Flowerlike Patterns Generated by a Laser Beam Transmitted through a Rubidium Cell with Single Feedback Mirror

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A spatial instability is observed when a laser beam at 795 nm is transmitted through a rubidium cell with single plane feedback mirror. The emitted beams have patterns looking like flowers. The dependence of the number of petals in the pattern with the distance between the cell and the mirror is studied and interpreted using an expansion of the instability in Laguerre-Gauss modes. This experiment shows the influence of the saturation of the nonlinearity in the pattern selection.

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There is presently a large interest in the study of pattern formation. The initial studies were done in hydrodynamics [1], but similar phenomena were also encountered in many other areas including nonlinear optics [2]. Even though the first studies made with passive nonlinear optical systems mainly rediscovered effects that were already known in fluid mechanics such as the formation of hexagonal patterns [3-5], it is clear that light has properties that are markedly different from those of the fields of hydrodynamics and that these properties should lead to original developments. For instance, the fact that light is a quantum field leads to quantum complementarity between the near and far fields patterns [6]. Another characteristic of light is its polarization which is preserved or not at the onset of instability [7,8]. In nonlinear optics, the three most popular systems are as follows: A Fabry-Perot cavity containing a nonlinear medium [9], a nonlinear medium interacting with counterpropagating beams [3], and a thin nonlinear medium with single feedback mirror [4,5,10-12]. The typical patterns predicted and observed in all these systems are *hexagons, rings, and stripes.* The system consisting of a thin slab of nonlinear medium and a feedback mirror is particularly interesting because it appears to be relatively easy to handle both from a theortical and an experimental point of view. In particular, the possibility to vary the distance between the cell and the nonlinear medium [5] is an easy and reliable way to check the validity of a model. We present the results of an experiment where the nonlinear medium consists of rubidium atoms. A striking result of our experiment is the observation of *new patterns* completely different from those considered before for passive nonlinear media. These patterns look like *flowers* with a number of petals which varies with the distance between the feedback mirror and the rubidium cell. We show that this experiment strongly differs from those performed with Kerr media because *the nonlinearity is very different from a quadratic nonlinearity and the pattern selection is associated with the strong saturation of this nonlinearity.* This experiment shows how pattern formation is modified in situations where there is *no perturbative expansion* of the nonlinearity.

Consider a nonlinear medium interacting with two counterpropagating pump beams $E_f e^{i(\phi_f + kz)}$ and $E_b e^{i(\phi_b - kz)}$. For a weak incident field $E' e^{i(\phi' - kz)}$, the preceding system acts as a mirror: the reflected field $E e^{i(\phi + kz)}$ is the sum of two components, one proportional to $E' e^{i\phi'}$ and one proportional to $E' e^{-i\phi'}$ (phase-conjugate term). As a result of the interference between these two components, *phase variations in the incident field are transformed into amplitude variations in the reflected field* as shown experimentally in [13] (hence the name "phase-contrast mirror" given to this system). Because a phase-contrast mirror can give reflection coefficients much larger than 100% [14], a cavity consisting of a feedback mirror and a phase-contrast mirror can spontaneously oscillate. This is precisely how the instability for a slab of nonlinear material with a feedback morror, first described by Firth [11], was interpreted in a different way in [12]. For the optical pumping nonlinearity considered in [13,14], E and E' have a linear polarization *orthogonal* to that of E_f and E_b and, at first order in E', E is given by

$$E e^{i\phi} = \frac{a e^{i(\phi_f + \eta)} E_f E_b E'}{1 + b(|E_f|^2 + |E_b|^2)} \sin(\phi' - \phi_b) , \qquad (1)$$

where $ae^{i\eta}$ is a coefficient which characterizes the nonlinear medium: a is proportional to the product of the atomic density by the length of the nonlinear medium and η characterizes the nonlinearity ($0 \le \eta \le \pi$). In the case of a *dispersive* nonlinearity, η is equal to 0 when the laser is tuned below resonance and equal to π above resonance. The saturation term $b(|E_f|^2 + |E_b|^2)$ is the ratio of the optical pumping rate divided by the relaxation rate of the ground state associated with other processes. The medium can be considered as a Kerr medium when $b|E_f|^2 \ll 1$.

The distance between the thin nonlinear medium and the mirror (which has an intensity reflection coefficient equal to R) is equal to d (Fig. 1). We suppose that the forward pump can be described as a Gaussian beam ($E_f = \sqrt{I_f} e^{-r^2/w^2}, \phi_f = -kr^2/2R_f$) and we assume that $d/kw^2 \ll 1$. In the case where the mirror is flat (which is

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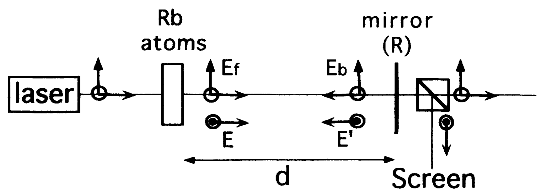

FIG. 1. Scheme of the experiment. A linearly polarized incident beam is transmitted through a cell containing rubidium atoms which is set at a distance d from a feedback mirror. An instability cross polarized with the incident beam can appear between the rubidium cell and the mirror. The instability can be separated from the incident beam using a Glan prism. The pattern of the instability is observed on a screen.

the situation of the experiment), this implies that the diameter of the reflected field in the nonlinear medium is nearly equal to the diameter of the incident field. The phase of the reflected field is $\phi_b = -(kr^2/2R_b)+\varphi_{00}$ with $\varphi_{00}=2(kd-2d/kw^2)$, the last term being the contribution of the Gouy phase. We now study in which conditions a Laguerre-Gauss mode having the same waist w and the same curvature R_f as the forward pump at the exit of the nonlinear medium can oscillate between the nonlinear material and the feedback mirror. We thus consider

$$E_{lm}=\sqrt{I}e^{-r^2/w^2}\left(\frac{\sqrt{2}r}{w}\right)^m L_l^m\left(\frac{2r^2}{w^2}\right)\times\cos(m\theta)e^{-eikr^2/2R_f}e^{i(\varphi+kz)}, \quad (2)$$

where L_l^m is a generalized Laguerre polynomial. A solution varying as $e^{im\theta}$ or $e^{-im\theta}$ is not consistent with the phase properties of the phase-contrast mirror which would mix these two modes. By contrast, a standing wave of the form $\cos[m(\theta-\theta_0)]$ is compatible with the boundary conditions on the nonlinear mirror. Consider now the field E'_{lm} obtained from E_{lm} after propagation between the nonlinear medium and the mirror and reflection on the mirror. The amplitude of E'_{lm} exhibits the same variations as E_{lm} versus r and θ, and its magnitude is just multiplied by $\sqrt{R}$. Its phase ϕ' is equal to $(-kr^2/2R_b)+\varphi+\varphi_{lm}$ where $\varphi_{lm}=2[kd-(2l+m+1)\times 2d/kw^2]$. Note that *the wave fronts of E and E' match the wave fronts of the pump beams E_f and E_b, respectively.* The oscillation threshold for the mode E_{lm} alone is obtained by replacing E and E' of Eq. (1) by the detailed expressions of E_{lm} [formula (2)] and E'_{lm}. One thus obtains $\varphi=\eta$ and

$$\frac{aR}{b(1+R)}c_{lm}(I_f)\sin\left[\eta-(2l+m)\frac{4d}{kw^2}\right]\geq 1, \quad (3)$$

where $c_{lm}(I_f)$ is an overlap integral describing the projection of the reflected field on the incident mode l,m:

$$c_{lm}(I_f)=\frac{l!}{(l+m)!}bI_f(1+R)\times\int_0^{+\infty}du\frac{e^{-2u}u^m|L_l^m(u)|^2}{1+bI_f(1+R)e^{-u}}. \quad (4)$$

In particular, when the saturation by the pump beams is very efficient $[bI_f(1+R)\gg 1]$, $c_{lm}(I_f)$ is a decreasing function of l and m which is *close to 1* for the lowest values of l and m because the radial extension of these modes is relatively small and they are thus localized at points of large pump intensity. Considering modes for which $c_{lm}(I_f)\approx 1$, we can compare the dependence versus d of the threshold condition with the dependence found by Firth [11] for the case of an extended system. We see that the emission angle of the instability is replaced in the threshold condition by $2\sqrt{2l+m}/kw$. Whereas a *continuous* variation of the pattern versus d was predicted (and observed [5]) in the case of an extended system, a *discrete* variation with sudden jumps from one pattern to another is expected here. Such a discrete variation originates from the *finite* dimension of the incident beam. Note also that the theoretical predictions of [15] performed for an incident beam of finite size cannot be applied here because the models for the nonlinearity are completely different. In fact, even in the frame of our model, the predictions are quite different for a Kerr medium ($bI_f=0$) and for a highly saturated medium ($bI_f\gg 1$). In the first case, *the pattern selection is mostly determined by the overlap integral* which permits the oscillation of a very few number of modes having small l and m values. In the second case, *the saturation diminishes the effect of the overlap* and the sine function of Eq. (3) has then a more dramatic importance in the pattern selection allowing modes of high l and m values to be observed. Finally, it is interesting to note that the Laguerre-Gauss patterns do not arise from the boundaries conditions imposed by the feedback mirror as in a usual cavity but that they are associated with the field distribution in the incident beam. If this field is not Gaussian, another basis may be more suited to describe the instability [16].

Our experimental setup consists of a $l=3$ cm rubidium cell at a temperature of about 100 °C which is placed in front of a flat multidielectric mirror with a reflection coefficient $R=96\%$. The distance d between the exit of the cell and the mirror can be continuously varied between 7 cm and 35 cm. The light source consists of a home-made single mode Ti-Sa laser pumped by an Ar^+ ion laser. For the experiments described in this paper its output was 200 mW. The beam is transmitted through a spatial filter and adjusted so that its focus is inside the rubidium cell. The beam diameter at the entrance of the cell is 0.8 mm which corresponds to a Rayleigh length $kw^2/2$ equal to 1.8 m. The laser beam was tuned on the high-frequency side of the $5S_{1/2}(F=3)$-$5P_{1/2}(F'=2)$ transition of 85 Rb at about 150 MHz from the center of

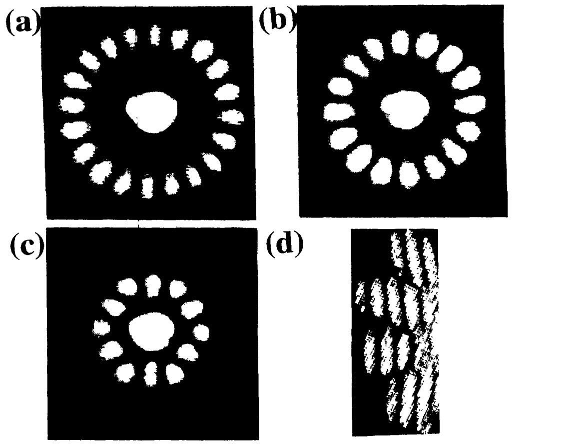

FIG. 2. Far-field patterns observed for $d=11$ (a), 17 (b), and 24 cm (c). The central part of the pattern corresponds to the pump beam which is slightly depolarized by the windows of the cell. These patterns look like flowers and can be described by Laguerre-Gauss modes with $l=0$ and $m=10$ (a), $m=8$ (b), and $m=6$ (c). The relative phase between consecutive petals is studied using an interference with an auxiliary beam. As shown in (d), a black fringe in one petal is followed by a white fringe in the next petal. This π phase shift, shown here for the case of a 12 petals pattern, is also observed for the other patterns.

this transition. It is clear that this power density allows a complete saturation of the electronic transition near the beam center and gives also very large optical pumping rates in the wings of the distribution of intensity [the condition $bI_f \exp(-2r^2/w^2) \gg 1$ is thus certainly satisfied in a large range of values of r]. A Glan prism is placed behind the reflection mirror to split the two polarizations components. The patterns for the two polarizations are observed on a screen located at a distance equal to 4 m from the cell. As expected from the mechanism of nonlinearity, the spatial instability is associated with a polarization instability [7] and the transverse pattern is mostly cross-polarized with the incident laser beam. Pictures of the far-field patterns observed for three different values of d are shown in Figs. 2(a)–2(c). These patterns look like flowers and are similar to Gauss-Laguerre modes with $l=0$ (the central part of the pattern mostly corresponds to the pump beam which is slightly depolarized by the windows of the cell [17]). We have compared the pictures of the near-field and far-field patterns and observed that they correspond to similar intensity distribution. To check the $\cos(m\theta)$ law of Eq. (2), we have studied the relative phase of the field for two neighbor petals using an interference with a reference beam. A shift of the interference fringes between consecutive petals is observed, a dark fringe in one petal corresponding to a white fringe in the next petal [Fig. 2(d)]. This shows the occurrence of a π phase shift as expected for the $\cos(m\theta)$ law. The observation of these stable fringes also shows that the instability has the same frequency as the laser. The number of petals in the pattern and its size evolve by jumps occurring for well-defined values of d. The variation of the number of petals N in the pattern versus $1/d$ is plotted in Fig. 3. We remark using Eq. (3) that, when $c_{lm}(I_f)=1$ and for $l=0$, the number of petals $N=2m$ should in average be proportional to $1/d$, the proportionality coefficient being $(\eta-\pi/2)(kw^2/2)$. Using $\eta=\pi$ and $kw^2/2=1.8$ m, we have plotted, with dashed lines, in Fig. 3 this theoretical straight line which is in agreement with the experimental points for low values of N. For larger values of N, the shift between the straight line and the experimental points can be interpreted from the decrease of $c_{lm}(I_f)$ when m increases. Actually, in this range of parameters, it should also be noticed that the nonlinear terms couple Laguerre-Gauss modes of different l values so that a description in terms of pure Laguerre-Gauss modes is probably inappropriate. Moreover, the understanding of what occurs for large values of m requires a good knowledge of the incident field distribution for $r \gg w$, i.e., in a range where the Gaussian distribution may no longer be correct. The Laguerre-Gauss modes are thus probably a good description of what occurs for low m values but a more complex description may be necessary for high m. One can also wonder why the only modes that are observed correspond to small values of l. We think that these modes are less affected by the strong absorption occurring in the rubidium cell. As shown by several authors [18], when the incident light is nearly resonant with a D_1 transition of alkalis, filaments of circularly polarized light propagate with a much weaker absorption. In the present experiment, the superposition of the oscillating beam with the laser beam gives a total field which is mostly σ^+ circularly polarized in one petal and mostly σ^- polarized in the next petal (because of the π phase shift between consecutive petals). As a result, the petals constitute a regular array of filaments which should lead to a maximum transmission.

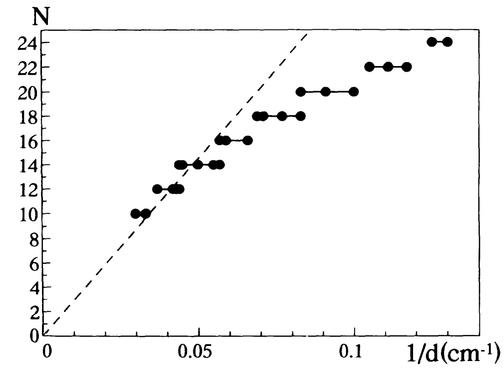

FIG. 3. Number of petals N vs $1/d$. The dots correspond to the experimental points. As expected, the observed patterns have an even number of petals. The dashed line corresponds to the theoretical prediction in the case of complete saturation. Saturation is more efficient for modes of low m values which have a smaller radial extension.

Although the present study is devoted to the understanding of the static properties of patterns, it can be noticed that temporal instabilities, similar to those reported earlier for counterpropagating beams [8,19], can also be observed for the scheme described in this paper by slightly changing the experimental conditions. A temporal variation of the intensity emitted in the petals can then be detected. This instability is most easily found when the backward beam is misaligned and for patterns having a large number of petals. The typical frequency is in the range 10 kHz–1 MHz which suggests that these temporal instabilities are also driven by optical pumping.

In conclusion, we have shown the occurrence of new stable patterns for a system consisting of a nonlinear medium and a feedback mirror. A key point in our analysis is the occurrence of saturation terms that cannot be expended in powers of the intensity and which lead to patterns very different from those found in Kerr media. This first experiment should stimulate new developments. First, a more precise theoretical analysis is certainly necessary. It should permit us to understand the behavior of patterns having a large number of petals and more generally to predict the pattern and its intensity as a function of the various parameters (beam waist, distance to mirror, incident intensity, and frequency detuning). Second, *these flowerlike patterns may be quite general:* We expect them to be found for other types of feedback system (counterpropagating beams for example) and other nonlinear media, the main requirement being the occurrence of an efficient saturation of the nonlinearity.

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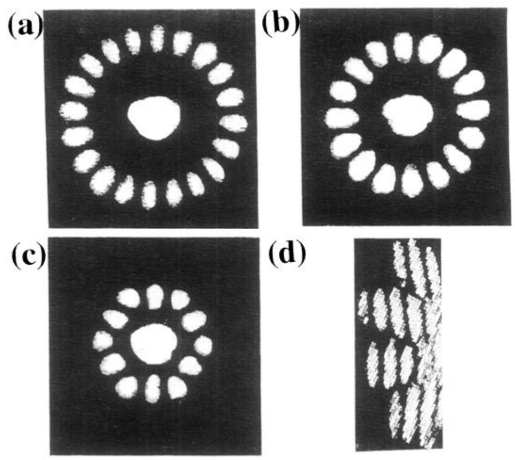

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