Giant Resonance Effects on Heavy-Ion Fusion

M.S. Hussein and A.F.R. de Toledo Piza
Instituto de Física, Universidade de São Paulo, C.P. 20516, 01452-990 São Paulo, São Paulo, Brazil
(Received 8 December 1993)

The effect of the coupling to a resonant state in one of the partners of a heavy-ion fusion reaction on the fusion cross section is discussed. We conclude that the width of the resonance could either enhance or hinder the fusion, depending on the relative importance of the spreading to escape widths. General comments on the fusion of neutron-rich nuclei are made.

PACS numbers: 25.70.Ef, 24.30.Cz, 25.70.Jj

Coupled-channel effects on the fusion of heavy ions have been widely discussed in the literature in recent years [1]. The overall picture that has emerged from these studies is that the fusion cross section is enhanced, as long as the coupling is restricted to normal channels. By normal, we mean excited states and transfer. The effect of the coupling to breakup channels (where the projectile or target could disintegrate during the fusion process) has only recently been considered in cases involving low Q values, usually encountered in loosely bound neutron-rich projectiles [2]. Conflicting conclusions have been reached concerning this latter effect [3,4]. A more natural framework in which to discuss this issue is to consider the effect of the coupling to a resonant state in one of the nuclei, which eventually decays into open channels (breakup).

Our aim in this paper is to elucidate the problem of the effect of the coupling to resonant states on the fusion cross section. Since, according to Ref. [5], the width of a resonance, Γ, of a many-body system such as the nucleus is the sum of a spreading width $\Gamma^{\downarrow}$ which measures the degree damping of the resonance due to its coupling to more complicated states in the *same* nucleus, plus the escape width, $\Gamma^{\uparrow}$, which measures the *actual* fragmentation of the nucleus into the open channels (except for the γ-emission contribution, which we do not consider), it is natural to expect that the effect of the coupling on σ_F should depend on the ratio $\mu \equiv \Gamma^{\downarrow}/\Gamma$. If this ratio is close to 1, we expect an *enhanced* fusion probability, since there are *many routes* (excited states in the *same* nucleus) for fusion to occur. The other limit, $\mu \ll 1$, should result in a smaller fusion probability, since the resonance (nucleus) could "break up" before fusion occurs.

We use the exit doorway model [6] to treat first the effect of the spreading of an excited, collective state, on the fusion cross section. The entrance channel couples to the compound nucleus (fusion) either directly or via the bunch of excited channels that are modulated by the doorway. To reach these "fine structure" channels the system has to pass through the doorway.

We take for the Hamiltonian of the system $H = H_0+V$, where H_0 is diagonal in open channel space. Here $H_0 = h_0 + K + U$, where h_0 is the intrinsic Hamiltonian, K is the kinetic energy operator, and U is the optical potential which contains the complex nuclear plus the Coulomb parts. The coupling among these channels is represented by V. We ignore the couplings among the excited, fine structure, channels scattered around the doorway. We now write

$$h_0 = | \varphi_0\rangle E_0 \langle\varphi_0 | + \sum_i | i\rangle e_i \langle i | + | d\rangle E_d \langle d | + \sum_i [| i\rangle \Delta_i \langle d | + | d\rangle \Delta_i \langle i |]. \quad (1)$$

The last term in Eq. (1) represents the spreading of $| d\rangle$. Notice that $| d\rangle$ is *not* an eigenstate of H_0. Only the states with *no* widths are so

$$h_0 | \varphi_0\rangle = E_0 | \varphi_0\rangle, \quad h_0 | i\rangle = E_i | i\rangle. \quad (2)$$

The full Schrödinger equation of the system reads

$$[E - (H_0 + V)] | \Psi\rangle = 0, \quad (3)$$

which, upon projection onto the different channels, gives the usual set of coupled-channel equations:

$$\begin{aligned} (E - E_0 - H_0)\Psi_0^{(+)} &= \sum V_{0i}\, \Psi_i^{(+)}, \\ (E - E_i - H_i)\Psi_i^{(+)} &= V_{i0}\, \Psi_0^{(+)}. \end{aligned} \quad (4)$$

The "exit-doorway" hypothesis implies

$$V_{0i} = V_{0d}\alpha_{di}, \quad V_{i0} = V_{d0}\alpha^*_{id} \,. \quad (5)$$

The coefficients $\alpha_{di} = \langle d | \varphi_i\rangle$ are obtained from Eqs. (1) and (2). One generally finds

$$| \alpha_{di} |^2 = \frac{\Gamma_d^{\downarrow}/2\pi}{(E_i - E_d)^2 + \frac{\Gamma_d^{\downarrow 2}}{4}}, \quad (6)$$

where $\Gamma^{\downarrow}$ is related to the Δ_i factors, viz.,

$$\Gamma_d^{\downarrow} = 2\pi \, | \bar{\Delta} |^2 \, \rho \, ,$$

where the bar denotes an average over the states i and ρ is the average density of fine structure states.

Equation (4) can be solved for $\Psi_0^{(+)}$, the *exact* wave function in the elastic channel (remember $V_{ii'} = 0$, and $E_0 = 0$):

0031-9007/94/72(17)/2693(4)$06.00

$$\left(E - H_0 - \sum_i V_{0i}\frac{1}{E - E_i - K_i - U_i + i\varepsilon}V_{i0}\right)\Psi_0^{(+)} = 0\,. \tag{7}$$

With (5) and (6), it is easy to reduce Eq. (7) to the following:

$$\left(E - H_0 - V_{0d}\frac{1}{E - E_d - H_d + \frac{i\Gamma_d^{\downarrow}}{2}}V_{d0}\right)\Psi_0^{(+)} = 0. \tag{8}$$

In deriving Eq. (8), we have assumed $U_i \equiv U_d$, and $H_d = K_d + U_d$, with U_d being the optical potential in the doorway channel. It is interesting to note that Eq. (8) can be rewritten as two coupled equations,

$$\begin{aligned}(E - K_0 - U_0)\,\Psi_0^{(+)} &= V_{0d}\Psi_d^{(+)},\\ (E - K_d - U_d)\,\Psi_d^{(+)} &= V_{d0}\Psi_0^{(+)} + (E_d - i\Gamma_d^{\downarrow}/2)\Psi_d^{(+)}.\end{aligned} \tag{9}$$

Equations (8) and (9) are the starting point for the discussion to follow. The total optical potential in (8) contains two pieces. A background one, U_0, which, in our model, arises from the coupling to the fusion channels, and a second piece coming from the coupling to the channels where one of the nuclei is in the doorway resonance state. We call this part of the optical potential the intermediate dynamic polarization potential (IDPP)

$$\begin{aligned}V_{\rm IDPP} &= V_{0d}\left(E - E_d - H_d + \frac{i\Gamma_d^{\downarrow}}{2}\right)^{-1}V_{d0}\\ &\equiv V_{0d}G_d(E)V_{d0}.\end{aligned} \tag{10}$$

It is instructive to discuss the reactive content of $V_{\rm IDPP}$. For this purpose we calculate its imaginary part,

$$\begin{aligned}{\rm Im}V_{\rm IDPP} &= V_{0d}\,G_d^{\dagger}\,{\rm Im}\,U_d\,G_d\,V_{d0}\\ &\quad + V_{0d}\,\Omega_d^{(-)}\,\breve{G}_d^{(+)^{\dagger}}\,\Gamma_d^{\downarrow}\,\breve{G}_d^{(+)}\,\Omega_d^{(-)^{\dagger}}\,V_{d0},\end{aligned} \tag{11}$$

where the bare intermediate Green's function $\breve{G}_d$ and the full intermediate Möller operator Ω_d are given by

$$\breve{G}_d = \left(E - E_d - K_d + \frac{i\Gamma^{\downarrow}}{2}\right)^{-1}, \tag{12}$$

$$\Omega^{(-)} = (1 + U_d^{\dagger}\,G_d^{\dagger})\,. \tag{13}$$

The first term in (11) represents flux lost to fusion channels *via the doorway*, while the second term accounts for flux lost in *directly* exiting the doorway. In fact, the operator $\frac{1}{2\pi}\,\breve{G}_d^{\dagger}\,\Gamma_d^{\downarrow}\,\breve{G}_d$ is nothing but a finite-width version of the usual delta function that describes on-shell processes.

The total fusion cross section is calculated from the elastic channel matrix element of ${\rm Im}U_0 + V_{0d}G_d^{\dagger} \times {\rm Im}U_d\,G_dV_{d0}$. We obtain [7]

$$\sigma_F = \frac{k}{E}\left[\langle\Psi_0^{(+)}\mid {\rm Im}\,U_0\mid\Psi_0^{(+)}\rangle + \langle\Psi_d^{(+)}\mid {\rm Im}\,U_d\mid\Psi_d^{(+)}\rangle\right]. \tag{14}$$

To get insight into the effect of the finite width of the resonance on σ_F, we consider the very schematic model of Dasso, Landowne, and Winther [8] where the coupling potentials are taken to be constant, and diagonalize the two coupled equations for $\mid\Psi_0^{(+)}\rangle$ and $\mid\Psi_d^{(+)}\rangle$, Eq. (9), after setting $U_d = U_0$, $K_d = K_0$, and $V_{0d} = V_{d0} = v$. This is accomplished by introducing an appropriate biorthogonal basis [9],

$$\begin{pmatrix} 0 & v \\ v & E_d - \frac{i\Gamma_d^{\downarrow}}{2}\end{pmatrix} = (\chi_+\ \chi_-)\begin{pmatrix}\lambda_+ & 0\\ 0 & \lambda_-\end{pmatrix}\begin{pmatrix}\tilde{\chi}_+^{\dagger}\\ \tilde{\chi}_-^{\dagger}\end{pmatrix}, \tag{15}$$

where

$$\begin{aligned}(\chi_+\,,\ \tilde{\chi}_+^{\dagger}) &= 1 = (\chi_-\,,\ \tilde{\chi}_-^{\dagger}),\\ (\chi_+\,,\ \tilde{\chi}_-^{\dagger}) &= 0 = (\chi_-\,,\ \tilde{\chi}_+^{\dagger}),\end{aligned}$$

$$\lambda_\pm = \frac{1}{2}\left[E_d - \frac{i\Gamma_d^{\downarrow}}{2} \pm \sqrt{\left(E_d^2 - \frac{\Gamma_d^{\downarrow 2}}{4} + 4v^2\right) - i\,E_d\Gamma_d^{\downarrow}}\right]. \tag{16}$$

Calling the eigenchannel wave functions Ψ_+ and Ψ_-, we obtain the two decoupled equations,

$$\begin{aligned}(E - K_0 - U_0 - \lambda_+)\Psi_+ &= 0,\\ (E - K_d - U_d - \lambda_-)\Psi_- &= 0\,.\end{aligned} \tag{17}$$

The transformation matrix reads

$$\begin{aligned}\begin{pmatrix}\Psi_0\\ \Psi_d\end{pmatrix} &= \begin{pmatrix}\frac{v\lambda_+}{\lambda_+^2+v^2} & \frac{v^2}{\lambda_-^2+v^2}\\ \frac{\lambda_+^2}{\lambda_+^2+v^2} & \frac{v\lambda_-}{\lambda_-^2+v^2}\end{pmatrix}\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix}\\ &\equiv M\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix}.\end{aligned} \tag{18}$$

The fusion cross section, Eq. (14), can be written as

$$\begin{aligned}\sigma_F &= (\Psi_+\ \Psi_-)M^{\dagger}\begin{pmatrix}{\rm Im}\,U & 0\\ 0 & {\rm Im}\,U\end{pmatrix}M\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix}\\ &= (M^{\dagger}\,M)_{++}\breve{\sigma}_F^{(+)} + \breve{\sigma}_F^{(-)}(M^{\dagger}\,M)_{--} + 2{\rm Re}[\breve{\sigma}_F^{(+)(-)}\,(M^{\dagger}\,M)_{+-}],\end{aligned} \tag{19}$$

where $\breve{\sigma}_F^{(\pm)}$ is the fusion cross section in the eigenchannel $(\pm)$ while $\breve{\sigma}_F^{(+)(-)}(M^{\dagger}\,M)_{+-}$ is an interference term that contains the matrix elements $\langle\Psi_+^{(+)}\mid{\rm Im}\,U\mid\Psi_-^{(+)}\rangle$ and $\langle\Psi_-^{(+)}\mid{\rm Im}\,U\mid\Psi_+^{(+)}\rangle$.

The matrix elements are evaluated using the incoming wave boundary condition. The full details of ${\rm Im}U$ are not needed. Only the penetrabilities of the complex barriers ${\rm Re}U(r) + \hbar^2\ell(\ell+1)/(2\mu\,r\,2) + \lambda_\pm$ are needed (once the flux penetrates the barrier, it is fully absorbed). Thus we have for $\breve{\sigma}_F^{(\pm)}$

$$\breve{\sigma}^{(\pm)} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\frac{1}{1+\exp 2\mathrm{Re}[\int_{r_0^\pm}^{r_1^\pm} k_\pm^\ell(r)dr]}, \quad (20)$$

where the complex turning points $r_0^\pm$ and $r_1^\pm$ are solutions of

$$k_+^\ell\begin{pmatrix} r_0^+ \\ r_1^+ \end{pmatrix} = k_-^\ell\begin{pmatrix} r_0^- \\ r_1^- \end{pmatrix} = 0.$$

The action $\int_{r_0}^{r_1} k(r)dr$ is complex, but only the real part enters in the calculation of $\breve{\sigma}_F$. The same model would imply an insignificant contribution of the cross term $\breve{\sigma}_F^{(+)(-)}$, since there is a mismatch in the phases of Ψ_+ and Ψ_-^*. We now use a parabolic barrier approximation, and approximate the sum over ℓ by an integral. Upon integration, we obtain the simple extension of Wong's formula [10],

$$\breve{\sigma}_F(V_B + \mathrm{Re}\lambda_\pm) = \frac{\hbar\omega R_B^2}{2E}\ln\left\{1+\exp\frac{2\pi}{\hbar\omega}[E-\mathrm{Re}\lambda_\pm - V_B(R_b)]\right\},$$

where V_B is the Coulomb barrier, R_B is its radius, and $\hbar\omega$ is related to the barrier curvature $\hbar\omega = (\hbar/\mu)d^2V_\ell/dr^2\,|_{r=R_B}$.

The total fusion cross section we obtain has the following form:

$$\sigma_F = A(\lambda_+)\,\breve{\sigma}_F\,(V_B + \mathrm{Re}\,\lambda_+) + A(\lambda_-)\,\breve{\sigma}_F\,(V_B + \mathrm{Re}\,\lambda_-)\,, \quad (21)$$

$$A(\lambda) = \left[\frac{v^4}{|\lambda^2+v^2|^2} + \frac{|\lambda v|^2}{|\lambda^2+v^2|^2}\right],$$

and λ is given in (16). The finite width of the resonance effectively reduces the Q-value effect and thus $\sigma_F(\Gamma_d^\downarrow)/\sigma_F(\Gamma_d^\downarrow = 0) \equiv E(\Gamma_d^\downarrow)$ should be larger than 1. To be specific we consider the system ^{11}Li+^{208}Pb. The barrier height and curvature were taken to be 26.0 and 3.0 MeV, respectively [3]. We consider the coupling to a normal giant dipole resonance (excitation of the core ^{9}Li) whose excitation energy is $E_d \cong 16$ MeV. We take for v = 3 MeV [3]. Effectively, the presence of $\Gamma^\downarrow$ produces a slight increase ($\sim$ 10%) in σ_F. This increase depends on the Q value (E_d). For large E_d, the effect of the coupling is insignificant. As E_d is lowered σ_F is increased when $\Gamma^\downarrow$ is taken into account. This is expected on physical grounds since the resonance is reached even if the energy transfer is smaller than E_d. As we see clearly in the figure, the effect is basically restricted to $E_{\mathrm{c.m.}} < V_B$.

In our discussion so far we have considered only the spreading width of the doorway. The approximation $\Gamma_{\mathrm{GR}} \sim \Gamma_{\mathrm{GR}}^\downarrow$ is quite reasonable in heavy nuclei such as ^{208}Pb. For light nuclei the opposite limit is usually attained, $\Gamma_{\mathrm{GR}} \sim \Gamma_{\mathrm{GR}}^\uparrow$. In fact the soft giant dipole resonance in ^{11}Li has its width 100% escape since complex excited states in the vicinity of the resonance do not exist. It is of importance therefore to consider the effect of $\Gamma_{\mathrm{GR}}^\uparrow$ on the fusion cross section. For simplicity we assume the giant resonance escapes by coupling to one channel which we call the "breakup" channel. The wave function of this three-body channel (e.g., ^{9}Li+$2n$+^{208}Pb) is denoted by $\Psi_b^{(+)}$.

We assume that this channel is reached directly from the ground state and indirectly via the doorway. The set of equations (9) is now modified to read

$$[E - K_0 - U_0 - V_0^{\mathrm{Pol}}(b)]\;\Psi_0^{(+)} = V_{0d}\,\Psi_d^{(+)},$$

$$[E - K_d - U_d - V_d^{\mathrm{Pol}}(b)]\;\Psi_d^{(+)} = V_{d0}\,\Psi_0^{(+)} + \left(E_d - \frac{i\Gamma_d^\downarrow}{2}\right)\Psi_d^{(+)}, \quad (22)$$

where we have introduced the usual dynamic polarization potential that accounts for the coupling of $\Psi_0^{(+)}$ to $\Psi_b^{(+)}$ and $\Psi_d^{(+)}$ to $\Psi_b^{(+)}$. In deriving Eq. (22) we have employed the approximation $V_0^{\mathrm{pol}}(b) \equiv V_{0b}\,G_b^{(+)}\,V_{b0}$ and $V_d^{\mathrm{pol}}(b) = V_{db}\,G_b^{(+)}\,V_{bd}$, where $G_b^{(+)}$ represents the propagation in the breakup channel.

In principle, $\mathrm{Im}\,V_d^{\mathrm{pol}}(b)$ is related to $\Gamma_d^\uparrow$ and, naively speaking, it should be added to $\Gamma_d^\downarrow$ to obtain the total width of the resonance in Eq. (22). However, this is completely misleading since $\Gamma^\uparrow$ and thus $\mathrm{Im}\,V_d^{\mathrm{pol}}(b)$ describe the actual *loss* of the projectile (or target), whereas $\Gamma^\downarrow$ describes its *survival*. In the fusion process the effect of the breakup of one of the partners naturally leads to a reduction of the cross section [3]. This is so, since breakup couplings lead to a repulsive real part and an absorptive imaginary part of V^{pol}. Both of these lead to lower penetrabilities at energies in the vicinity of the Coulomb barrier.

Since V^{pol} is generally small compared to other potentials in the problem and is of longer range, its effect on

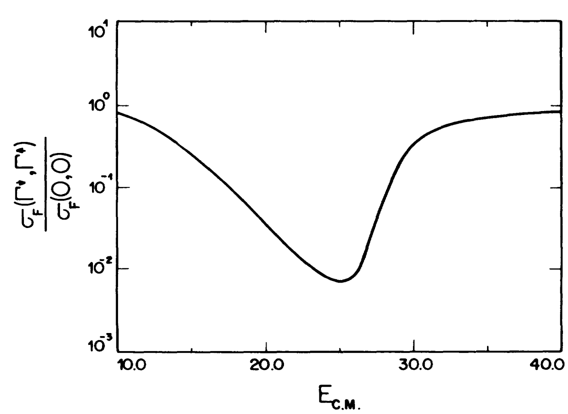

FIG. 1. The ratio $\sigma_F(\Gamma_d^\downarrow, \Gamma_d^\uparrow)/\sigma_F(\Gamma\downarrow_d = 0\,, \Gamma_d^\uparrow = 0)$ for the system ^{11}Li+^{208}Pb, $\Gamma^\downarrow = 2$ MeV , $\Gamma^\uparrow = 1$ MeV , and E_d = 0,2 MeV. See text for details.

the wave functions $\Psi_0^{(+)}$ and $\Psi_d^{(+)}$ can be conveniently expressed as a damping factor. In Refs. [3] and [4] it was shown that σ_F can be conveniently expressed as (after approximating $V^{\rm pol}$ by its local equivalent version, see Ref. [11] for details)

$$\sigma_F = \frac{\pi}{k^2}\sum_\ell (2\ell+1)\,T_\ell\,(V_{\rm pol}=0) \times \exp\left[\frac{-2}{\hbar}\int_0^\infty {\rm Im}\,V_{\rm pol}\,dt\right]. \tag{23}$$

It should be easy to convince oneself that the breakup survival probability $\exp\left[-(2/\hbar)\int_0^\infty {\rm Im}\,V_{\rm pol}\,dt\right]$ involves an appropriate energy scale $\Gamma_d^\uparrow$ and an appropriate time scale, the effective collision time τ_c. Thus we write $2\int_0^\infty {\rm Im}\,V_d^{\rm pol}\,dt = \Gamma_d^\uparrow\,\tau_c(\ell)$.

The treatment of $V_0^{\rm pol}(b)$ follows similar steps as above (the Q values in both cases are roughly equal); the difference resides in higher-order effects in $V_d^{\rm pol}(b)$. Thus we also write $2\int_0^\infty {\rm Im}\,V_0^{\rm Pol}\,dt \equiv \Gamma_0^\uparrow\,\tau_c(\ell)$, where $\Gamma_0^\uparrow$ may be called the "channel escape width." For simplicity we set $\Gamma_0^\uparrow = \Gamma_d^\uparrow$. We now introduce the mixing parameter considered earlier in the study of the decay of giant resonances [12] $\mu \equiv \frac{\Gamma_d^\downarrow}{\Gamma_d^\uparrow+\Gamma_d^\downarrow}$. Thus for a fixed Γ_d, $\Gamma_d^\uparrow = (1-\mu)\,\Gamma_d$, $\Gamma_d^\downarrow = \mu\Gamma_d$, we have for the fusion cross section

$$\sigma_F(\mu) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left\{\frac{A(\lambda_+)\,e^{-(1-\mu)\frac{\Gamma_d}{\hbar}\tau_c(\ell)}}{1+\exp\left\{\frac{2\pi}{\hbar\omega}\left[V_B+{\rm Re}\,\lambda_+(\mu)+\frac{\hbar^2\ell(\ell+1)}{2\mu R_B^2}-E\right]\right\}} + \frac{A(\lambda_-)\,e^{-(1-\mu)\frac{\Gamma_d}{\hbar}\tau_c(\ell)}}{1+\exp\left\{\frac{2\pi}{\hbar\omega}\left[V_B+{\rm Re}\,\lambda_-(\mu)+\frac{\hbar^2\ell(\ell+1)}{2\mu R_B^2}-E\right]\right\}}\right\}. \tag{24}$$

In Fig. 1, we show the ratio $\sigma(\Gamma_d^\downarrow,\Gamma_d^\uparrow)/\sigma(\Gamma_d^\downarrow=0,\Gamma_d^\uparrow=0)$ for $^{11}{\rm Li}+^{208}{\rm Pb}$, taking for $E_d = 0.2$ MeV. We took $\Gamma_d^\downarrow = 2$ MeV and $\Gamma_d^\uparrow = 1$ MeV. It is clear that now the fusion is strongly hindered by a factor of 100 in the barrier region. Thus the effect of $\Gamma_d^\uparrow$ is much more important than that of $\Gamma_d^\downarrow$. Considering now the realistic version of the soft dipole mode in ^{11}Li, its width is totally escape (to the $2n+^9{\rm Li}$ channel) and thus the fusion of ^{11}Li is hindered [3,13]. Finally, we mention that if there is no direct coupling between the entrance channel and the breakup one, $V_0^{\rm pol}(b)=0$, Eq. (24) is slightly modified. We leave the discussions of this case to a future publication.

In conclusion, we have developed a reaction theory that enables the inclusion of the width of the giant resonance in the fusion cross section. We found that the damping width only mildly enhances σ_F at lower than barrier energies, whereas the escape width strongly hinders it. More detailed application of the new theory will be published elsewhere. M.S.H. was supported in part by CNPq.

[1] See, e.g., M. Beckerman, Rep. Prog. Phys. **51**, 1047 (1988); R.G. Stokstad and E.E. Gross, Phys. Rev. C **23**, 281 (1981); H. Esbensen, Nucl. Phys. **A352**, 147 (1981); R.A. Broglia, C.H. Dasso, S. Landowne, and A. Winther, Phys. Rev. C **27**, 2433 (1983); H.J. Krappe, K. Mohring, M.C. Nemes, and H. Rossner, Z. Phys. A **314**, 23 (1983); A.B. Balantekin, S.E. Koonin, and J.W. Negele, Phys. Rev. C **28**, 1565 (1983).

[2] C.A. Bertulani, L.F. Canto, and M.S. Hussein, Phys. Rep. **226**, 281 (1993).

[3] M.S. Hussein, M.P. Pato, L.F. Canto, and R. Donangelo, Phys. Rev. C **46**, 377 (1992).

[4] N. Takigawa, M. Kuratani, and H. Sagawa, Phys. Rev. C **47**, R2470 (1993).

[5] A.F.R. de Toledo Piza, Ph.D. thesis, MIT, 1966; A. Bohr and B.R. Mottelson, *Nuclear Structure I* (Benjamin, New York, 1969).

[6] H. Feshbach, A.K. Kerman, and R.M. Lemmer, Ann. Phys. (N.Y.) **41**, 280 (1967).

[7] M.S. Hussein, Phys. Rev. C **30**, 1962 (1984).

[8] C.H. Dasso, S. Landowne, and A. Winther, Nucl. Phys. **A407**, 221 (1983).

[9] P. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1952).

[10] C.Y. Wong, Phys. Rev. Lett. **31**, 766 (1973).

[11] L.F. Canto, R. Donangelo, M.S. Hussein, and M.P. Pato, Nucl. Phys. **A542**, 131 (1992).

[12] H. Dias, M.S. Hussein, and S.K. Adhikari, Phys. Rev. Lett. **57**, 1998 (1986); A.F.R. de Toledo Piza and G.A. Foglia, in *Proceedings of the International Nuclear Physics Conference, São Paulo, Brazil*, edited by M.S. Hussein *et al.* (World Scientific, Singapore, 1990); R. Bonetti, M.B. Chadwick, P.E. Hodgson, B.V. Carlson, and M.S. Hussein, Phys. Rep. **202**, 121 (1991).

[13] We define fusion as that process which involves the absorption of the projectile intact. In Ref. [4], the fusion cross section was presented as the sum of the fusion of $^{11}{\rm Li}+^{208}{\rm Pb}$ plus that in the breakup channel, $^9{\rm Li}+^{208}{\rm Pb}$. If this is done the hindrance is obviously removed.