)^2 - \frac{|e| \hbar B}{2 m_{e,h} c} . \quad (4)$$

The electron and hole spin projections are equal to -1 and $+1$, respectively. This spin configuration corresponds to the lowest energy state in a high magnetic field.

The analysis of the properties of the ground state and its stability in the presence of the condensate of excitons is carried out by defining a new set of creation and annihilation operators which is known as the Bogoliubov canonical transformation,

$$a_{\mathbf{p}} = u_{\mathbf{p}} \tilde{a}_{\mathbf{p}} + v_{\mathbf{p}} \tilde{b}_{-\mathbf{p}}^\dagger, \quad b_{\mathbf{p}} = u_{\mathbf{p}} \tilde{b}_{\mathbf{p}} - v_{\mathbf{p}} \tilde{a}_{-\mathbf{p}}^\dagger , \quad (5)$$

where $\tilde{a}_{\mathbf{p}}$ and $\tilde{b}_{\mathbf{p}}$ are operators of quasielectrons and quasiholes, respectively, and $u_{\mathbf{p}}^2 + v_{\mathbf{p}}^2 = 1$. The functions $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are determined from the conditions of minimum energy and of stability of the ground vacuum state of the system.

Having transformed the Hamiltonian in accordance with (5), we can write out the condition of the stability of the ground state [3,17] with accuracy within terms of the order of $n^{3/2}$ inclusively, which gives the equation

$$\left[\varepsilon_e(\mathbf{p}) + \varepsilon_h(\mathbf{p}) - \mu - 2 \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} v_{\mathbf{p}'}^2 \right] u_{\mathbf{p}} v_{\mathbf{p}} - (u_{\mathbf{p}}^2 - v_{\mathbf{p}}^2) \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} u_{\mathbf{p}'} v_{\mathbf{p}'} = 0 . \quad (6)$$

This equation is reduced in the first approximation in $v_{\mathbf{p}0} \sim n^{1/2}$ to the Schrödinger equation for the single exciton bound state in a high magnetic field,

$$[\varepsilon_e(\mathbf{p}) + \varepsilon_h(\mathbf{p}) - \mu] v_{\mathbf{p}} - \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} v_{\mathbf{p}'} = 0 . \quad (7)$$

The chemical potential in this approximation coincides with the energy of the ground state, $\mu_0 = -E_0$. In the next order in perturbation theory, we can obtain the correction to the chemical potential. We have

$$v_{\mathbf{p}} = v_{\mathbf{p}0} + \delta v_{\mathbf{p}}, \quad \mu = -E_0 + \delta\mu , \quad (8)$$

with $\delta v_{\mathbf{p}}$ being orthogonal to $v_{\mathbf{p}0}$. Taking (7) and (8) into account, we rewrite, accurate to terms linear in $\delta v_{\mathbf{p}}$ and terms of the order of $n^{3/2}$, the condition of the stability of the ground state (6),

$$[\varepsilon_e(\mathbf{p}) + \varepsilon_h(\mathbf{p}) + E_0] \delta v_{\mathbf{p}} - \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} \delta v_{\mathbf{p}'} = -\beta_{\mathbf{p}} + v_{\mathbf{p}0} \delta\mu , \quad (9)$$

where

$$\beta_{\mathbf{p}} = \frac{1}{2} \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} v_{\mathbf{p}'0} [4 v_{\mathbf{p}0}^2 - 4 v_{\mathbf{p}'0} v_{\mathbf{p}0} + v_{\mathbf{p}'0}^2] .$$

Projecting (9) onto $v_{\mathbf{p}0}$, we obtain the correction to the chemical potential,

$$\delta\mu = \frac{n}{2} \sum_{\mathbf{p},\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} \Psi_0(\mathbf{p}') \Psi_0(\mathbf{p}) \times [4\Psi_0^2(\mathbf{p}) - 4\Psi_0(\mathbf{p}')\Psi_0(\mathbf{p}) + \Psi_0^2(\mathbf{p}')] . \quad (10)$$

After integrating (10), the final result for $\delta\mu$ is

$$\delta\mu = (16 n \mathcal{R}_{\rm ex}/B^*) \Phi(B) , \quad (11)$$

where $\Phi(B)$ is a dimensionless slowly varying function of the magnetic field. It changes from 0.42 at $B^* = 10$ to 0.6 at $B^* = 100$.

The excitation spectrum of excitons in the condensed state is determined by the poles of a two-particle Green's function. In fact, we must introduce two exact Green's functions: the normal Green's function $G_2(P;p,p')$

describing the propagation of an electron-hole pair through the medium and the anomalous Green's function $\tilde{G}_2(P;p,p')$ describing the appearance of two electron-hole pairs from the condensate. Here $P=(\mathbf{P},E)$ is the summary momentum and frequency of the exciton; $p=(\mathbf{p},\omega)$ and $p'=(\mathbf{p}',\omega')$ are the relative momentum and frequency of the exciton. It is worth pointing out again that, if the pair interaction of excitons corresponds to the triplet ground state in a high magnetic field, the formation of a molecular state can happen under no circumstances. The vertex $\Sigma(P;p,p')$ (which describes the scattering of an exciton by excitons of the condensate) and $\tilde{\Sigma}(P;p,p')$ (which describes the creation of two excitons with opposite momenta from the condensate and their departure to the condensate) therefore have no pole character and the perturbation theory can still be applied in the present case. The analytical form of Dyson's equations can be written, accurate to terms linear in all three parameters n, E, and $\mathbf{P}^2/2M$, as follows [3]:

$$\begin{aligned}G_2(P;p,p')=&G_e(p+P/2)G_h(-p+P/2)\int\frac{id^4p_1}{(2\pi)^4}\Sigma(0;p_1,p')G_2(P;p_1,p')\\&+G_e(p)G_h(-p)\int\frac{id^4p_1}{(2\pi)^4}\tilde{\Sigma}(0;p_1,p')\tilde{G}_2(P;p_1,p')\\&+G_e(p+P/2)G_h(-p+P/2)\Sigma(p,p')G_e(p'+P/2)G_h(-p'+P/2)\,,\end{aligned}\tag{12}$$

$$\begin{aligned}\tilde{G}_2(P;p,p')=&G_e(p-P/2)G_h(-p-P/2)\int\frac{id^4p_1}{(2\pi)^4}\Sigma(0;p_1,p')\tilde{G}_2(P;p_1,p')\\&+G_e(p)G_h(-p)\int\frac{id^4p_1}{(2\pi)^4}\tilde{\Sigma}(0;p_1,p')G_2(P;p_1,p')+G_e(p)G_h(-p)\tilde{\Sigma}(p,p')G_e(p')G_h(-p')\,.\end{aligned}\tag{13}$$

Here $G_e(p)$ and $G_h(p)$ are the Green's functions of a free electron and hole in a high magnetic field. In the presence of an external field $G_e(p)$, for example, is a function of two momenta [18,19],

$$G_e(p)=G_e(\mathbf{p}_1;\mathbf{p}_2;\omega)=G_e(p_z,\mathbf{p}_{1\perp};p_z,\mathbf{p}_{2\perp};\omega)=\frac{D(\mathbf{p}_{1\perp},\mathbf{p}_{2\perp})}{2\pi\lambda^2[\omega-(p_z^2/2m_e-\mu_e)+i\delta]}\,,\tag{14}$$

where

$$D(\mathbf{p}_{1\perp},\mathbf{p}_{2\perp})=\int d^2r_{1\perp}d^2r_{2\perp}\exp\{-(i/\hbar)[\mathbf{r}_{1\perp}\cdot\mathbf{p}_{1\perp}-\mathbf{r}_{2\perp}\cdot\mathbf{p}_{2\perp}]\}\exp(-[\mathbf{r}_{1\perp}-\mathbf{r}_{2\perp}]^2/4\lambda^2+2i[x_1-x_2][y_1+y_2]/\lambda^2)\,.$$

Here $r_\perp=|\mathbf{r}_\perp|=\sqrt{x^2+y^2}$, $p_\perp=|\mathbf{p}_\perp|=\sqrt{p_x^2+p_y^2}$. The similar expression also holds for $G_h(p)$.

The solution of (12)-(13) indicates that the two-particle Green's function of an excitonic gas in a high magnetic field formally coincides with the single-particle Green's function of a dilute nonideal Bose gas [20] with the correction to the chemical potential $\delta\mu$ which strongly depends on the magnitude of the magnetic field. The excitonic spectrum in the presence of the condensate has the usual form for a low-density Bose gas,

$$E(\mathbf{P})=\sqrt{2\delta\mu\varepsilon(\mathbf{P})+\varepsilon^2(\mathbf{P})}\,.\tag{15}$$

The dependence of the energy of a free exciton in a high magnetic field $\varepsilon(\mathbf{P})$ on its momentum is [21]

$$\varepsilon(\mathbf{P})=P_z^2/2M+(P_x^2+P_y^2)/2M_0\,,\tag{16}$$

where $M=m_e+m_h$ is the total effective mass of the exciton, and the finite transverse mass appears as a result of the Coulomb interaction in the excitonic atom: $M_0=Ma_0^2a/\lambda^2=MB^*/\ln B^*$.

Thus, the spectrum of an exciton gas in a high magnetic field becomes

$$E(\mathbf{P})=\left[\frac{\delta\mu}{M}\left(\cos^2\theta+\frac{\ln B^*}{B^*}\sin^2\theta\right)P^2+\frac{P^4}{4M^2}\left(\cos^2\theta+\frac{\ln B^*}{B^*}\sin^2\theta\right)^2\right]^{1/2},\tag{17}$$

where θ is the angle between the direction of the magnetic field and the momentum $\mathbf{P}$. The excitation spectrum vanishes linearly as $P\to 0$, with a slope equal to a macroscopic speed of sound; i.e., this spectrum satisfies the Landau criterion for superfluidity. The macroscopic speed of sound in the superfluid state of the exciton gas depends on the direction of the magnetic field,

$$V_c=\left[\frac{\delta\mu}{M}\left(\cos^2\theta+\frac{\ln B^*}{B^*}\sin^2\theta\right)\right]^{1/2}.\tag{18}$$

Thus, the superfluidity arises in the excitonic system in a high magnetic field and, as could be expected, it is likely to appear in directions close to the direction of the magnetic field. Our analysis shows that a high magnetic field helps to resolve those main problems of stability of the many-exciton system at high density, the problems which became the main obstacles on the way to achieving an exciton quantum liquid.

The system of excitons allows the formation of the Bose condensate in a high magnetic field and behaves itself as an almost ideal Bose gas does. The excitons can transfer their excitation energy and also their angular, electric, and magnetic moment if there is any. Therefore, the superfluidity of excitons could imply, for instance, the existence of a superflow of energy in the absence of a temperature gradient and a chemical-potential gradient; that is, the existence of superthermal conductivity. One can also expect that after the transition into the superfluid state, the anomalous diffusion of the excitons will take place. The expansion rate of the excitons from the point where they were created can be 2–3 orders of magnitude greater than that predicted by simple diffusion. The excitons can penetrate into the crystal over a distance which is a few orders of magnitude larger than the classical diffusion length. Besides the interesting kinetic and thermal properties, it is perfectly natural to expect a manifestation of the excitonic Bose condensate in unusual optical properties of a semiconductor. For instance, one possible experimental evidence for the Bose-Einsetein condensed state can be the appearance of a sharp spike in the luminescence spectrum at zero wave vector of excitons.

As was pointed out [5], in the presence of the exciton condensate the expression for the dielectric polarizability can be approximately written as

$$\chi(\omega,\mathbf{P})\sim-\left[\frac{1+N_{\mathbf{P}}}{\omega+\mu_0-E(\mathbf{P})+i\delta}-\frac{N_{\mathbf{P}}}{\omega+\mu_0+E(\mathbf{P})+i\delta}\right], \tag{19}$$

where ω is the frequency of an external electromagnetic field and $N_{\mathbf{P}}=[\delta\mu+\varepsilon(\mathbf{P})-E(\mathbf{P})]/ZE(\mathbf{P})$ is the distribution function with respect to $\mathbf{P}$ of the supercondensate excitons. The imaginary part of the dielectric polarizability which determines the absorption is

$$\chi''(\omega,\mathbf{P})\sim(1+N_{\mathbf{P}})\delta(\omega+\mu_0-E(\mathbf{P}))-N_{\mathbf{P}}\delta(\omega+\mu_0+E(\mathbf{P}))\,. \tag{20}$$

The complex dielectric constant can be represented in the form of the dispersion formula

$$\varepsilon(\omega)=1+C\chi\,, \tag{21}$$

where C is the constant depending on the properties of the medium. We see that in the presence of the condensate there is an increase in the absorption at the frequency $\omega=E_0+E(\mathbf{P})$. This absorption comes from the process in which the energy of the absorbed quantum goes into the formation of an exciton. Besides the increasing absorption, amplification of the light at the frequency $\omega=E_0-E(\mathbf{P})$ takes place, where $E(\mathbf{P})$ is given by expression (17). The amplification of the light comes from stimulated annihilation of excitons in the presence of the condensate. Finally, the dielectric constant is equal to unity at the frequency $\omega=E_0-\delta\mu-\varepsilon(\mathbf{P})$, where $\varepsilon(\mathbf{P})$ is given by (16). In effect, the crystal would seem to be totally transparent. The "laser" effect, supertransparency, and other exotic properties of semiconductors would take place only in the presence of the exciton condensate.

In conclusion, we would like to emphasize that the Bose condensation and superfluidity for hydrogenlike excitons in a high magnetic field become quite realistic even at room temperatures because these phenomena are not suppressed in a high magnetic field by the collective effects leading to an electron-hole liquid without a magnetic field at a high density of the system. It opens new possibilities of studying and creating a great number of new phenomena relating to hydrogenlike quantum liquids.

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