Magnetotunneling Spectroscopy of a Quantum Well in the Regime of Classical Chaos

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Resonant tunneling spectroscopy is used to study the energy-level spectrum of a new chaotic dynamical system, an electron in a trapezoidal potential well in the presence of a high magnetic field tilted relative to the confining barriers. Distinct series of quasiperiodic resonances are observed in the current-voltage characteristics which change dramatically with tilt angle. These resonances are related to unstable closed orbits within the chaotic domain. The experimental results are explained by identifying and studying the properties of periodic orbits accessible to the tunneling electrons.

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Our theoretical understanding of the quantum properties of systems which display chaotic classical dynamics has advanced considerably in recent years [1]. Despite the complexity of the energy-level spectrum of such systems, universality has been identified in the distribution of energy levels [2] and in the response to an external perturbation [3]. In the semiclassical limit the energy-level pattern and spatial eigenfunctions have been related to the occurrence of closed classical orbits within the chaotic sea [1,4]. However, the experimental studies of quantum phenomena in nonintegrable systems have been few. Most investigations have focused on hydrogenic atoms in a high magnetic field close to the ionization threshold [5,6]. In this regime, the classical motion is chaotic but distinct unstable periodic orbits exist, which give rise to series of periodic quasi-Landau resonances in the photoabsorption spectrum. In the solid state, chaotic motion in two-dimensional quantum-dot stadia [7] and antidot superlattices [8] has been studied and periodic components in the magnetoconductance related to unstable periodic orbits in the structures.

In this Letter we study a new dynamical system, an electron in a trapezoidal potential well, which we show is classically chaotic in the presence of a high magnetic field tilted with respect to the confining barriers. When the tilt angle θ, relative to the direction normal to the barriers, is 0° or 90° the motion is regular corresponding respectively to helical orbits or skipping orbits between the barriers [9]. For intermediate tilt angles, when the cyclotron radius is sufficiently small, the orbit segments between successive collisions with the barriers rapidly become uncorrelated giving rise to strongly chaotic motion. As shown in Fig. 1, unstable periodic orbits also occur in this chaotic dynamic sea, for particular values of the initial electron velocity.

We have investigated the quantized energy-level spectrum associated with this classically chaotic system by incorporating the potential well into a double-barrier semiconductor heterostructure as in Fig. 1. In our experiments the GaAs quantum well (QW) of width $w=120$ nm is enclosed between $Al_{0.4}Ga_{0.6}As$ tunnel barriers of width $b=5.6$ nm surrounded by weakly n-doped (2×10^{16} cm^{-3}) contact layers [9]. Under bias, charge accumulates and a bound state is formed at the emitter-barrier interface. At liquid-helium temperatures this gives rise to a degenerate two-dimensional electron gas (2DEG). In the presence of an applied magnetic field $\mathbf{B}=(B_x,0,B_z)$, the component $B_x=B\cos\theta$ perpendicular to the plane of the 2DEG quantizes the 2D in-plane motion into Landau levels [10]. The magnetic field is sufficiently large (11.4 T) that, except for θ close to 90°, only the lowest Landau level is occupied. Resonant tunneling occurs when the energy of this discrete emitter state coincides with the energy ϵ_n of a subband in the QW [11]. As the bias voltage V is varied the tunneling electrons thus scan the energy-level spectrum of the QW. Continuity of the current I is maintained by leakage through the collector barrier.

The resonant tunneling characteristics are shown in Fig. 2 for a range of tilt angles at $B=11.4$ T. Second derivative plots are used to suppress the monotonically varying background. At $\theta=0°$ a single series of resonance peaks is observed with a voltage period $\Delta V\cong30$

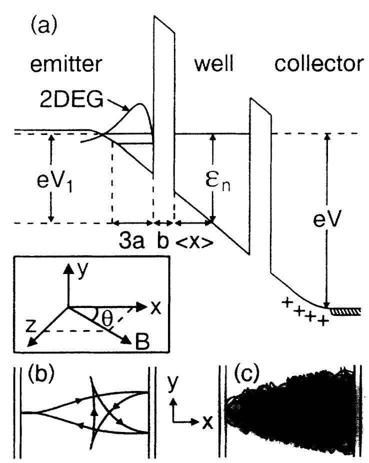

FIG. 1. (a) Conduction band profile of double-barrier structure under bias voltage V, showing resonant tunneling of electrons from emitter 2DEG into subband level ϵ_n in the well. Inset shows tilt angle θ of magnetic field $\mathbf{B}$ relative to tunneling (x) direction. (b) Projection in x-y plane of closed periodic orbit ($V=440$ mV, $B=11.4$ T, $\theta=20°$). (c) Chaotic orbit resulting from 0.1% change in initial electron velocity.

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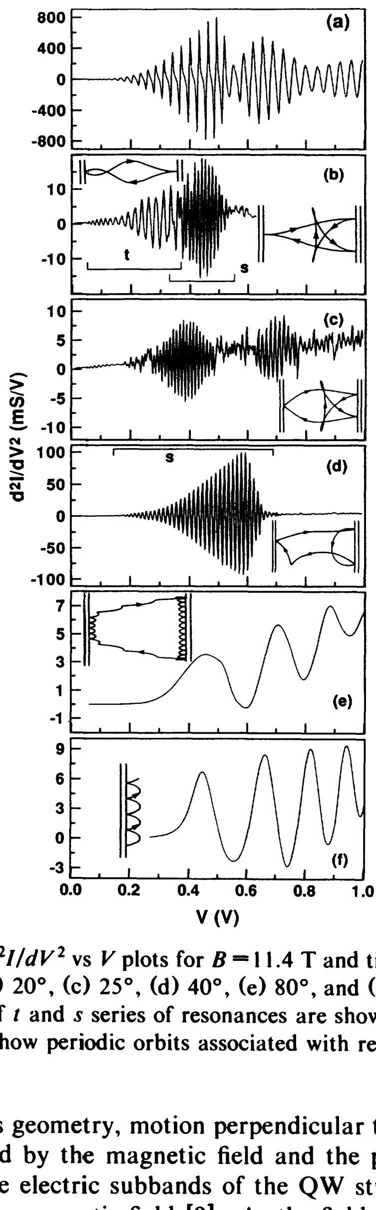

FIG. 2. d^2I/dV^2 vs V plots for $B=11.4$ T and tilt angle θ set to (a) 0°, (b) 20°, (c) 25°, (d) 40°, (e) 80°, and (f) 90°. Voltage ranges of t and s series of resonances are shown in (b) and (d). Insets show periodic orbits associated with resonant structure.

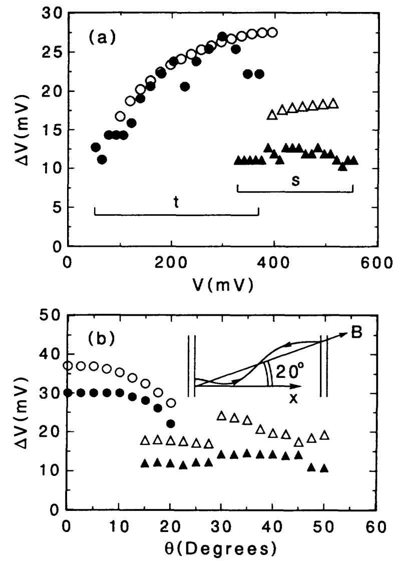

FIG. 3. (a) Variation of spacing ΔV with V for t series (circles) and s series (triangles) of resonances at $B=11.4$ T, $\theta=20°$, showing experimental values (filled circles and triangles) and theoretical values based on closed-orbit periods (open circles and triangles). (b) Variation of ΔV with θ for t series ($V=375$ mV) and s series (mean values over observed voltage range) at $B=11.4$ T. Experimental and theoretical values labeled as in (a). Inset shows projection on x-z plane of traversing orbit for $V=120$ mV, $\theta=20°$.

mV. In this geometry, motion perpendicular to the layers is unaffected by the magnetic field and the peaks correspond to the electric subbands of the QW studied previously at zero magnetic field [9]. As the field is tilted, the resonant structure changes dramatically. For $10° < \theta \lesssim 20°$, two distinct series of resonances are observed, labeled s and t in Fig. 2(b). The t series emerges smoothly from the resonant structure at $\theta=0°$ while the s series appears at higher voltages. The changeover between the two series is characterized by a sudden reduction in ΔV by a factor ~ 2. The variation of ΔV with applied bias V is shown in Fig. 3(a) for both series when $\theta=20°$. For $\theta \gtrsim 20°$ the s series dominates the entire voltage range as shown in Figs. 2(c), $\theta=25°$ and 2(d), $\theta=40°$. For $50° < \theta < 60°$ no regular oscillatory structure is observed and for $\theta > 60°$ widely spaced oscillations appear in d^2I/dV^2, as shown in Fig. 2(e). The spacing of these oscillations decreases steadily as θ is increased to 90°. In this case the tunneling electrons are injected into quantized skipping states, in which an electron bounces periodically along the emitter-barrier interface [see inset in Fig. 2(f)]. Magneto-oscillations of this type have been previously studied for both single-barrier and wide-well double-barrier structures [11,12].

When $\theta=0°$ or 90° the classical electron orbits in the QW are all regular and periodic which gives rise to an energy spectrum of discrete, well-separated levels. In the semiclassical regime of large quantum numbers n, the correspondence principle requires that the splitting $\Delta\epsilon_n = \epsilon_{n+1} - \epsilon_n$ between neighboring energy levels is related to the periodic time T of the classical orbit by $\Delta\epsilon_n = h/T$. For a nonintegrable system the energy-level spectrum is very dense but each periodic, unstable orbit in the chaotic sea produces a regular clustering of the levels [1]. A particular orbit with period T_P leads to an oscillatory variation in the smoothed density of states with an energy period $\Delta\epsilon_P = h/T_P$ [13]. This modulation of the density of states will only be resolved in our experiments if $\Delta\epsilon_P > \hbar/\tau$, where τ is the electron scattering time. We have used $\tau \sim 0.1$ ps to take account of optic phonon emission [14] and impurity and interface scattering. We have therefore searched for periodic orbits in the QW with periods $T_P \leq 2\pi\tau \sim 0.6$ ps.

To ensure correspondence between the classical and quantum descriptions of the system the classical orbits must be accessible to the tunneling electrons. The tunneling matrix element involves the overlap of the wave functions of the QW states and emitter-2DEG states in the emitter barrier. For $\theta=0°$ this requires that the Landau level index for the in-plane motion is conserved [10]. For $\theta=90°$ both the emitter and QW states have plane-wave character in the transverse y,z directions. The tunneling matrix element only couples states with a velocity difference equal to that supplied by the Lorentz force as the electron traverses the mean distance between the 2DEG and nearside QW interface [11,12]. This requirement and that of energy conservation determine the allowed skipping states into which an electron can tunnel.

In our experiments the B field is sufficiently large that, for $\theta \leq 70°$, electrons in the 2DEG occupy the lowest in-plane Landau level. There is thus a distribution of transverse momentum components with a mean square value $p_y^2+p_z^2=e\hbar B\cos\theta$. We therefore consider a set of classical electrons with a range of transverse momenta given by the above equality. On calculating the momentum change due to the Lorentz force as the electron traverses the mean distance between the 2DEG and nearside QW interface, we obtain a range of initial velocities v_y,v_z for electrons entering the well. The kinetic energy of an injected electron is obtained from the potential drop across the 2DEG and barrier, which is modeled using the Fang-Howard variational wave function [15] for the emitter bound state.

Our search for accessible periodic orbits in the QW starts with the equations for the electron trajectory in parametric form, which are obtained by analytic solution of the classical law of motion for a particle of mass $m^*=0.067m_e$ in constant electric and magnetic fields. The times at which successive specular collisions with the well walls occur must then be calculated numerically. It may be shown that if the electron returns to the emitter barrier with no change in the in-plane velocity components v_y,v_z then it also returns to the same position and the orbit is periodic. We therefore consider 1440 initial velocity values, within the allowed range, and look for minimal change in v_y,v_z upon return to the emitter wall. Periodic orbits can be approximately located very quickly by this technique. The trajectory is then plotted graphically and the initial velocity can be adjusted so that there are no visible deviations from periodicity after fifty periods.

For $\theta=20°$ we have found accessible periodic orbits in which the electron traverses the well, colliding with each barrier in turn, as in Fig. 2(b), left inset. These traversing (t) orbits are topologically the same as the helical orbits at $\theta=0°$. It is only above a threshold voltage $\cong 100$ mV that these orbits have periods <0.6 ps. Above $V\cong 400$ mV the t orbits are inaccessible to the injected electrons since the Lorentz force prevents electrons returning to the emitter wall after just one collision with the collector wall. A topologically distinct closed orbit is now manifest, in which the electron makes two successive collisions with the collector wall for each collision with the emitter wall. These s orbits, which look star shaped in projection on the x-y plane [see right inset in Fig. 2(b)], have periods <0.6 ps above $V\cong 400$ mV and do not exist above a cutoff voltage $\cong 515$ mV, when the electron has a sufficiently large normal component of velocity v_x to surmount the collector barrier potential. These results are consistent with the observed sudden reduction in ΔV at $V\cong 350$ mV shown in Fig. 2(b), since the s orbits have almost doubled periods owing to the extra loop performed in the middle of the well between the two collisions with the collector barrier. For $\theta=25°$, only s orbits are accessible to the injected electrons and hence no sudden change in ΔV is observed, as in Fig. 2(c). The region of diminished oscillatory amplitudes around $V\cong 550$ mV corresponds to a small voltage range found theoretically where the electrons pass over the collector barrier.

To compare these theoretical results more quantitatively with experiment we estimate the voltage period ΔV of the oscillatory structure from the calculated orbit period T_P, using a simple model of the potential distribution across the structure. Since the tunneling probability is small, the emitter 2DEG is almost in thermal equilibrium with the weakly doped contact layer. The emitter Fermi level, which lies in the occupied Landau level, is then close to the conduction band edge as shown in Fig. 1. We define ϵ_n to be the energy of the nth level in the well, measured from the conduction band edge at the mean position $\langle x\rangle$ for this state. If the potential drop between the emitter Fermi level and this position $\langle x\rangle$ is V_1, the resonance condition is $eV_1=\epsilon_n$. Successive resonances correspond to an interval $\Delta V_1=\Delta\epsilon_n/e=h/eT_P$ in the chaotic regime. The corresponding interval ΔV is obtained using $V_1/V=(3a+b+\langle x\rangle)/L$, where $3a$ is the width of the 2DEG [15], and $L\cong 187$ nm is the effective electrical width of the structure given by capacitance measurements. Taking $\langle x\rangle=\frac{1}{2}w$ for s and t orbits then gives $f(V)=\Delta V_1/\Delta V\cong 0.4$ for 200 mV $\leq V\leq 700$ mV.

A comparison of experimental ΔV values with the values $\Delta V=h/feT_P$ obtained from the calculated periods T_P of the t and s orbits is shown in Fig. 3. In Fig. 3(a), at $\theta=20°$ ΔV increases with V for the t orbits as the electrons are accelerated to higher speeds in the well. The changeover from t to s orbits occurs at a slightly higher voltage than found experimentally and the reduction in ΔV is underestimated. We attribute these features to neglect of band nonparabolicity which would significantly increase the effective mass of the electron since its kinetic energy can reach several hundred meV. The predicted variation of ΔV with θ, shown in Fig. 3(b), is also consistent with experiment. For small θ the t orbits are helixlike about the magnetic field axis, as in Fig. 3(b), so the total path length and hence T_P increase with θ. This leads to ΔV values which decrease smoothly with increasing θ. For s orbits the ΔV values are broadly independent of θ and are much smaller than those for t orbits reflecting the change in topology.

The experiments may be used to construct domains in θ-V space within which the t- or s-type resonances are observed. Theoretically, these domains are defined by the accessibility of the unstable t or s orbits to the tunneling

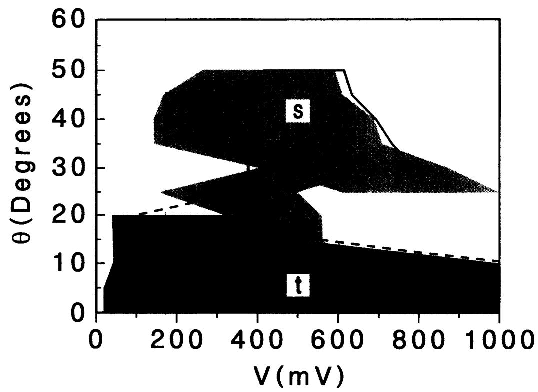

FIG. 4. θ-V domains within which t series (heavy shading) and s series (light shading) of resonances are observed experimentally. Black region indicates overlap. Theoretical domains are deduced from unstable t orbits (broken line) and s orbits (solid line).

electrons and the limitations imposed by scattering and by escape over the collector barrier. A comparison is made in Fig. 4 where, despite the simplicity of our model, there is a remarkable similarity between the shapes and sizes of the observed and predicted domains. This strongly supports our theory of the origin of the oscillatory structure and identification of the unstable orbits.

Finally, we discuss the widely spaced oscillations in the d^2I/dV^2 vs V plots for $60° < \theta < 90°$ (Fig. 2), which we associate with skipping motion along the emitter-barrier interface. At $\theta = 90°$ [Fig. 2(f)] the skipping orbits are stable and successive minima arise from tunneling into the lowest discrete skipping levels [11,12]. For $\theta < 90°$ we have located closed orbits which exhibit skipping motion at both barriers, shown in Fig. 2(e) for $\theta = 80°$. The tunneling density of states will be dominated by the almost periodic motion at the emitter barrier, which accounts for the similarity of the oscillatory structure in Figs. 2(e) and 2(f). Modulation of the density of states associated with the long period of the complete closed orbit is not resolvable.

In summary, we have used resonant tunneling spectroscopy to study the manifestations of classical chaos in the electronic states of a trapezoidal potential well subjected to a tilted magnetic field. The important role of unstable periodic orbits in understanding the structure of the energy spectrum has been emphasized and we have identified the relevant closed orbits in the chaotic part of phase space. The orbits should lead to a "scarring" of the eigenfunctions [4] accessible to the tunneling electrons but numerical calculations of wave functions in the regime of strong classical chaos have not yet been made for this system.

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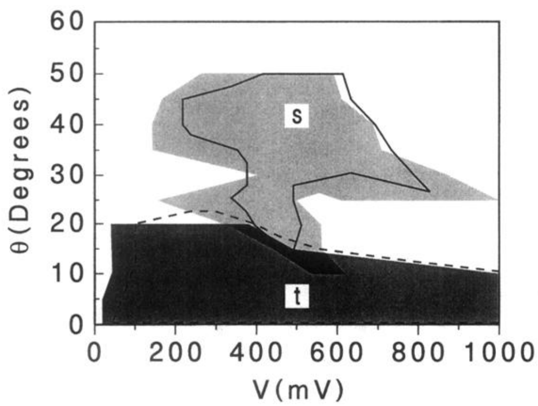

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