Large Larmor Radius Stability of the z Pinch

T. D. Arber and M. Coppins
Blackett Laboratory, Imperial College of Science, Technology and Medicine, Prince Consort Road, London SW7 2BZ, United Kingdom

J. Scheffel
Fusion Plasma Physics, Alfvén Laboratory, Royal Institute of Technology, S-100 44 Stockholm, Sweden
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The linear $m=0$ stability of the z pinch in the collisionless, large ion Larmor radius regime is examined using the Vlasov fluid model. The results reveal a strong equilibrium dependence. The uniform current density equilibrium shows a reduction in growth rate when the average ion Larmor radius is about one-fifth of the pinch radius. However, finite Larmor radius effects cannot in themselves produce a stabilized z pinch.

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Early z pinches, for which ideal magnetohydrodynamics (MHD) was an appropriate model, were exceptionally susceptible to instabilities with growth times comparable to the ion thermal transit time. Modern z-pinch experiments (including those in which the pinch is formed from a cryogenic fiber of hydrogen or deuterium [1–2]) operate in a region of parameter space in which ideal MHD stability theory is inapplicable (see Ref. [3] and references therein). Instead they have collisionless, large ion Larmor radius (LLR) plasmas. It has been suggested that MHD instabilities may be suppressed in such pinches. In this Letter we present results for the linear $m=0$ stability of the z pinch in this regime.

Assuming the plasma is collisionless, its properties will depend critically on a parameter ϵ which is the ratio of the average ion Larmor radius to a, the pinch radius. It is well known [4] that $\epsilon=5.71\times10^8\sqrt{A/N}$, where N (the pinch line density, i.e., the number of ions per unit length) is given in m^{-1}. For fiber pinches N is proportional to r_f^2, where r_f is the initial fiber radius. In the case of hydrogen $\epsilon\simeq2.0/r_f$, where r_f is in microns. It is extremely difficult to make cryogenic fibers of less than 10 μm in radius. Thus for fiber pinches we are restricted to $\epsilon<0.2$. In principle, higher values of ϵ are attainable in other classes of z pinches, although, as we shall see, this is not necessarily desirable.

For systems with straight or nearly straight field lines, increasing ϵ gives rise to improved stability [5–8]. Analytic solutions showing significant large Larmor radius effects have also been found for electrostatic perturbations in a dilute plasma [9]. In order to investigate if this behavior will also occur in the z pinch, with its strongly curved field, the ions must be treated kinetically, and all particle orbit types must be included (e.g., singular [4] and resonant particles).

Previous work on the collisionless, linear stability of the z pinch has included the zero Larmor radius limit [10,11] using the Chew-Goldberger-Low (CGL) equations, approximate solutions for the finite Larmor radius (FLR) case [12,13], and an exact solution for the skin current equilibrium [14].

Previously nonlinear theory includes a 3D particle-in-cell (PIC) code (SPLASH) which simulates the evolution of a z pinch using both particle ions and electrons [15]. This work was constrained by computational expense to artificial mass ratios and the inclusion of an axial magnetic field. Consequently, the code was incapable of comparing growth rates with the appropriate fluid model in order to assess the quantitative importance of kinetic effects, e.g., finite and large Larmor radius, on the stability properties of z pinches. The SPLASH code has since evolved into the TRISTAN code designed in Ref. [16]. However, this newer code has not been used to investigate z-pinch stability. The present work is restricted to linear modes, and thus deals with a simplified subset of the problem, but all of the essential ion physics is included and a direct and quantitative assessment of large ion Larmor radius stability is possible.

We have developed two alternative formulations of the linearized kinetic stability problem which include all orbit types, all values of ϵ, and can be applied to any equilibrium. Both use the Vlasov fluid model [17], which treats the ions fully kinetically (via the Vlasov equation), and the electrons as a cold background fluid maintaining quasineutrality. The main restriction is the neglect of electron temperature. Since the $m=0$ mode in a z pinch does not involve a displacement (or perturbed magnetic field) parallel to the equilibrium magnetic field B_0, the inclusion of finite electron pressure would only affect the component of the perturbed electric field perpendicular to B_0. In the limiting case of a pure skin current z pinch this was shown to have no significant effect [14]. The inclusion of electron pressure for the diffuse profiles is currently being undertaken.

Two classes of equilibria are considered in detail in this Letter. Both assume a Maxwellian ion distribution function, with uniform temperature (T_{i0}). The first (*parabolic*) corresponds to a uniform current density, and has a magnetic field $B_0(x)\propto x$ and ion number density $n_0(x)\propto 1+\alpha-x^2$, where $x=r/a$ and α is a constant. For this equilibrium fixed (internal) and free boundary modes have been considered. Internal modes assume that the

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plasma boundary is not deformed in the instability. This requires that $\alpha > 0$ because the eigenfunction is not well behaved for $m=0$ internal modes if $n_0(x=1)=0$ [11]. For free boundary modes, i.e., a deformable plasma-vacuum interface, the present analysis is restricted to equilibria without a skin current and consequently $\alpha=0$. The second equilibrium (*Bennett*) corresponds to a uniform electron velocity. In this case $B_0(x) \propto x/(1+\delta^2 x^2)$ and $n_0(x) \propto (1+\delta^2 x^2)^{-2}$ where δ is a constant. Only internal modes are considered for the Bennett profile. However, for large δ the equilibrium extends out into a sufficiently extended, low density region that the vacuum region may be ignored.

The first of the two methods which we use employs an initial value approach. In this formulation we assume harmonic functions of θ and z, but make no assumption about time dependence. Instead, we retain the time derivatives in the linearized Vlasov fluid equations. A random perturbation is applied and its time evolution is followed. After a few growth times the fastest growing mode will dominate the behavior, and the solution will converge to exponential growth at a well defined growth rate, and with a well defined structure of the perturbed variables.

This formulation has been implemented numerically in the FIGARO code. In this approach the linearized Vlasov equation is advanced in time using the method of characteristics. This amounts to integrating Vlasov's equation along unperturbed equilibrium ion trajectories. This differs from PIC simulations in that particles are only used as labels of equilibrium phase space trajectories along which the distribution function is updated. In PIC simulations the distribution function is found from the actual density of computational particles in phase space. In FIGARO each particle carries with it a value of f_1 (perturbed ion distribution function), the value of which is calculated by advancing the linearized Vlasov equation. At any time step, therefore, values of f_1 are known at the particle phase space positions. Moments are taken to obtain fluid variables, and $\mathbf{B}_1$ and $\mathbf{E}_1$ are calculated from Faraday's law and the electron fluid equation. $\mathbf{E}_1$ is then used to advance f_1 to the next time step, and so on. The fluid variables are specified in a set of 1D (radial) cells (typically 50). It should be noted that there is no discretization of velocity space in this model. Ions are distributed uniformly and randomly in phase space (up to four thermal speeds) according to a Maxwellian distribution.

The second approach uses the variational nature of the Vlasov fluid dispersion functional [18–20]. The unknown eigenfunction $\boldsymbol{\xi}$ (the electron fluid displacement) is expanded in a truncated series of orthogonal functions (MHD eigenfunctions, CGL eigenfunctions, or Bessel functions) and eigenvalues are determined from the requirement that the determinant of the dispersion functional matrix [19] must vanish. Typically, only three MHD or CGL eigenfunctions are required for convergence, the same eigenvalue being recovered with eight Bessel functions.

Figure 1 shows results from both FIGARO (points) and the variational method (solid lines) for internal modes. The $(ka=10)$ growth rate is plotted against ϵ for equilibria of the two classes discussed above. The $\epsilon=0$ limit is given by CGL, and the growth rates plotted here are normalized to the relevant CGL values. The parabolic equilibrium $(\alpha=0.1)$ shows a reduction of growth rate as ϵ is increased up to about 0.2, beyond which the growth rate increases with ϵ. The minimum growth rate occurs at approximately the same value of ϵ for all ka. For $\epsilon > 0.4$ the Vlasov fluid growth rate is actually higher than the CGL value. In the case of the Bennett equilibrium $(\delta=3.0)$, large Larmor radius *destabilization* actually occurs over the whole range of ϵ, and increasing ϵ always increases the growth rate. The points from FIGARO were produced using relatively few (10^4) ions. Convergence of the two methods has been verified over the range of ka and ϵ by increasing the number of ions in FIGARO and using more expansion functions in the variational method. The fact that the two methods provide a mutual check is extremely valuable. The only previously known result against which the solution can be benchmarked is the $\epsilon=0$ (CGL) limit. Both methods tend to this limit, and this provides a second, independent confirmation of the results. We have chosen to present results from the largest consistent data set, i.e., that using 10^4 ions in FIGARO and three expansion functions in the variational method. With these values the codes are consistent to about 10%.

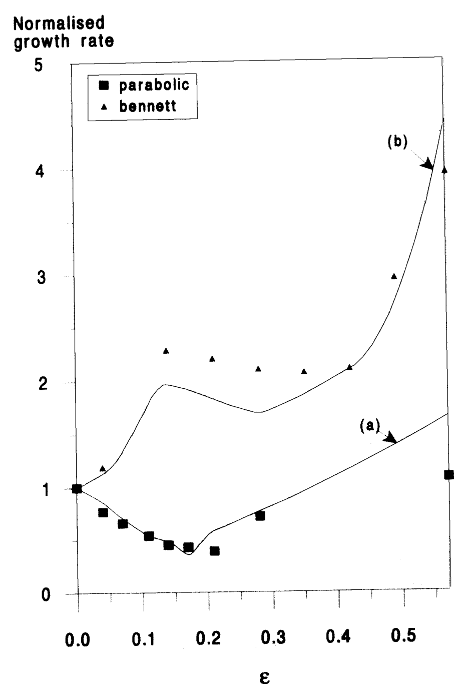

FIG. 1. Growth rate (normalized to the CGL value) against ϵ for $ka=10$. Lines, variational code results; (a) parabolic equilibrium $(\alpha=0.1)$; (b) Bennett equilibrium $(\delta=3)$. Points: FIGARO results (see legend on graph).

The extreme sensitivity to the form of the equilibrium, even to the extent that large Larmor radius effects appear to improve stability in one case but destabilize the other, is one of the most surprising aspects of this work. Previous Vlasov fluid work has identified regions where a particular process is dominant; e.g., Ref. [8] showed that it was the resonant ions which prevented absolute finite Larmor radius (FLR) stabilization of the theta pinch. At present we have no conclusive physical explanation of the detail in Fig. 1 and we refrain from speculation. One of the principle difficulties in attempting an explanation for the z pinch is that there is no satisfactory FLR fluid model. This is due to the strong field curvature and the existence of a magnetic field null at the origin.

Figure 2 shows results only for the parabolic equilibrium ($\alpha=0.1$). Here the growth rate, normalized to the radial ion thermal transit time $(a/\sqrt{2k_BT_{i0}/m_i})$, is plotted against ka for various values of ϵ. In the very small Larmor radius case ($\epsilon=0.01$) the Vlasov fluid curve reproduces the CGL result and is not presented in the figure. In both the very small and the large Larmor radius cases the growth rate is not greatly changed from the CGL values for all ka considered. However, there is an intermediate regime, around $\epsilon=0.2$, for which the growth rate curve has a maximum at $ka\simeq 6$.

The variational method has also been extended to free boundary modes for equilibria which do not contain a skin current. For the parabolic profile ($\alpha=0$) free boundary growth rates are higher than those for purely internal modes. However, since the corresponding CGL fluid growth rate is also larger for free boundary modes the graph of growth rate against ϵ normalized to the fluid limit growth rate shows the same structure as Fig. 1. The Bennett profiles cannot exist surrounded by a vacuum without an equilibrium skin current. For this case free boundary modes have been simulated by studying the internal modes of the Bennett profile with large δ. With $\delta=10$ the equilibrium extends out into a sufficiently low density region that the vacuum may be ignored. Renormalizing to the CGL growth rate reproduces the structure presented in Fig. 1 with large Larmor radius destabilization over the whole range of ϵ. We conclude that allowing for a deformable plasma-vacuum interface has no significant effect on the character of the kinetic stability of the $m=0$ mode in a z pinch. From considerations of the energy principle it is clear that, since there is no vacuum contribution to the potential energy for $m=0$ modes, all of the essential physics is in fact included in the study of internal modes.

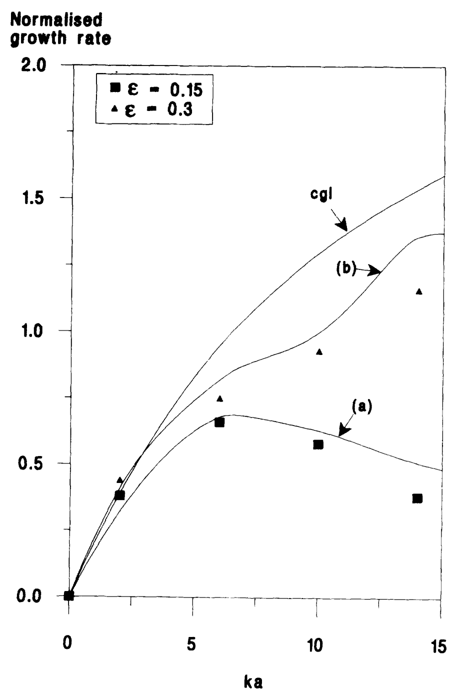

FIG. 2. Parabolic equilibrium ($\alpha=0.1$). Growth rate (normalized to the radial ion thermal transit time) against ka for various values of ϵ. Lines, CGL; (a) and (b) variational code results [(a) $\epsilon=0.15$, (b) $\epsilon=0.3$]. Points: FIGARO results (see legend on graph).

Broadly speaking, the eigenfunction structure remains largely CGL-like for all ϵ. The Bennett equilibrium shows a slight variation with ϵ. The CGL eigenfunction is obtained for ϵ above about 0.4, but for smaller Larmor radius the perturbation is less localized near the pinch edge, and the peak of $|\xi_r|$ is shifted inwards by about 10%.

It can be shown that if a z pinch is unstable to ideal MHD then it is also unstable in the Vlasov fluid model if the equilibrium distribution function is a monotonically decreasing function of the particle energy [18]. The inherent inclusion of resonance effects renders earlier results on finite Larmor radius stabilization (e.g., [13]) too optimistic. For the z pinch there is an important exception to this rule in that the zero Larmor radius $m=0$ stability threshold is given by the CGL model [11] which is easier to satisfy than the corresponding ideal MHD case. However, for all finite ϵ the stability threshold is that of ideal MHD even for the $m=0$ mode. This apparent contradiction is resolved by noting that in the limit of ϵ tending to zero the CGL model gives an accurate estimate of the growth rate. Consequently, for equilibria which are CGL stable but ideal MHD unstable, the growth rate must tend to zero in the limit of ϵ tending to zero (i.e., for small ϵ the Vlasov fluid model predicts small growth rates if the equilibrium is CGL stable). In order to investigate the situation we have studied the parabolic profile with $\alpha=1$, an equilibrium which is stable in CGL but unstable in MHD [10]. For all finite ϵ the stability threshold of Vlasov fluid theory is the same as that for MHD. Therefore this equilibrium should be unstable in the Vlasov fluid model. For $0.01<\epsilon<0.4$ neither code was able to identify an instability and we conclude that the growth rate is too small to be resolved by our methods (i.e., $<1\%$ of the ideal MHD growth rate). Such a small growth rate would be of no practical importance in the z pinch and consequently this result suggests that for the $m=0$ mode the *practical* stability threshold is found from the CGL model, even though the *exact* threshold for all finite ϵ is given by the less favorable ideal MHD model. For a parabolic equilibrium to be CGL stable the pressure at the boundary must be at least 50% of the pressure on axis. Such an equilibrium is unobtainable in a cryogenic fiber pinch.

Parabolic number density profiles have been found in experiments (e.g., [21]) and one dimensional simulations (e.g., [22]). The present work therefore suggests that real fiber z pinches will probably show improved $m=0$ stability due to large Larmor radius effects, up to $\epsilon=0.2$. From the stability point of view, therefore, there is little point in using significantly lower values of line density. In the case of fiber pinches, the optimum ϵ corresponds to the minimum technically feasible fiber radius. However, even for this optimal line density the presence of instabilities with growth rates comparable to those of MHD or CGL for $ka \sim 6$ would still disrupt the pinch unless they saturated nonlinearly at finite amplitude. In order for linear stability to predict a pinch in which instabilities are of no practical importance a reduction in growth rate of at least an order of magnitude more than that found for $ka=6$ must be obtained for all values of ka.

Considerable care must be taken in the practical interpretation of this work. First, only linear $m=0$ modes have been considered. It is well known that these modes are a singular case in the z pinch as they do not involve a displacement parallel to the equilibrium magnetic field. Such modes are found to be strongly sensitive to the compressibility of the plasma in fluid models. Other m number modes are little affected by compressibility and the stability results presented in this Letter for $m=0$ cannot automatically be considered symptomatic of the stability properties of other mode numbers. Indeed, other numerical studies [23] of FLR effects in systems with strongly curved field lines have shown that when only incompressible displacements are allowed considerable stabilization does occur. This is in keeping with earlier results for systems with straight or nearly straight field lines [5-8]. Since the present analysis treats the ions by Vlasov's equation, it follows that the kinetic equivalent of the fluid compressibility is correctly and exactly included in this analysis. It is possible that for $m=1$, where compressibility is unimportant, the z pinch may exhibit more of the stability properties found for similar systems, e.g., Ref. [23]. The generalization of the present work to include free boundary $m=1$ modes is currently being undertaken. The disappointing FLR results for the $m=0$ mode may not be typical of other modes.

In conclusion, we have shown that the class of profiles which are stable to $m=0$ modes is larger than would be suggested from ideal MHD theory, in which only Kadomtsev profiles are free of $m=0$ instabilities. However, profiles which are not CGL stable, such as the parabolic and Bennett profiles, are not stabilized by FLR or LLR effects. Thus FLR effects alone are not sufficient to guarantee stability to $m=0$ modes and it is only if the z pinch relaxes to a CGL stable profile that $m=0$ modes will be absent.

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