Formation of Solitonic Stars through Gravitational Cooling

Edward Seidel[1] and Wai-Mo Suen[2]
[1]National Center for Supercomputing Applications, Beckman Institute, 405 N. Mathews Avenue, Urbana, Illinois 61801
[2]McDonnell Center for the Space Sciences, Washington University, St. Louis, Missouri 63130
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We studied the formation of compact bosonic objects through a dissipationless cooling mechanism. Implications of the existence of this mechanism are discussed, including the abundance of bosonic stars in the Universe, and the possibility of ruling out the axion as a dark matter candidate.

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It has long been known that there exist compact self-gravitating solitonlike equilibrium configurations of bosonic fields [1]. The recent surge of interest in these solitonic objects is largely due to the suggestion that the dark matter could be bosonic in nature. Many particle theories predict that weakly interacting bosons are abundant in the universe [2], and may have played a significant role in the evolution of the Universe.

There are two types of self-gravitating compact solitonic objects made up of bosonic fields known as boson stars [1] and oscillating soliton stars [3] (oscillatons). In many ways, they are similar to neutron stars. The simplest example of a boson star is made up of a complex massive Klein-Gordon scalar field, with no self-interaction except through gravity:

$$\Box\phi+m^2\phi=0, \quad G_{\mu\nu}=8\pi T_{\mu\nu}(\phi)\,, \tag{1}$$

where $T_{\mu\nu}$ is the usual stress energy given by ϕ and its derivatives. The total mass of a boson star described by Eq. (1) ranges from 0 to a maximum of $M_c=0.633\times M^2_{\rm Planck}/m$, which is typically smaller than stellar mass. However, if a quartic self-coupling term is included, even for a small coupling constant, the mass can be comparable to a neutron star [4].

The major difference between a boson star and an oscillaton is that the latter is made up of a bosonic field without a conserved current. The simplest example of an oscillation is made up of a real massive Klein-Gordon field, again described by Eq. (1), but now with ϕ being a real field. The nonlinearity of these coupled Einstein-Klein-Gordon equations again gives rise to a nontopological solitonic solution. However, unlike the complex field case, this soliton solution is time dependent, with both the spacetime geometry $g_{\mu\nu}$ and ϕ oscillating in time, in a way similar to the breather solution to the sine-Gordon equation. The existence of a solitonic solution in this case is particularly interesting, because the axion, being a real (pseudo) scalar field, is one of the most promising candidates of dark matter [5]. When the axion's coupling to other matter fields and self-couplings can be neglected when the density is low, it is described by Eq. (1).

However, the existence of a solitonic solution is not a guarantee that such an object can actually be formed in the Universe. It is well known [6] that for self-gravitating fields described by Eq. (1), there is a gravitational instability analogous to the Jeans' instability. However, if there is no efficient cooling mechanism to get rid of the excess kinetic energy, as is apparently the case for a system described by Eq. (1), collapse initiated by this instability generally only leads to a diffuse viralized "cloud," but not a compact object. This is an outstanding question in the study of the bosonic compact objects.

It is of particular interest to look at the case of axions. It is generally believed that axions, having no effective cooling mechanism, remain in the form of an extended gas cloud after separation from the Hubble flow [7,8]. Hogan and Rees [8] argued that the initial isocurvature perturbation leads to "axion miniclusters" of mass $\sim 10^{27}$ g, with diameters of order 10^{14} cm, which subsequently undergo classical hierarchical clustering.

In this paper we show that there *is* a dissipationless cooling mechanism which very efficiently leads to the formation of compact bosonic objects. This mechanism, which we call gravitational cooling, is similar to the violent relaxation of collisionless stellar systems [9]. A collisionless stellar system, quite independent of the initial conditions, collapses to a centrally dense system by sending some of the stars to large radius, and settles into an equilibrium configuration with a more or less definite distribution. Likewise, quite independent of the initial conditions, a scalar field configuration described by Eq. (1) will collapse to form a compact soliton star (boson star for the complex field case, oscillaton for the real field case), by ejecting part of the scalar field, carrying out the excess kinetic energy. In retrospect, it should not be surprising that there is such a cooling mechanism similar to the violent relaxation. The evolution of a massive scalar field under its self-gravity is in many ways similar to that of ordinary material bodies, e.g., in the Jeans' instability analysis [6]. Perturbed boson stars and oscillatons can evolve back to their equilibrium configurations by radiating part of the scalar field [3,10]. Furthermore, such scalar radiation can drive the equilibrium configurations on the unstable branch to the stable branch [10].

On the other hand, gravitational cooling is different from violent relaxation in many ways. Gravitational cooling is (1) efficient even when the ratio of the initial kinetic to potential energy is large, and (2) much more thorough as a relaxation process, ending in a unique final state independent of the initial conditions. These features

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will be demonstrated below, where we present results of a numerical study of the cooling mechanism.

The immediate implication of the existence of the dissipationless gravitational cooling mechanism is that bosonic solitonic stars can be formed in our Universe. If the dark matter is described by a classical bosonic field, it could be made up of such stars, provided that the localized clouds separated from the Hubble flow are not too massive (otherwise a black hole may form). These clouds will begin to collapse under their own self-gravity, at first in free fall. Then, as nonlinear gravitational effects become important at higher densities, the gravitational cooling process becomes important, allowing the configuration to settle into a compact bosonic object.

The abundance of such solitonic stars with astrophysical mass but microscopic size has interesting consequences on galaxy formation, microwave background power spectrum, galaxy dynamics, and formation of the first stars, among other things. A particularly intriguing possibility is that such a mechanism may rule out the axion as a dark matter candidate. The axion miniclusters of Hogan and Rees [8] have a density of 10^7 g/cm^3. At such a density, the annihilation ($AA \rightarrow \gamma\gamma$) and other dissipative processes are not effective [7], and the evolution is accurately described by Eq. (1). Therefore the gravitational cooling mechanism studied here is applicable to the case of axion miniclusters. For axions with mass $m \sim 10^{-5}$ eV, the maximum oscillaton mass is 10^{28} g, which is bigger than the total mass of a minicluster. Hence one might expect that the end point of the collapse is an oscillaton (but not a black hole). If there is no further fragmentation during the collapse of the minicluster, the resulting oscillaton has a density of $\rho = 10^{24}$ g/cm^3 (phase space number density $n_p \simeq 10^{62}$ cm^{-3}). However, at such high density, the axion is no longer described by a free Klein-Gordon field. In particular, we expect the stimulated decay of axions to be important (single axion decay rate is extremely small, $r \sim 10^{-49}$ sec^{-1}, for $m_a \sim 10^{-5}$ eV). The amplification coming from stimulated decay gives a factor of $\exp(D)$, with [11]

$$D \sim \frac{\Gamma_\pi M_P^2 V_e}{R m_\pi^4} \frac{f_\pi}{f_a}, \tag{2}$$

where $\Gamma_\pi \sim 8$ eV, $f_\pi \sim 1$ GeV, $f_a \sim 10^{12}$ GeV (for $m_a \sim 10^{-5}$ eV), $m_\pi = 135$ MeV, V_e is the escape velocity ($\sim \sqrt{2GM/R}$), and R the radius of the bound object. For an oscillaton with the mass of a minicluster, $D \sim 10^7$, which simply implies that the collapse driven by the gravitational cooling ends up in a bright flash [12]. This suggests that it might not be self-consistent to have the axions as a dark matter candidate: The axion has a tendency to form compact objects (oscillatons) in a short dynamical time scale, but it is unstable in such a state.

Here we present the results of a numerical study of the gravitational cooling mechanism for the Klein-Gordon scalar fields, described by Eqs. (1) (see Refs. [3,10] for

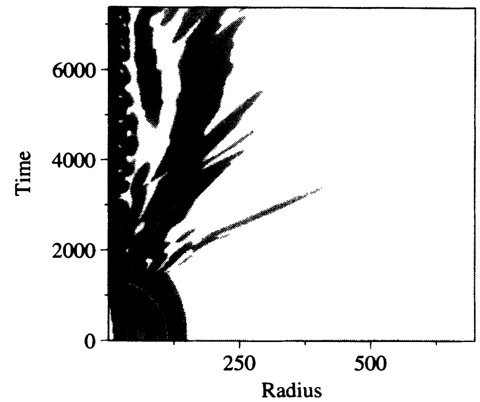

FIG. 1. The evolution of $r^2\rho$ for a massive, self-gravitating complex scalar field is shown. Because of the self-gravitation, the field collapses quickly and a perturbed boson star is formed at the center. The star oscillates and begins to settle down as scalar material is radiated through the gravitational cooling process discussed in the text.

the full set of equations and a description of the numerical methods.) Spherical symmetry is assumed throughout, as our aim is to demonstrate the existence of the cooling mechanism but not the detailed modeling of the formation of a soliton star in the astrophysical environment. We do not expect the spherical evolution to reflect the details of a realistic collapse, as processes like fragmentation, formation of a pancake, etc., have been left out. A study of the 3D case is under way.

In Fig. 1, we show the evolution of a coherent complex Klein-Gordon scalar field with self-gravitation. The initial configuration is taken to be a Gaussian distribution with $\phi = 0.0025 e^{-r^2/(90)^2}$ and $\dot{\phi} = 0.9 i \phi$ (all lengths are in units of $1/m$; m is the mass of the scalar field). Darker areas represent higher field strength. We see that the field collapses and settles down to a bound state by ejecting part of itself at each bounce. The ejected "scalar radiation" is shown as the black strips to the upper right hand corner. As time goes on, the ejection is less energetic, and the strips becomes more vertical as they are emitted with a smaller speed. By about $t = 4000(1/m)$, which is about 4 times the free falling time of the initial configuration, it settles down into a perturbed boson star on the stable equilibrium branch [10] with a total mass of about $0.56 M_P^2/m$. Both the field distribution and the oscillation frequency match those of a boson star with the same mass studied in Ref. [10]. The overall scalar field ejected by the gravitational cooling process is given by $(M_{\text{initial}} - M_{\text{final}})/M_{\text{initial}} \simeq 0.13$. We note that the initial mass M_{initial} of this configuration was $0.644 M_P^2/m$, which is greater than the maximum mass of a stable boson star. Without the mass loss due to scalar radiation this configuration can only form a black hole.

To see that an initial collapse does not necessarily lead to the formation of a compact object, we show the evolution of a *massless* scalar field with the same initial condi-

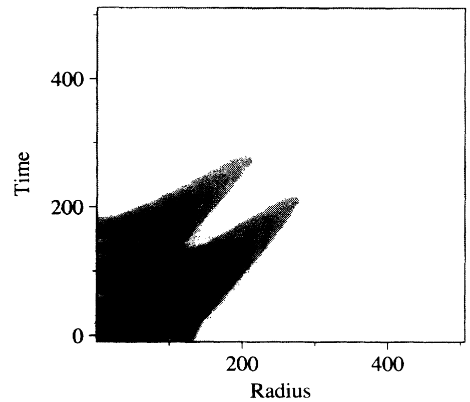

FIG. 2. To contrast the formation process for a massive field, the evolution of the energy density ρ for a massless, self-gravitating complex scalar field is shown. No boson star can form for a massless field.

tion. The field collapses, rebounces, and completely disperses to infinity, as shown in Fig. 2. No nonsingular self-gravitating solitonic object can be formed with a massless Klein-Gordon scalar field [13]. To show that the cooling process is due to nonlinear effects of the self-gravitation of the field, we have also studied evolutions with the self-gravitation turned off by setting the gravitational constant G to zero. In such cases the field simply disperses away, and no compact object is formed.

The above evolutions are with the fully relativistic systems. To better compare the gravitational cooling to violent relaxation, which is studied usually in Newtonian terms, we take the weak field post-Newtonian expansion of Eq. (1). The leading order equations in the expansion describe a Schrödinger type field under its own self-gravitation [10]. The kinetic energy K and the potential energy W of the configuration can be computed, with the total energy $E=K+W$ a conserved quantity.

We begin with an initial configuration $\phi=0.00045 \times e^{-(r/210)^2}[1+0.5\sin(\pi/15r)](1+i)$. This is spatially much larger than the previous case, with a strong spatial perturbation on the initial packet. In spite of the very different initial conditions, the subsequent evolution is similar. A boson star is formed with a smaller total mass. In Fig. 3, we plot the change of the total energy in time. The energy is positive and remains a constant until the scalar radiation comes out from the boundary of the grid at $r=1500(1/m)$. After this time it settles down to a bound state with negative binding energy (Fig. 3 inset). The ratio of the kinetic energy K to potential energy W is shown as a solid line with a scale given by the left vertical axis. We see that initially $K/|W|$ is larger than 1. The ratio decreases as the outward traveling scalar radiation with large kinetic energy is emitted from the system in the gravitational cooling process. It is interesting to compare this to the violent relaxation, which is inefficient for an initial value of $K/|W|$ so large, as shown by van Albada [14] (albeit in a 3D case). Also in Fig. 3, the total

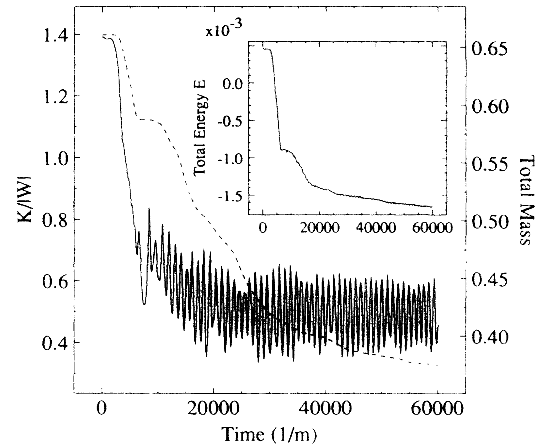

FIG. 3. The evolution of a Newtonian scalar field configuration is shown. The ratio $K/|W|$ of the kinetic to potential energy is shown as a solid line, and the total mass of the system is shown as a dashed line. The inset shows the evolution of the total energy $E=K+W$. Initially this configuration is unbound with positive E, but as part of the field is radiated away the energy of the remaining configuration becomes negative as a bound, boson star is formed.

mass of the system is shown as the dashed line. The gravitational cooling process emits about 55% of the initial mass by the end of the simulation ($t=60000$). This configuration had an initial radius of about 10 times that of the final compact object. We have also performed simulations with much larger configurations, with initial radii 1000 times that of the final configuration (a ratio of 10^{-9} in density). In all cases the result is the same, initial free fall followed by gravitational cooling and the formation of a compact bosonic object [15].

The above cases are all with a complex field. To be relevant to the axion (and other bosonic dark matter candidates described by real fields, e.g., pseudo Higgs [16]), we show in Fig. 4 the evolution of a real scalar field with the same initial data profile as in Fig. 1. (In this case, as the field is real, we chose $\dot{\phi}=0$.) The gravitational cooling process is the same. The difference is that the final state has intrinsic oscillations [3], i.e., it is an oscillaton instead of a boson star.

We demonstrated that there is a dissipationless cooling mechanism which is efficient in forming compact soliton objects out of coherent bosonic field configurations. The mechanism is not specific to a particular kind of field: There is no need to introduce any particular coupling or damping mechanism. As long as the field is massive, its self-gravitational interaction provides the relaxation towards a bound state. In the self-generated time-dependent gravitational potential, energy can be transferred to a part of the field, ejecting it out of the system, carrying with it the excess kinetic energy, in a way similar to the violent relaxation of stellar systems.

The existence of this gravitational cooling mechanism means that if the dark matter is described by a coherent

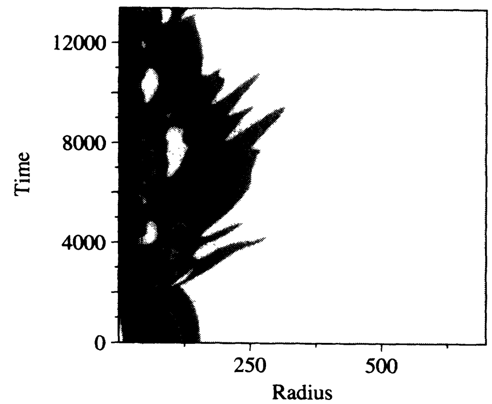

FIG. 4. The evolution of $r^2\rho$ for a massive, self-gravitating real scalar field is shown. The evolution is very similar to that shown in Fig. 1 for a complex configuration with the same initial profile for ϕ.

bosonic field, it is very possible for it to be in the form of soliton stars. In turn it implies that those bosonic particles that are unstable when the density is high are likely to be ruled out as dark matter candidates.

The biggest restriction in our present study is that it is spherically symmetric, and hence cannot address the question of fragmentation at the later stage of collapse (when the pressure is large), which will be the subject of a 3D numerical study [15]. With fragmentation, the gravitational cooling mechanism should lead to more than one solitonic object with smaller masses, starting from one collapsing cloud. These objects with smaller masses have larger radius and hence lower density. This leads one to wonder if the axion can be saved by fragmentation [17]. We note that for an axionic oscillaton to be nonluminous, D in Eq. (2) cannot be much larger than 1 [11]. This means that the maximum mass of axionic oscillatons as dark matter can only be of order 10^{24} g, having a radius 10^4 cm. Hence in order to avoid being ruled out as a dark matter candidate, a single minicluster of Hogan and Rees [8] has to be broken into so many pieces that even after a cosmological time scale of merging, there are still more than 10^3 oscillatons left. That is not likely to happen in the generic case.

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Note added.—Kolb and Tkachev [18] have recently proposed a different mechanism for forming axionic stars.

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[15] One might ask how going away from spherical symmetry may affect the picture described here. We note that the scalar field is basically evolving according to the free Klein-Gordon equation with a background metric determined by the total mass of the system. The field can be decomposed into spherical Y_{lm} components. Each component basically evolves independently in the background metric. Our spherical study shows that the $l=0, m=0$ component, which is the most significant piece describing the initial configuration just separated out from the Hubble flow, will have a large collapse ratio. If we depart from perfect spherical symmetry the presence of some small amplitude, high lm components, which do not collapse, will not affect the overall picture of the bulk part of the collapse. However, the assumption of spherical symmetry is significant in the late stage of the collapse. We expect fragmentation of the collapsing originally spherically symmetric object to occur when pressure becomes large through a nonradial instability. See, e.g., C. Hunter, Astrophys. J. **136**, 594 (1962); **139**, 570 (1964). This implies more than one boson star will be formed through the gravitational cooling process in the generic situation, a subject presently being studied.

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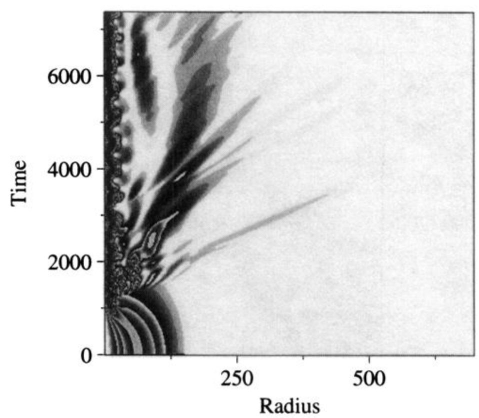

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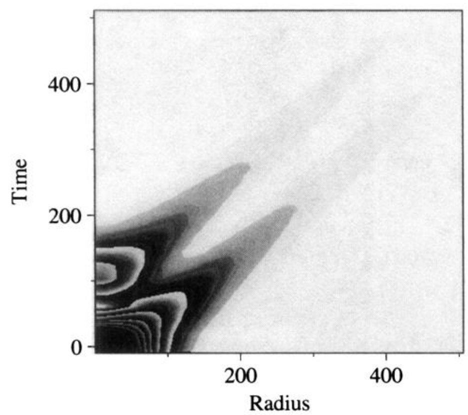

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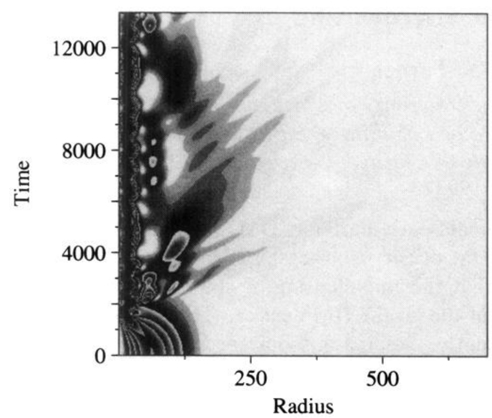

FIG. 4. The evolution of $r^2\rho$ for a massive, self-gravitating real scalar field is shown. The evolution is very similar to that shown in Fig. 1 for a complex configuration with the same initial profile for ϕ.