Stopping Power Theory for Screened Coulomb Binary Collisions in a Nondegenerate Plasma

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The stopping power of a nondegenerate, quasineutral plasma is evaluated both numerically and analytically for energetic charged particles which undergo screened Coulomb collisions as a result of Debye shielding. The numerical evaluation is exact within the framework of the binary collision approximation and accurately takes into account both the plasma particle velocity distribution and the profile of the screened Coulomb intertaction potential. The stopping power of plasma electrons is found to be considerably smaller than reported by Li and Petrasso [Phys. Rev. Lett. **70**, 3059 (1993)].

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The stopping power is a useful parameter used in a variety of disciplines providing the average energy loss rate for fast-moving ions or electrons (hereafter referred to as "test particles") in matter. A significant amount of work has been carried out in the development of stopping power theories for solids and gases (a historical review is found in Ref. [1]). For nondegenerate, quasineutral plasmas, stopping power theories have also been developed which consider a collective plasma response to the test particle [2] or which consider binary collisions [3]. The utility of a stopping power theory for nondegenerate plasmas is diverse. Applications include studies involving inertial confinement fusion [2,3], magnetic confinement fusion associated with two-component plasma concepts [4,5], neutron generation [6], and stellar plasmas [7].

In the work presented here, the stopping power of a nondegenerate, quasineutral plasma is evaluated within the framework of the binary collision approximation. The binary collision approximation is suitable for this primarily when the test particle speed is smaller than the electron thermal speed. In this case, the interaction potential between the test particle and a plasma particle is a screened Coulomb potential which results from Debye shielding. Within the binary collision approach, the stopping power of each species of particles in the plasma is computed separately. For example, for a fully ionized hydrogen plasma, the stopping power of the plasma ions and the stopping power of the plasma electrons are evaluated separately and the total stopping power of the plasma is the sum of the two.

In the binary collision approach, energy lost by the test particle as it travels through the plasma is approximated as a cumulative sum of energy lost (or gained) in successive, independent binary collisions. For a test particle which undergoes N binary collisions while traveling a distance δl, the energy lost by the test particle is $\delta E=\sum_{i=1}^{N}\Delta E_i$, where ΔE_i is the energy lost in the ith collision. The stopping power, dE/dl, is defined such that $\int_0^{\delta l}(dE/dl)dl=\delta E$. With the approximation that only a small change in energy occurs during N collisions where N is a large number, $\int_0^{\delta l}(dE/dl)dl\rightarrow(dE/dl)\delta l$ and

$$\frac{dE}{dl}=\frac{\delta E}{\delta l}. \tag{1}$$

With this, the stopping power is

$$\frac{dE}{dl}=\frac{N}{\delta l}\left(\frac{1}{N}\sum_{i=1}^{N}\Delta E_i\right)=n\sigma_{\max}\langle\Delta E\rangle$$
$$=\int d^3v_2 f(\mathbf{v}_2)\int d\sigma\Delta E\,, \tag{2}$$

where $N/\delta l$ is the average number of binary collisions per unit distance traveled due to the forward motion of the test particle, $\langle\Delta E\rangle$ is the average energy lost by the test particle per collision, and $f(\mathbf{v}_2)$ is the velocity distribution function of the plasma particles (subscript 2) which is normalized to the plasma density (n). This relation assumes that binary collisions have impact parameters less than $b_{\max}=\sqrt{\sigma_{\max}/\pi}$. The limit $b_{\max}\rightarrow\infty$ can be taken when a screened Coulomb interaction potential is used but not when the long range pure Coulomb interaction potential is used. The latter would cause the Coulomb logarithm to diverge. The fundamental difference between the approach in the present work and that used previously [8] is the definition used for the stopping power. Referring back to Eq. (1), the stopping power might also be defined (approximately) as

$$\frac{dE}{dl}\approx\left\langle\frac{\Delta E}{\Delta l}\right\rangle=\int d^3v_2 f(\mathbf{v}_2)\frac{u}{v_1}\int d\sigma\Delta E\,, \tag{3}$$

where ΔE is the energy lost by the test particle per collision, Δl is the distance traveled per collision due to the relative motion of the test particle, and $u=|\mathbf{u}|=|\mathbf{v}_1-\mathbf{v}_2|$ is the relative speed between the test particle and a plasma particle during a binary collision. Equation (3) is used by Trubnikov (see Eq. 7.29 of Ref. [8]) and also provides the basis for the kinetic theory of stopping developed by Sigmund [see Eqs. (5) and (13) of Ref. [9]]. The stopping powers predicted by Eqs. (2) and (3) have been compared to experimental stopping power values for stopping by electrons in copper (which is predominantly due to stopping by the degenerate electron

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plasma in copper) and for stopping by atomic electrons in hydrogen [10]. In both cases, Eq. (2) provides better agreement with the experimental data particularly when the mean electron speed is comparable to the test particle speed. In the present work, Eq. (2) is used for the first time to predict the stopping power of a nondegenerate plasma.

The energy lost as a result of a single binary collision with a plasma particle is

$$\Delta E = 2m_r\left[v_1(v_1-v_{2\parallel})-\frac{m_r}{m_1}u^2\right]\sin^2\left(\frac{\theta_c}{2}\right) - m_r v_1 v_{2\perp}\sin\theta_c\,, \tag{4}$$

where subscript 1 denotes the test particle, $m_r=m_1m_2/(m_1+m_2)$ is the reduced mass, and θ_c is the center-of-mass scattering angle in the plane of the collision with values between π and $-\pi$. Here, the velocity of the plasma particle is separated into perpendicular ($\perp$) and parallel ($\parallel$) components with respect to the direction of travel of the test particle prior to the collision. To arrive at Eq. (4), the velocity vectors for the test particle and the plasma particle which it collides with are initially considered in the laboratory frame of reference. A laboratory coordinate system with the z axis parallel to the initial direction of travel of the test particle is chosen. The two velocity vectors are then transformed to a coordinate system which is aligned with the laboratory coordinate system but which is moving such that the plasma particle is at rest. A rotation of this coordinate system is then necessary in order to realign the velocity vector of the test particle with the z axis. Following this, a transformation (without rotation) is made to a center-of-mass coordinate system. The asymptotic velocity vectors after the collision are then found in terms of the center-of-mass scattering angle of the test particle by requiring conservation of momentum and energy. The transformations are reversed and the velocity vectors for the two particles after the collision are evaluated with respect to the laboratory coordinate system. With the initial and final speed for either particle, the energy transfer as a result of the collision is found. For convenience, the value of the energy transferred, denoted ΔE, is chosen to be positive when energy is lost by the test particle and negative when energy is gained. Consequently, calculated stopping power values presented here are positive (although formally they should be negative). There is equal probability for positive and negative values for θ_c and $\int\sin\theta_c\,d\sigma\rightarrow 0$. Consequently, the stopping power is

$$\frac{dE}{dl}=4\pi\lambda_D^2 m_r\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\sin^2\left(\frac{\theta_c}{2}\right)\beta\,d\beta\left[v_1(v_1-v_{2\parallel})-\frac{m_r}{m_1}u^2\right]f(v_{2\perp},v_{2\parallel})\,dv_{2\perp}\,dv_{2\parallel}\,. \tag{5}$$

Here, the differential cross section for the collision has been written as $d\sigma=2\pi b\,db$ and then a change of variables has been made given by $b=\lambda_D\beta$, where λ_D is the Debye screening length.

It should be kept in mind that the applicability of Eq. (5) is restricted to situations where the binary collision approximation is valid. Along with the restriction that the test particle speed be smaller than the electron thermal speed, the Coulomb logarithm (λ) should have a sufficiently large average value. For an average value near or below unity, correlated many-body interactions become increasingly important and the binary collision approximation may no longer be valid. The Coulomb logarithm is present in Eq. (5) as

$$\lambda=\Lambda^2\int_0^{\infty}\sin^2\left(\frac{\theta_c}{2}\right)\beta\,d\beta\,, \tag{6}$$

where $\Lambda=\lambda_D m_r u^2/\kappa$ and $\kappa=|q_1q_2|$ in cgs units. The center-of-mass scattering angle for a binary collision is given by [11]

$$\theta_c=\pi-2b\int_{r_0}^{\infty}\frac{dr}{r^2}\left[1-\frac{V(r)}{E_c}-\frac{b^2}{r^2}\right]^{-1/2}, \tag{7}$$

where

$$1-\frac{V(r_0)}{E_c}-\frac{b^2}{r_0^2}=0\,. \tag{8}$$

Here, $r_0(b)$ is the distance of closest approach and $V(r)$ is the interaction potential energy. The interaction potential due to Debye shielding is the screened Coulomb potential, $V(r)=(\kappa/r)\exp(-r/\lambda_D)$, where λ_D is the Debye screening length [8]. For efficient computation of the stopping power, a fit to a numerical evaluation of Eq. (6) is used which is accurate to within 4% for $\lambda\geq 0.1$. The fit is given by

$$\lambda(\Lambda)=\frac{(1+\Lambda_f)^2\ln(1+\Lambda_f)}{(2+\Lambda_f)^2}-\frac{\Lambda_f}{2(2+\Lambda_f)}+0.15\Lambda_f e^{-0.5\Lambda_f}\,, \tag{9}$$

where $\Lambda_f=1.17\Lambda$. In comparison, the conventional formula for the Coulomb logarithm ($\ln\Lambda$) is accurate to within 4% only for λ values greater than ~ 10. In terms of the Coulomb logarithm, Eq. (5) is now written as

$$\frac{dE}{dl}=\frac{4\pi\kappa^2}{m_r}\int_{-\infty}^{\infty}\int_0^{\infty}\frac{\lambda(u)}{u^4}\left[v_1(v_1-v_{2\parallel})-\frac{m_r}{m_1}u^2\right]f(v_{2\perp},v_{2\parallel})\,dv_{2\perp}\,dv_{2\parallel}\,. \tag{10}$$

This equation provides the stopping power of each component of a nondegenerate, quasineutral plamsa and is exact

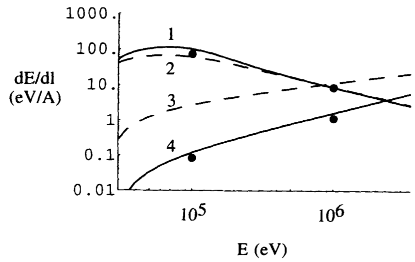

FIG. 1. The stopping of alpha particles by the ion component (curves 1 and 2) and the electron component (curves 3 and 4) of a 10^{26} cm^{-3}, 20 keV thermalized D-T plasma as predicted by Eq. (10) (solid circles), Eq. (11) (solid curves), and previous theory [3] (dashed curves).

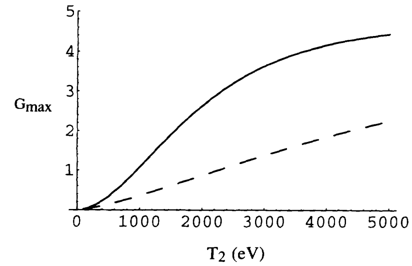

FIG. 2. The fusion energy multiplication factor for energy-clamped 100 keV deuterons within a 5×10^{13} cm^{-3} thermalized tritium plasma. The evaluations shown use the stopping power as predicted by Eq. (11) (solid curve) and as predicted by previous stopping power theory [3] (dashed curve).

within the framework of the binary collision approximation. For an analytic evaluation of Eq. (10), an approximation is made by replacing the Coulomb logarithm with its approximate average value. This is done by using $\Lambda=\lambda_D m_r(v_f^2+3T_2/m_2)\kappa$, where T_2 is the plasma temperature in energy units. For a Maxwellian plasma, evaluation of Eq. (10) then gives

$$\frac{dE}{dl}=\frac{2\pi n\kappa^2\lambda m_2}{m_r T_2}\left[\left(\frac{1}{2}-\frac{m_r}{m_1}+\chi\right)\left(\frac{\pi}{\chi}\right)^{1/2}e^{-\chi}\mathrm{erfi}(\sqrt{\chi})-1\right], \tag{11}$$

where $\chi=m_2 v_f^2/2T_2$ and $\mathrm{erfi}(x)=-i\,\mathrm{erf}(ix)$ is the imaginary error function.

An example calculation of the binary-collision-based stopping power is shown in Fig. 1 using parameters relevant to inertial confinement fusion. Both Eq. (11) and an exact numerical evaluation of Eq. (10) are shown. These are found to agree well. Also shown are stopping power calculations using the theory developed by Li and Petrasso [Eqs. (1) and (2) in Ref. [3]] which is based on Trubnikov's [8] work. It is apparent that the rate at which fast ions lose energy to plasma ions as predicted by Eq. (11) agrees well with the theory of Li and Petrasso. However, the stopping power of plasma electrons is significantly smaller. This is an important result because it signifies that the alpha particles will transfer a greater fraction of their energy to the plasma ions as they slow down within the plasma and that the alpha particles will have a larger range within the plasma. The smaller stopping power is also important for studies associated with two-component, magnetically confined fusion plasmas. Consider a magnetically confined tritium plasma into which 100 keV deuterium is deposited by neutral-beam injection. The transfer of energy from the deuterium to the plasma can be compensated for by magnetic compression. As a result, the energy of the deuterium can be "clamped" near 100 keV where the peak of the D-T fusion cross section occurs. For this scenario, the fusion energy multiplication factor, $G_{\max}$, is evaluated as the ratio of the fusion energy production rate to the energy transfer rate between the energetic deuterium ions and the plasma. (A more detailed discussion of $G_{\max}$ along with its relation to Q is found in Ref. [5].) Figure 2 shows a calculation of $G_{\max}$ using Eq. (11). For energy clamping at 100 keV, $G_{\max}=1$ at a plasma temperature and density of 950 eV and 5×10^{13} cm^{-3}, respectively. The temperature at which $G_{\max}=1$ is significantly lower than previously reported [5].

In summary, a new stopping power theory for a nondegenerate, quasineutral plasma has been presented. The theory is exact within the framework of the binary collision approximation when evaluated numerically. An analytic relation which compares well with the exact result has also been given which is suitable when the Coulomb logarithm is slowly varying ($\lambda\gtrsim2$). An important prediction of the new theory is that the stopping power of plasma electrons is considerably smaller than previously thought. This suggests that a greater fraction of energy deposited into a plasma by an energetic charged particle is transferred to the plasma ions. The applicability of the new theoretical result has been briefly explored considering parameters relevant to both inertial confinement fusion and magnetic confinement fusion. For an inertially confined fusion plasma, a smaller stopping power of the plasma electrons implies a larger fraction of alpha particle energy will be deposited into the ion component of the D-T plasma as the alphas slow down. For a two-component, magnetically confined fusion plasma, a smaller stopping power results in a value for $G_{\max}$ which exceeds unity at a plasma temperature less than 1 keV.

[1] J. F. Ziegler, J. P. Biersack, and U. Littmark, *The Stopping and Range of Ions in Solids* (Pergamon Press, New

York, 1985), p. 1.
[2] J. D'Avanzo, M. Lontano, and P. F. Bortignon, Phys. Rev. E **47**, 3574 (1993), and references therein.
[3] C. K. Li and R. D. Petrasso, Phys. Rev. Lett. **70**, 3059 (1993).
[4] J. M. Dawson, H. P. Furth, and F. H. Tenney, Phys. Rev. Lett. **26**, 1156 (1971); R. F. Post, T. K. Fowler, J. Killeen, and A. A. Mirin, Phys. Rev. Lett. **31**, 280 (1973); C. A. Ordonez, Fusion Tech. **22**, 388 (1992).
[5] H. P. Furth and D. L. Jassby, Phys. Rev. Lett. **32**, 1176 (1974).
[6] F. H. Coensgen, T. A. Casper, D. L. Correll, C. C. Damm, A. H. Futch, B. G. Logan, and A. W. Molvik, Nucl. Sci. Eng. **106**, 138 (1990).
[7] L. Vriens, Phys. Rev. A **8**, 332 (1973).
[8] B. A. Trubnikov, *Reviews of Plasma Physics 1* (Consultants Bureau, New York, 1965), p. 105.
[9] P. Sigmund, Phys. Rev. A **26**, 2497 (1982).
[10] D. Semrad and P. Bauer, Nucl. Instrum. Methods Phys. Res., Sect. B **12**, 24 (1985); C. A. Ordonez (to be published).
[11] D. N. Ruzic, Phys. Fluids B **5**, 3140 (1993).