Transitions between Hall Plateaus in the Presence of Strong Landau Level Mixing

Ziqiang Wang,[1] Dung-Hai Lee,[2] and Xiao-Gang Wen[3]

[1]*Center for Materials Science and Theoretical Division T-11, Los Alamos National Laboratory, Los Alamos, New Mexico 87545 and Department of Physics, Boston University, Boston, Massachusetts 02215*

[2]*IBM Research Division, T. J. Watson Research Center, Yorktown Heights, New York 10598 and Department of Physics, University of California, Berkeley, California 94720*

[3]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

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We study the effects of Landau level mixing on the critical properties of plateau transitions in the quantum Hall effect. Combining numerical results with analytical arguments, we conclude that for noninteracting electrons the universality class of the plateau transition is *unchanged* in the presence of a strong Landau level mixing.

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The plateau transition in the quantum Hall effect has attracted much attention recently [1–5]. In brief, the phenomenology of the plateau transitions is the following. If one sweeps the perpendicular magnetic field applied to a two-dimensional electron gas, a series of plateaus in σ_{xy} are observed [6]. At the same time, σ_{xx} exhibits pronounced Shubnikov–de Haas oscillations when the locations of its minima coincides with that of the plateaus. Between these plateaus, σ_{xy} interpolates between the quantized values, and σ_{xx} peaks in the middle of the transitions [6]. By analyzing the temperature (T) and sample size (W) dependence of the transition widths (ΔB), Wei *et al.* [7] and Koch *et al.* [8] have demonstrated experimentally that $\Delta B = B_0 \max[(T/T_0)^{1/z\nu}, (W/W_0)^{-1/\nu}]$, where B_0, T_0, and W_0 are nonuniversal magnetic field, temperature, and length, respectively. For a *spin-resolved*, integer-plateau transition $1/z\nu \approx 0.42$ [7] and $\nu \approx 2.3$ [8]. (In the rest of this paper we will concentrate only on integer-plateau transitions.) This scaling behavior indicates the existence of a continuous phase transition (hereto referred as the plateau transition) at $T=0$ [9]. The single diverging length at the transition $\xi \propto |B - B_c|^{-\nu}$ is the quasiparticle localization length.

Experimentally the Hall plateaus associated with opposite spin orientations are not always resolved. This situation usually occurs when the magnetic field is relatively weak or when the disorder is relatively strong [10,11]. In a transition between two spin-unresolved plateaus $\Delta\sigma_{xy} = \pm 2e^2/h$ and σ_{xx} shows a single peak. In a nice experiment Wei *et al.* [10] demonstrated that for a spin-unresolved transition $1/z\nu \approx 0.21$. Therefore, it raised the possibility that the spin-resolved and spin-unresolved transitions might belong to two *different* universality classes. In this paper we address this possibility by studying the effects of Landau level mixing on the critical properties of the plateau transition. (In this paper we use the phrases "Landau level index" and "S_z spin index," and "Landau level mixing" and "spin mixing" interchangeably.)

Theoretically, Levine, Libby, and Pruisken [12] made the first breakthrough in understanding the plateau transition. They pointed out that for *noninteracting* electrons the long wavelength "transport action" for the spin-resolved plateau transition is an $N \to 0$ $U(2N)/U(N) \times U(N)$ σ model *with a topological term.* (In the rest of the paper we also restrict ourselves to noninteracting electrons.) They conjectured that with the topological term, the theory possesses stable fixed points at $(\sigma_{xx}, \sigma_{xy}) = (0,n)e^2/h$ (corresponding to the quantized plateaus) and critical points at (const, $n+\frac{1}{2})e^2/h$ (corresponding to the critical points between the plateaus). Unfortunately, calculation of the critical properties (either analytical or numerical) using the σ model is extremely difficult [13].

Recently, considerable numerical progresses have been made in studying the spin-resolved plateau transition [1]. Among them the "quantum percolation" model of Chalker and Coddington [2] is particularly interesting. In that work the authors demonstrated an electron delocalization transition with $\nu \approx 2.5 \pm 0.5$. This result was recently confirmed and refined by Lee, Wang, and Kivelson [4] who obtained $\nu \approx 2.4 \pm 0.2$. More recently, one of us [5] proved that the network model can be mapped onto an antiferromagnetic SU($2N$) spin chain, whose coherent-state path-integral action is the σ model. As a result, it indirectly proves that the latter can produce a ν that is consistent with the experiments.

While a consistent picture is emerging for the spin-resolved transition, the experiment by Wei *et al.* [10] raised questions concerning the effects of spin mixing on the critical properties of the plateau transitions. In the present Letter we address this issue by combining analytical and numerical results. Our conclusion is that for *noninteracting* electrons, in spite of a strong mixing, the plateau transitions associated with each Landau level remain distinct and the universality class of each transition remains unchanged.

To simplify the problem, let us consider a situation in which two Landau levels (two spins) can mix. The appropriate network then has *two* edge states (one for each Landau level; see Fig. 1). The direction of the drift velocities for these states is the *same.* As discussed in Ref.

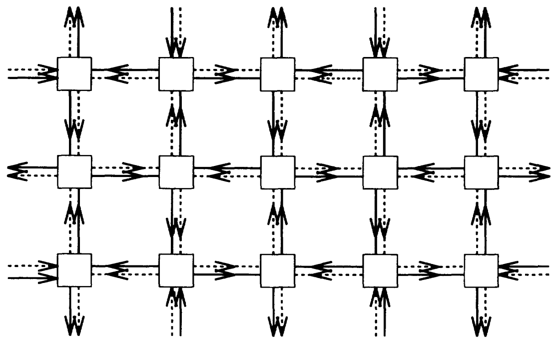

FIG. 1. The network model for spin-unresolved transition. The arrows on the solid and dashed links indicate the directions of the edge velocities, and the open squares (nodes) enclose the tunneling points.

[5], this network model can be mapped onto *two* coupled SU(2N) spin chains in the representation where each spin is characterized by a single column of the Young tableau with height N. The Hamiltonian is given by

$$\hat{H} = \sum_{x,\alpha} J_\alpha(x)\,\mathrm{Tr}[\hat{S}_\alpha(x+a)\hat{S}_\alpha(x)] - \sum_x J_{12}(x)\,\mathrm{Tr}[\hat{S}_1(x)\hat{S}_2(x)]\,. \tag{1}$$

Here a is the lattice constant, $\alpha = 1,2$ is the Landau level index, and $\mathrm{Tr}[\hat{S}\hat{S}'] \equiv \sum_{a,b}\hat{S}_a^b\hat{S}_b'^a$ where $\hat{S}_a^b(a,b=1,\ldots,2N)$ is the SU(2N) generator. In Eq. (1) all the J's are positive; hence the intrachain interaction is *antiferromagnetic* while the interchain one is *ferromagnetic*. Moreover, while J_{12} is translation invariant J_1 and J_2 are not. They satisfy $J_\alpha(x) = J_\alpha(x+2a)$. In this model J_α measures the strength of quantum tunneling between edge states that have the *same* Landau level index α, and J_{12} measures the strength of Landau level mixing. Following Ref. [5] we define

$$R_\alpha \equiv [J_\alpha(x+a) - J_\alpha(x)]/[J_\alpha(x+a) + J_\alpha(x)]$$

and

$$J_\perp \equiv 4[J_{12}(x)] \Big/ \left[\sum_\alpha J_\alpha(x+a) + J_\alpha(x)\right].$$

When $J_\perp = 0$ there are two transitions as we tune R_1 and R_2 while keeping $R_1 > R_2$. All three massive phases break the translation symmetry and correspond to three "spin-Peierls" phases [see Fig. 2(a)]. We identify these three phases with three adjacent plateaus, and the two transitions with the corresponding plateau transitions. For $J_\perp \gg 1$ (strong Landau level mixing) the vertical spin pairs couple to form the $M=2$ representation of SU(2N). At long wavelength and low energy the remaining degrees of freedom are those of a *single* $M=2$, SU(2N) antiferromagnetic chain. In this case there are also two transitions as we tune the ratio between the strengths of two nearest-neighbor bonds. Among the three massive phases, two break translation symmetry and they correspond to two spin-Peierls phases [see Fig. 2(b)]. The

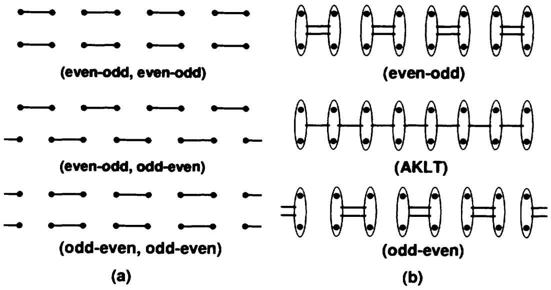

FIG. 2. The phases of two ferromagnetically coupled spin chains: (a) Three "spin-Peierls" phases at $J_\perp = 0$ and (b) two "spin-Peierls" phases and an AKLT phase for $J_\perp \gg 1$.

phase which remains translation invariant is the SU(2N) analog of the Haldane phase [14]. Thus the phase structure and the universality class of the phase transition are *unchanged* in the presence of a *strong* Landau level mixing.

This result can also be derived from the following considerations of the free electron Hamiltonian,

$$H = \int dx \left[-\frac{1}{2m}\psi_\alpha^\dagger(\partial_i + iA_i)^2\psi_\alpha + V(x)\psi_\alpha^\dagger\psi_\alpha + \mathbf{H}(x)\cdot\psi_\alpha^\dagger\sigma_{\alpha\beta}\psi_\beta \right], \tag{2}$$

where ψ_α is the spin-$\frac{1}{2}$ electron annihilation operator, $\epsilon_{ij}\partial_i A_j$ is the strength of the applied magnetic field, and $V(x)$ is the one-body random potential. The third term in Eq. (2) describes the Zeeman splitting/spin mixing. When $H_{x,y} = 0$ and $\langle H_z\rangle \neq 0$ the spin up and down electrons are decoupled and undergo *separate* plateau transitions. When $\langle H_z\rangle = 0$ these two transitions coincide. Our question is, If $\langle H_z\rangle = 0$ while $H_{x,y}$ are big, how many transitions are there, and what are their critical properties? For simplicity, let us consider $\mathbf{H}(x)$ as a random 3D vector with a large fixed norm. Let us write $\mathbf{H}(x) = |\mathbf{H}(x)|z^\dagger(x)\sigma z(x)$. Here $z(x) = (z_1(x), z_2(x))$ is a spinor field chosen so that $z^\dagger(x)\sigma z(x)$ lies in the x-y plane. After a local SU(2) gauge rotation $\psi \to U\psi$ with $U = \begin{pmatrix} z_1 & z_2^* \\ z_2 & -z_1^* \end{pmatrix}$, the Hamiltonian becomes

$$H = \int dx \left[-\frac{1}{2m}\psi^\dagger(\partial_i + iA_i + U^\dagger\partial_i U)^2\psi + V(x)\psi^\dagger\psi + |\mathbf{H}(x)|\psi^\dagger\sigma_z\psi \right]. \tag{3}$$

If we ignore $U^\dagger\partial_i U$, Eq. (3) reduces to the $H_{x,y} = 0$ case discussed above, and hence describes two well-separated transitions both belonging to the universality class of the spin-resolved transitions. When the direction of $\mathbf{H}(x)$ changes *slowly* in space, $(U^\dagger\partial_i U)_{\alpha\beta} \ll 1$. The diagonal part of $U^\dagger\partial_i U$ induces a small effective random magnetic field which is irrelevant in the presence of a strong uniform external field. The remaining off diagonal parts of

$U^\dagger \partial_i U$ have two effects: (a) They slightly renormalize $V(x)$ and $|\mathbf{H}(x)|$ and (b) they mix the two spin components. Thanks to the large $|\mathbf{H}(x)|$, both effects are irrelevant. (This is supported by our numerical results that a *weak* mixing is irrelevant in the presence of a strong Zeeman splitting.) Again, the conclusion is that strong Landau level mixing does not change the universality class of the plateau transitions.

To reconcile the above result with experiments, we present numerical results for the network model. We show that even for the *strongest* Landau level mixing, it is possible to mistake the case of two nearby transitions each having $\nu \approx 2.3$ for the case of a single transition with a significantly larger ν.

In Fig. 1 each link is associated with a 2×2 unitary matrix $U = \exp(i\phi)\exp(i\chi\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})$, where ϕ and χ are random phases in the interval $[0,2\pi)$, $\hat{\mathbf{n}}$ is a random three-component unit vector, and $\boldsymbol{\sigma}$ are the Pauli matrices. As an electron traverses a particular link, we multiply $\binom{\psi_1}{\psi_2}$, the probability amplitudes of finding an electron in the pair of edge states, by U. We found that it is sufficient to parametrize $\hat{\mathbf{n}} = (0, \sin\eta, \cos\eta)$. Moreover, we have checked that the randomness in the tunneling strength is irrelevant as in the single channel case [4]. Thus, we restrict the transfer matrices at all tunneling points to be the same, and have the form

$$T = \begin{pmatrix} T_1, & 0 \\ 0, & T_2 \end{pmatrix}, \quad T_a = \begin{pmatrix} \cosh\gamma_a, & \sinh\gamma_a \\ \sinh\gamma_a, & \cosh\gamma_a \end{pmatrix}. \tag{4}$$

In the actual calculations we let $\gamma_a = \gamma_c + (E - E_a)$ where $\gamma_c = \ln(\sqrt{2}+1)$, $|E_1 - E_2| \equiv \Delta$, and $-W/2 < \eta < W/2$. (Physically Δ should be identified with the Landau level separation or spin Zeeman splitting, while W is the Landau level mixing.) The case of maximum coupling corresponds to $W = 0.5\pi$. We follow the standard procedures of finite size scaling analysis [2,4]. The normalized localization lengths $\xi_M(E)/M$ (ξ_M is the localization length on a cylinder of width M) for different values of W, Δ, and E are calculated numerically using the transfer matrix method. The "thermodynamic" localization length ξ_∞ is obtained by demanding that all data collapse onto a single curve when $\xi_M(E)/M$ is plotted vs $\xi_\infty(E-E_c)/M$. This curve represents the scaling function, and E_c marks the critical point. By requiring $\xi_\infty \propto |E - E_c|^{-\nu}$ in the vicinity of E_c, one can extract the critical exponent ν.

In the following we present these results for $W = 0.5\pi$, $\Delta = 0.0$ [Fig. 3(a)] (the spin degenerate case) and $W = 0.1\pi$, $\Delta = 0.4$ [Fig. 3(b)]. If the transition is governed by a fixed point with one relevant operator and a number of *nondegenerate* irrelevant operators [3], the critical ξ_M/M should become *M independent* for sufficiently large M's. Unfortunately, we have not achieved this in the present calculations. For $M = 8 \rightarrow 128$ we observed a slight residue M dependence. We take it as an indication that we are still seeing the effects of irrelevant operators. In general, the presence of such operators will prevent data collapsing. However, we show in Figs. 3(a) and 3(b) that this is *not* the case. In each case there are two transitions as a function of E at $E_{c1} > E_{c2}$ (data shown are for $E > E_{c1}$ and $E < E_{c2}$). In the insets of Figs. 3(a) and 3(b), we show that the exponent deduced from $\xi_\infty(E - E_c)$ agrees with that of the spin-resolved transition, $\nu = 2.4 \pm 0.2$. A more stringent test of whether the universality class remains *unchanged* is to compare the scaling functions. If we compare the scaling curves obtained in Figs. 3(a) and 3(b) (symbols) with that of the spin-resolved transitions (solid line), we find that they *do not* coincide. As discussed above, we attribute the discrepancy to finite size effects caused by irrelevant operators. Empirically, we find that to a very good approximation the scaling function $\mathcal{F}(\xi_\infty/M, \xi_{\mathrm{irr}}/M)$ deduced from Figs. 3(a) and 3(b) is related to the spin-

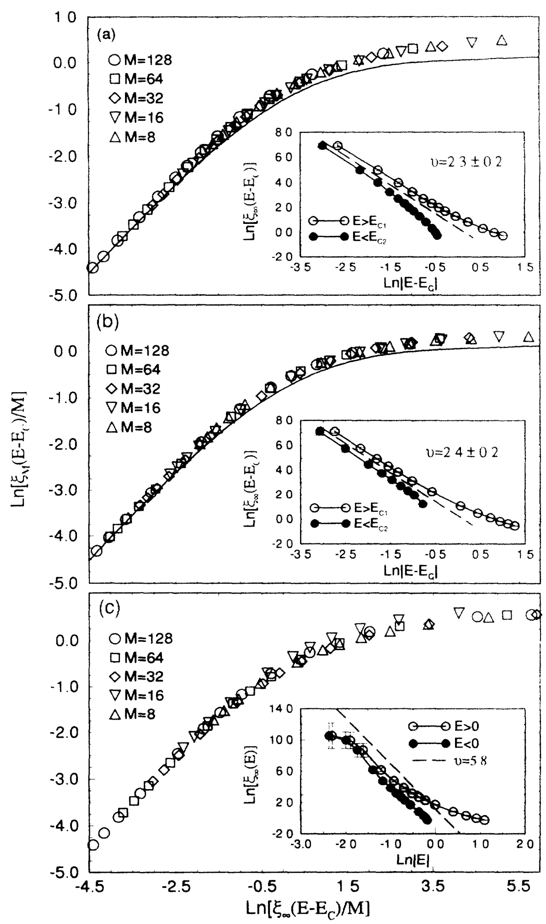

FIG. 3. The scaling plots. Insets: $\ln\xi_\infty(E - E_{ca})$ vs $\ln|E - E_{ca}|$. The data are taken above the upper (E_{c1}) and below the lower (E_{c2}) transitions. The two critical points are found at (a) $E_{c1} = 0.23$ and $E_{c2} = -0.28$ for $W = 0.5\pi$ and $\Delta = 0.0$ and (b) $E_{c1} = 0.45$ and $E_{c2} = -0.02$ for $W = 0.1\pi$ and $\Delta = 0.4$. The solid lines in (a) and (b) are the spin-resolved scaling function [4]. (c) *Assuming* a single transition at $E_c = 0$ for $W = 0.5\pi$ and $\Delta = 0$.

resolved scaling function $\mathcal{G}(\xi_\infty/M)$ via $\mathcal{F}(x,y)=\lambda(y)\times\mathcal{G}(\epsilon(y)x)$. Here λ and ϵ are scale factors, and $\xi_{\rm irr}$ is the crossover length associated with the leading irrelevant operator. The fact that despite the presence of irrelevant operators *data collapsing works* implies $d\lambda/dy, d\epsilon/dy \ll 1$. Universality requires that $\lim_{y\to 0}\lambda(y)=\lim_{y\to 0}\epsilon(y)=1$, which can be realized when either $\xi_{\rm irr}(E)\ll 1$ or $M\to\infty$. An indication that this is indeed happening can be obtained by noting that as W/Δ is reduced in going from Fig. 3(a) to 3(b), the scaling curve gets closer to the spin-resolved result. To prove that both λ and ϵ do approach unity for the cases studied, one needs to study systems with a much larger M, a task that is beyond our current computing capability. The fact that the correction to scaling has more effects on ξ_M/M than on ν is consistent with the experimental findings that ν seems to be universal while the critical conductivities are not [15]. For both cases studied, if the energy falls between the upper and lower critical points (i.e., $E_{c2}<E<E_{c1}$), $\xi_M(E)/M$ becomes quite insensitive to the system sizes studied [16]. Were it not for the analytical arguments, one could mistake those as indicating the existence of a whole interval of critical points or a metallic phase.

To address the apparent inconsistency between the theory and experiment, in Fig. 3(c) we show the scaling plot and ξ_∞ vs E for $W=0.5\pi$ and $\Delta=0$ *assuming there is only a single transition* at $E=0$. The quality of the data collapsing is noticeably worse than that in Fig. 3(a). Moreover, the resulting ξ_∞ shows *saturation* near $E=0$. However, if one uses the limited linear portion of the $\ln\xi_\infty$ vs $\ln|E|$ plot to determine an exponent, one gets $\nu\approx 5.8$. This value is in reasonable agreement with the current experimental result, if one assumes that the dynamical exponent remains unchanged between the spin-resolved and spin-unresolved transitions [10]. Hence *if the Coulomb interaction does not qualitatively change the present results, we conclude that the apparent "new universality class" observed in the spin-unresolved plateau transitions is an artifact due to the limitation in the experimental resolution.* In that case, with improved resolution one should be able to show that the spin-unresolved transition actually consists of two consecutive plateau transitions, each having the same ν as in a spin-resolved transition.

The conclusion that *Landau level mixing does not change the universality class of the plateau transition* bears an important implication on the topology of the quantum Hall effect phase diagram [3]. Specifically, in Ref. [3] the authors *assumed* that when the system makes a transition between one plateau and another, it has to pass by all intervening plateaus. For example, in the integer quantum Hall effect this "selection rule" implies that in order to get to $\sigma_{xy}=0$ from $\sigma_{xy}=2e^2/h$, the system must pass the $\sigma_{xy}=e^2/h$ plateau [17]. Our work adds supports to this assumption.

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[15] Since ξ_M/M determines the transmission and reflection probabilities [4], it also determines σ_{xx} and σ_{xy}.

[16] More precisely, for $W=0.5\pi$, $\Delta=0$ we see a very slow decrease in ξ_M/M as M is increased. For $W=0.1\pi$ and $\Delta=0.4$, ξ_M/M first increases with M for small M's. This trend is *reversed* when M gets large enough.

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