# Vortex Glass Phase and Universal Susceptibility Variations in Planar Arrays of Flux Lines

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Some of the thermodynamic properties of the low temperature vortex-glass phase of randomly pinned flux lines in 1+1 dimensions are studied. The flux arrays are found to be sensitive to small changes in external parameters such as the magnetic field or temperature. These effects are captured by the variations in the magnetic response and noise, which have universal statistics and should provide an unambiguous signature of the glass phase.

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Flux lines in a clean type-II superconductor form an Abrikosov lattice at low temperatures [1]. However, the flux lattice is destroyed by random microscopic impurities in the material [2]. Recently, it has been suggested that the disordered flux array may form a new thermodynamic phase at low temperatures, called the "vortex glass" phase, in which flux lines are *collectively pinned* by the impurities [3–5]. Although a glasslike behavior has been reported experimentally [6], a quantitative theoretical description of the vortex-glass phase is still lacking except in the special case of flux lines confined to a plane (1+1 dimensions). As first shown in Ref. [3], such a 1+1 dimensional flux array undergoes a phase transition at a finite temperature $T_g$. Below $T_g$, the flux array is pinned by the random impurities and forms a glass phase. However, the properties of the glass phase have not yet been elucidated, and a number of contradictory results exist in the literature [7–13]. In this paper, we analyze the vortex glass phase using the renormalization-group method of Cardy and Ostlund [14]. We find the glass phase to be characterized by anomalous variations in the magnetic responses of the flux array, and extreme sensitivity to small changes in the applied field, impurity potential, and temperature. Such glassy behavior has been previously conjectured for spin glasses [15] and one flux line [16–18], and is expected to be generic to a wide class of randomness dominated phases. However, the 1+1 dimensional flux array is one of the very few systems where analytic results can be obtained.

We consider an array of flux lines confined to the $(x,z)$ plane, with an applied field $\mathbf{H}=H_z\hat{\mathbf{z}}$ and repulsive interactions which we model by linear elasticity [19,20]. Impurities yield a random potential $V(x,z)$. Labeling the transverse displacement of the $n$th line by $r_n(z) =[n-\phi_n(z)/2\pi]/\rho$, where $\rho\sim H_z$ is the average line density, we can describe the large scale fluctuations of the flux array by the Hamiltonian [3,7,8,20]

$$\mathcal{H}=\int dx\,dz\left\{\frac{\kappa}{2}[(\partial_x\phi)^2+(\partial_z\phi)^2]-V(x,z)\partial_x\phi \right.$$
$$\left. -4\pi\rho V(x,z)\cos[2\pi\rho x+\phi]\right\}, \qquad (1)$$

where $\phi(x,z)$ is the coarse-grained displacement field. In Eq. (1), the $x$ and $z$ dimensions have been rescaled to make the quadratic part isotropic. The elastic coefficient $\kappa\sim(d\rho/dH)^{-1}$ is weakly temperature dependent. The cosine term in Eq. (1) comes from the invariance of the system to an overall shift in the labeling index $n$ of the lines. It picks out the *discrete* nature of the flux lines and is crucial to the formation of a glass phase. The model (1) also describes the phase fluctuation of Josephson vortex lines in a planar Josephson junction as derived in Ref. [21].

Upon renormalization, one generates a term of the form $V'(x,z)\partial_z\phi$, which randomly biases the local *tilt* of the flux lines. It is found that the variances of $V$ and $V'$ are renormalized in the same way, so that the inherent spatial anisotropy and frustration present in Eq. (1) disappear at large length scales. It is then more convenient to work with the isotropic Hamiltonian

$$\mathcal{H}[\phi]=\int_{\mathbf{r}}\left\{\frac{\kappa}{2}(\nabla\phi)^2-\boldsymbol{\mu}\cdot\nabla\phi-W(\phi(\mathbf{r}),\mathbf{r})\right\}, \qquad (2)$$

where $\mathbf{r}=\{x,z\}$, $\boldsymbol{\mu}(\mathbf{r})=V(\mathbf{r})\hat{\mathbf{x}}+V'(\mathbf{r})\hat{\mathbf{z}}$ is Gaussian distributed with mean zero and variance of the component $\mu_i$

$$\overline{\mu_i(\mathbf{r})\mu_j(\mathbf{r}')}=\sigma\delta_{ij}\delta^2(\mathbf{r}-\mathbf{r}'), \qquad (3)$$

and $W(\phi,\mathbf{r})\propto\cos[\phi(\mathbf{r})-\beta(\mathbf{r})]$ is a random potential, describing the effect of a random phase $\beta(\mathbf{r})$, with

$$\overline{W(\phi,\mathbf{r})W(\phi',\mathbf{r}')}=2g\cos[\phi-\phi']\delta^2(\mathbf{r}-\mathbf{r}'). \qquad (4)$$

Denoting the bare parameters by the subscript 0, we have $g_0\sim\sigma_0\rho^2$. A renormalization-group analysis [14,22] yields the recursion relations under a change of scale by $b=e^l$,

$$d\kappa/dl=0, \qquad (5)$$

$$d\sigma/dl=Ag^2, \qquad (6)$$

$$dg/dl=\varepsilon g-Cg^2. \qquad (7)$$

The coefficients $A$ and $C$ are cutoff dependent, however the ratio $A/(\kappa C)^2=2\pi+O(\varepsilon)$ is *universal.* Equations (6) and (7) are valid to leading order in $g$, which will be sufficient provided

$$\varepsilon\equiv 2-T/2\pi\kappa \qquad (8)$$


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is small. This is true even though $\sigma$ can flow to large values by Eq. (6), because the random potential $\boldsymbol{\mu}$ in Eq. (2) can be shifted away by the transformation $\phi'(\mathbf{r}) = \phi(\mathbf{r}) - u(\mathbf{r})$, with $\kappa\nabla^2 u = \nabla\cdot\boldsymbol{\mu}$, regardless of the magnitude of $\boldsymbol{\mu}$. The resulting potential $W'(\phi',\mathbf{r}) = W(\phi'+u,\mathbf{r})$ has the same statistics as $W(\phi,\mathbf{r})$ since the latter is uncorrelated in $\mathbf{r}$. Consequently, the flow of $g$ cannot be affected by $\boldsymbol{\mu}$. A similar use of the statistical symmetry of $\mathcal{H}$ shows that the result that $\kappa$ is unrenormalized in Eq. (5) is *exact* [22,23].

Clearly, $\varepsilon = 0$ is a special point; it defines a critical temperature $T_g = 4\pi\kappa$ through Eq. (8). For $T > T_g$ where $\varepsilon < 0$, $g$ renormalizes to zero and at long scales the system is described by the Gaussian part of the Hamiltonian (2) with a *finite* renormalized $\sigma$. This is the flux liquid phase with the disorder causing only short wavelength modifications of the pure system [19]. But for $T < T_g$ where $\varepsilon > 0$, there is a nontrivial phase controlled by a fixed line $g^*(T)$, with $\sigma$ renormalizing as in Eq. (6). Close to the transition, we have $g^* \approx \varepsilon/C \propto (T_g - T)$, and on scale $L$, $\sigma(L) \approx A(g^*)^2 \log \rho L$. This is a vortex glass phase [24].

The existence of a perturbatively accessible fixed line allows us to study the thermodynamic properties of the vortex glass phase quantitatively. The nonrenormalization of $\kappa$ implies a simple form for the mean square thermal fluctuations of $\phi$, i.e., $\overline{\langle[\phi(\mathbf{r}) - \phi(\mathbf{r}')]^2\rangle_c} = T/(\pi\kappa)\log[\rho|\mathbf{r}-\mathbf{r}'|]$ the same as in the absence of randomness [23]. The glass phase is instead distinguished by more strongly divergent static distortions. For example, the mean square (thermally averaged) displacement is $\overline{\langle\phi(\mathbf{r}) - \phi(\mathbf{r}')\rangle^2} \approx 2\varepsilon^2 \log^2[\rho|\mathbf{r}-\mathbf{r}'|]$ due to the logarithmic divergence of $\sigma$ [22]. However, this is not a unique feature of a glass phase, as systems with long-range correlated $\boldsymbol{\mu}$'s can also give rise to anomalous mean square displacement even if $g = 0$, in which case the system is harmonic and trivial.

We therefore consider other thermodynamic quantities whose behavior is unique to a glass phase. We first study the magnetic response of the flux array. We change the applied external field by an amount $\delta\mathbf{H} = \delta H_x \hat{\mathbf{x}} + \delta H_z \hat{\mathbf{z}}$, which tends to compress and/or rotate the flux array. For an isotropic system, the Hamiltonian becomes

$$\mathcal{H}_{\mathbf{h}}[\phi] = \mathcal{H}[\phi] - \int_{\mathbf{r}} \mathbf{h}\cdot\nabla\phi\,, \tag{9}$$

where $\mathbf{h} = (\delta H_z \hat{\mathbf{x}} + \delta H_x \hat{\mathbf{z}})\Phi_0/8\pi^2$, $\Phi_0$ being the magnetic flux quantum. The change in the flux density is $\langle\partial_x\phi\rangle/2\pi$, and in the "tilt angle" is $\langle\partial_x\phi\rangle/2\pi\rho$. The linear response on which we focus is $\chi_{i,j} \equiv (\partial/\partial h_i)\langle\partial_j\phi\rangle$. For the isotropic system (2), we have $\chi_{ij} = \chi\delta_{ij}$, and the magnetic permeability is just $(\Phi_0^2/16\pi^3)\chi$. Simple rescaling yields a similar result for the anisotropic system.

Consider the high temperature phase where discreteness is irrelevant, i.e., $g = 0$. Then the last term in (9) can be simply shifted away by the transformation

$$\phi'(\mathbf{r}) = \phi(\mathbf{r}) - \mathbf{h}\cdot\mathbf{r}/\kappa\,, \tag{10}$$

yielding a free energy

$$F(\mathbf{h}) = \frac{h^2}{2\kappa}L^2 - \frac{1}{\kappa}\int_{\mathbf{r}} \mathbf{h}\cdot\boldsymbol{\mu}\,, \tag{11}$$

and hence a response $\overline{\chi} = 1/\kappa$. Since the random part of $F(\mathbf{h})$ is linear in $\mathbf{h}$, the response will be *sample independent* as in a pure system, with $\overline{(\Delta\chi)^n} = 0$ for $n > 2$ where $\Delta\chi \equiv \chi - \overline{\chi}$. This is solely a consequence of the quadratic nature of the Hamiltonian with $g = 0$.

The magnetic response in the low temperature phase, where the random phase term in (2) is relevant, is much more interesting. Since the transformation (10) does not change the statistics of the Hamiltonian $\mathcal{H}$ [22,23], except for generating an extra quadratic term as in Eq. (11), the quenched-averaged free energy $\overline{F(\mathbf{h})}$ is the *same* as for $g = 0$. Thus, $\overline{\chi} = 1/\kappa$ independent of $g$. Furthermore, the average of higher order nonlinear susceptibilities $\overline{(\Delta\chi)^n}$ all vanish due to the statistical symmetry [23]. Thus average response functions are identical in the glass and liquid phases. This result has led some to mistakenly doubt the existence of the glass phase [12]. However we will show that the glassy effects are manifested in sample-to-sample variations of the susceptibility and its extreme sensitivity to small perturbations.

Let us compute the effect of the random potential $W$ on the susceptibility variation, perturbatively at first. After the transformation (10), the correlations between the free energy at two different fields $\mathbf{h}_1$ and $\mathbf{h}_2$ can be calculated to the lowest order in $g$. For $\Delta F(\mathbf{h}) \equiv F(\mathbf{h}) - \overline{F(\mathbf{h})}$, we have

$$\overline{\Delta F(\mathbf{h}_1)\Delta F(\mathbf{h}_2)} = 2g(\rho L)^{-T/2\pi\kappa}\int_{\mathbf{r}} \cos[(\mathbf{h}_1 - \mathbf{h}_2)\cdot\mathbf{r}/\kappa] \tag{12}$$

for a system of size $L \times L$, with the $(\rho L)^{-T/2\pi\kappa}$ factor arising from averaging over thermal fluctuations. Differentiating with respect to $\mathbf{h}_1$ and $\mathbf{h}_2$ leads to nontrivial sample-to-sample variations of the magnetic susceptibility, with variance

$$\overline{(\Delta\chi)^2} = (Dg/\rho^2\kappa^4)(\rho L)^{\varepsilon} \tag{13}$$

to first order in $g$, with $D$ being a sample-geometry dependent coefficient.

For $T > T_g$ (i.e., $\varepsilon < 0$), Eq. (13) gives the form of approach to the asymptotic liquid phase where $\overline{(\Delta\chi)^2} = 0$ as discussed above. For $T < T_g$, $\overline{(\Delta\chi)^2}$ diverges since $\varepsilon > 0$, indicating the failure of the small-$g$ expansion. Equation (13) does, however, suggest the form of the correct behavior: The term $g(\rho L)^{\varepsilon}$ should just be replaced by the renormalized $g_R(L)$. Explicit computation shows that this is indeed the case. For large systems in the vortex glass phase, $g_R(L) \to g^*$ hence we obtain a fractional variance

$$\frac{\overline{(\Delta\chi)^2}}{\bar{\chi}^2} \approx \frac{Dg^*}{\rho^2\kappa^2} = \hat{D}\varepsilon \tag{14}$$

with $\hat{D}$ a *universal* geometry and boundary condition dependent coefficient, that is independent of nonmeasurable bare parameters such as $g_0$. For a $L_x \times L_z$ rectangular section of a large isotropic sample, we have $\hat{D} \approx (8\pi/5)a^3$, where $a = L_x/L_z$ for the longitudinal susceptibility and $a = L_z/L_x$ for the transverse susceptibility. The large sample-to-sample variations of $\chi$ indicate that the vortex glass phase is radically different from the fluid phase [25]. The size-independent variations of $\chi$ in the vortex glass phase are reminiscent of "universal conductance fluctuations" in disordered metals [26].

Experimentally, variations of $\chi$ may be obtained by measuring the magnetic response of *one* sample at different applied fields $\mathbf{H}$. It will be particularly convenient to keep $|\mathbf{H}|$ and $T$ fixed, and follow the response as the *direction* $\hat{\mathbf{H}}$ is changed. The variance $\overline{(\Delta\chi)^2}$ only depends on $T$ and $|\mathbf{H}|$ (through $\kappa$). Then as $\hat{\mathbf{H}}$ is changed, say by rotating a sample in a fixed field, it effectively samples different "realizations" of the random potential, drawn from the *same* distribution since systems with different field directions $\hat{\mathbf{H}}$ are *statistically equivalent* [27]. For a system of size $L \times L$, the free energies and hence the susceptibilities become uncorrelated if $\hat{\mathbf{H}}$ is changed by an angle much greater than $(\rho L)^{-1}$ as can be guessed from Eq. (12) with $\kappa \sim \Phi_0 H/\rho$. In the glass phase, we thus expect to obtain a wildly varying susceptibility $\chi(\hat{\mathbf{H}})$, whose precise form is a property of the specific sample, but with universal statistics, in particular, $\overline{(\Delta\chi)^2}/\bar{\chi}^2$. Alternatively, one could monitor the magnetic noise as a function of $\hat{\mathbf{H}}$. This should exhibit universal variations like $\chi(\hat{\mathbf{H}})$, since the two are related by the fluctuation-dissipation relation. The susceptibility variations at a fixed $T$ and $|\mathbf{H}|$ provide "magnetic fingerprints" of the glassy flux phase. The reproducibility of the magnetic fingerprint for the same sample under identical conditions provides a probe of thermal equilibrium on long scales: Only samples small enough to equilibrate fully (see below) will show reproducible behavior.

From the above discussion, it is evident that the equilibrium state of the flux array depends sensitively on small changes in the applied field. As argued in Refs. [15] and [18] on general grounds, a wide class of random systems can exhibit such sensitivity to small changes of a variety of parameters such as a field or temperature. Large variations resulting from small changes in the random potential $V(\mathbf{r})$ have been studied numerically by Zhang [16] for a single flux line. In the remainder of this paper, we analyze explicitly the effect of such a small change in the random potential for the $1+1$ dimensional flux array. Sensitivity of the array to small temperature changes can be analyzed similarly. We merely quote the analogous result for this somewhat more complicated case.

We consider two noninteracting flux arrays, $\phi(\mathbf{r})$ and $\tilde{\phi}(\mathbf{r})$, in two *different* realizations of the random potential, $\{\boldsymbol{\mu}(\mathbf{r}), W(\phi,\mathbf{r})\}$ and $\{\tilde{\boldsymbol{\mu}}(\mathbf{r}), \tilde{W}(\phi,\mathbf{r})\}$, respectively. We take the random potentials to be statistically equivalent but slightly different form each other, so that $\overline{\tilde{\boldsymbol{\mu}}(\mathbf{r})\tilde{\boldsymbol{\mu}}(\mathbf{r}')}$ is given by Eq. (3) and $\overline{\tilde{W}(\phi,\mathbf{r})\tilde{W}(\phi',\mathbf{r}')}$ is given by Eq. (4). However, the cross correlators are

$$\overline{\mu_i(\mathbf{r})\tilde{\mu}_j(\mathbf{r}')} = \hat{\sigma}\delta_{ij}\delta^2(\mathbf{r}-\mathbf{r}')\,, \tag{15}$$

$$\overline{W(\phi,\mathbf{r})\tilde{W}(\phi',\mathbf{r}')} = 2\hat{g}\cos[\phi-\phi']\delta^2(\mathbf{r}-\mathbf{r}')\,, \tag{16}$$

with the bare values $\hat{\sigma}_0 < \sigma_0$ and $\hat{g}_0 < g_0$. The renormalization-group recursion relations Eqs. (5)-(7) must be unchanged as the systems are uncoupled. However, the cross correlations renormalize as

$$d\hat{\sigma}/dl = A\hat{g}^2\,, \tag{17}$$

$$d\hat{g}/dl = \hat{\varepsilon}\hat{g} - Cg\hat{g}\,, \tag{18}$$

with $\hat{\varepsilon} \equiv \varepsilon + (\hat{\sigma} - \sigma)/2\pi\kappa^2$.

To investigate the effect of weak decorrelation of the random potentials in the glass phase, we linearize the recursion relations around the vortex glass fixed point $g^*$. For small decorrelation $\delta_\sigma = \sigma - \hat{\sigma} \ll 1$ and $\delta_g = g - \hat{g} \ll 1$, we have

$$\frac{d\delta_\sigma}{dl} = 2Ag^*\delta_g\,, \tag{19}$$

$$\frac{d\delta_g}{dl} = \frac{1}{2\pi\kappa^2}g^*\delta_\sigma\,. \tag{20}$$

This flow has one positive eigenvalue

$$\lambda_\delta \approx g^*\sqrt{A/\pi\kappa^2} = \sqrt{2\varepsilon}\,. \tag{21}$$

Therefore, infinitesimally small decorrelations in the bare random potential grow under renormalization. On long scales $L \gg L_\delta \sim \delta^{-1/\lambda_\delta}$, with $\delta$ a linear combination of $\delta\sigma$ and $\delta_g$, $\hat{g}(L)$ vanishes and $\hat{\sigma}(L)$ saturates. The two systems then appear substantially different and will have essentially *independent* susceptibilities, with $\overline{\Delta\chi(L)\Delta\tilde{\chi}(L)} \to 0$ for large $L$. There will, however, be residual cross correlations associated with the finite renormalization of $\hat{\sigma}$.

These effects can best be probed by changing the temperature of one sample slightly by $\delta T$. The same exponent $\lambda_\delta$ in Eq. (21) controls the crossover, and for system sizes $L \gg L_\delta(\delta T)^{-1/\lambda_\delta}$, $\chi(T)$ and $\chi(T+\delta T)$ will be roughly independent. If $\delta T \ll T_g - T$, the temperature dependence of $\chi$ will probe statistically similar variations of $\chi$ as did the field direction dependence of $\chi(\hat{\mathbf{H}})$.

Physically the source of the sensitivity to $\mathbf{H}$ and $T$ changes are quite different. The former is due to the changes in mean position of the lines while the latter is more subtle: It is caused by the *entropic* contributions to the free energy, which drastically changes the effective random potential on long scales. Although this has been predicted for a variety of random systems [15,18] and supported by numerical and approximate renormalization group calculations, this to our knowledge, is the first time an analytic calculation has yielded the hypersensitivity to temperature changes.

We close with a comment on dynamics: Recently, a number of authors have claimed that free energy barriers in this system grow as various powers of $\log L$ [7,8,13]. An explicit dynamic renormalization-group calculation [11,28] found that the dynamic exponent $z \approx 2+1.8\varepsilon$ for $T \leq T_g$, yielding a nonlinear resistivity, $dE/dJ^{0.9\varepsilon}$, where $E$ is the emf generated by a uniform current $J$ applied perpendicular to the $(x,z)$ plane. However, because the $1+1$ dimensional vortex glass phase is controlled by finite temperature fixed line rather than a zero temperature fixed point, the barriers are not well defined by the form $\varepsilon(J)$ found. In the limit $T \rightarrow 0$, however, one finds [29] $z \sim 1/T$, which can then be correctly interpreted as barriers growing as $\log L$.

The dynamics can also be used to probe the length dependence of $\chi$. At finite frequency, $\omega$, scales of size $l_\omega \sim \omega^{-1/z}$ are probed. Since the susceptibility $\chi(\omega)$ for each correlation volume $l_\omega^2$ will be essentially independent, the variations in $\chi(\omega)$ for a sample of size $L \times L$ will be $\Delta\chi(\omega) \sim l_\omega/L \sim \omega^{-1/z}/L$, crossing over to the static result only when $l_\omega \sim L$.

In this paper, we have analyzed some of the glassy properties of randomly pinned flux arrays confined to a plane. In the vortex glass phase, the magnetic susceptibility is found to be strongly dependent on the external field, temperature and specific sample, exhibiting variations with universal statistics.

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