Meson Phase Space Density in Heavy Ion Collisions from Interferometry

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The interferometric analysis of meson correlations provides a measure of the average phase space density of the mesons in the final state. This quantity is a useful indicator of the statistical properties of the system, and it can be extracted with a minimum of model assumptions. Values obtained from recent measurements are consistent with the thermal value, but do not rule out superradiance effects.

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It would be interesting to know the average phase space density of the pions produced in ultrarelativistic heavy ion collisions. In the final state, the local phase space density is frozen (Liouville's theorem) and it gives a measure of the dynamics in the prior interacting region. If the system could be described by a local statistical equilibrium, the distribution function f would have the Bose-Einstein form, $f_T(p,r)=1/\{\exp[u(r)\cdot p/T]-1\}$. For massless particles the average of this quantity is a pure number,

$$\langle f\rangle_T=\frac{\int d^3p\,d^3r\,f_T^2}{\int d^3p\,d^3r\,f_T}=\frac{\zeta(2)}{\zeta(3)}-1\approx 0.37\,.$$

For massive bosons, the number is lower; for example, the pion average is

$$\langle f\rangle_{m,T}\approx 0.11\text{–}0.15 \tag{1}$$

for chemical freeze temperatures in the range 150–200 MeV. If the experimental $\langle f\rangle$ were close to Eq. (1), it would be welcome evidence for the existence of a local equilibrium. If $\langle f\rangle$ came out much larger, it would lend considerable support to the idea of superradiant pion states [1], which have been an object of renewed interest [2,3]. On the other hand, if $\langle f\rangle$ came out much smaller than thermal, it would point to entropy-generating processes such as the slow decay of heavy resonances or quark-gluon droplets.

In this Letter I want to point out that the measured two-particle correlations yield direct information about the phase space density of the mesons, when interpreted according to Pratt's interferometric formula [4] for the two particle correlation function,

$$C(p_1,p_2)\equiv\frac{d^6n^{(2)}/d^3p_1\,d^3p_2}{(d^3n^{(1)}/d^3p_1)(d^3n^{(1)}/d^3p_2)}=1+\frac{\int d^4x_1\,d^4x_2\,g(x_1,p)g(x_2,p)\cos q\cdot(x_1-x_2)}{\int d^4x\,g(x,p_1)\int d^4x\,g(x,p_2)}\,,$$

where $p=(p_1+p_2)/2$, $q=p_1-p_2$, and g is the source function for the mesons. I first write the formula as

$$\frac{d^6n^{(2)}}{d^3p_1\,d^3p_2}-\frac{d^3n^{(1)}}{d^3p}\frac{d^3n^{(1)}}{d^3p}=\int d^4x_1\,d^4x_2\,g(x_1,p)g(x_2,p)\cos q\cdot(x_1-x_2)\,. \tag{2}$$

I next convert the source function g to an equivalent source at a common time t_0 by the replacement

$$g(\mathbf{r},t,\mathbf{p})\to\delta(t-t_0)\int^{t_0}dt'\,g(\mathbf{r}-\mathbf{v}(t'-t_0),t',\mathbf{p})\equiv\delta(t-t_0)(2\pi)^3f(r,p)\,,$$

where v is the velocity associated with the momentum vector $\mathbf{p}$. This replacement does not affect the correlation function if $\mathbf{v}\cdot(\mathbf{p}_1-\mathbf{p}_2)=(E_1-E_2)$. The condition is satisfied for small momentum differences and for longitudinal motion of extreme relativistic particles, and seems rather safe for the present application. The coefficient of the δ function is then the phase space density at t_0, extrapolating the positions of the final state mesons to that time. I next integrate over the momentum difference d^3q, which produces a delta function $\delta^3(r_1-r_2)$ to eliminate one of the spatial integrals. The result is

$$\int\frac{d^3q}{(2\pi)^3}\int d^4x_1\,d^4x_2\,g(x_1,\mathbf{p})g(x_2,\mathbf{p})\cos q\cdot(x_1-x_2)=(2\pi)^3\int d^3r\,f^2(\mathbf{r},\mathbf{p})\,. \tag{3}$$

This quantity is the phase space density of particles of momentum $\mathbf{p}$ when normalized to $(2\pi)^3d^3n/d^3p$,

$$\langle f\rangle_{\mathbf{p}}=\frac{1}{d^3n/d^3p}\int d^3r\,f^2(\mathbf{r},\mathbf{p})\,.$$

Boson condensation or other strong Bose effects might show up in regions of momentum space where this quantity becomes of the order of unity [5]. For example, it would be particularly interesting to study the region of low transverse

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momentum, where bosonic enhancement effects have been suggested [6,7].

However, in this Letter I just examine an average which can be compared to the thermal value, Eq. (1). In the longitudinal expansion of the final state, different rapidity intervals become independent of each other, so I will consider an average in which the rapidity is fixed. Integrating over transverse momentum, the formula for a small rapidity interval reads

$$\langle f\rangle_{dy}=\frac{1}{dn/dy}\int\frac{d^2p_t}{p_0}\int d^3q\left[\frac{d^6n^{(2)}}{d^2p_{t1}dy_1d^2p_{t2}dy_2}\bigg|_{p+q/2,p-q/2}-\frac{d^3n^{(1)}}{d^2p_t\,dy}\bigg|_{p+q/2}\frac{d^3n^{(1)}}{d^2p_t\,dy}\bigg|_{p-q/2}\right]. \quad (4)$$

Since Eq. (4) is an integral measure of the correlation function, it should be less dependent on the accuracy of the momentum measurements than other observables.

The integral should undoubtedly be evaluated directly from the experimental data, but for an orientation I shall try to evaluate it from published NA44 parametrized distributions [8]. The NA35 experiment [9] obtained similar information, but did not quote the entire parametrization. One of the common parametrizations for these correlations is Gaussian with parameters λ and source sizes R_L, R_s, and R_0,

$$\frac{d^6n^{(2)}}{d^2p_{t1}dy_1d^2p_{t2}dy_2}=\{1+\lambda\exp[-\tfrac{1}{2}(q_L^2R_L^2+q_s^2R_s^2+q_0^2R_0^2)]\}\frac{d^3n^{(1)}}{d^2p_t\,dy}\bigg|_{p1}\frac{d^3n^{(1)}}{d^2p_t\,dy}\bigg|_{p2}.$$

The single-particle transverse momentum spectra can be parametrized by an exponential function,

$$\frac{d^3n^{(1)}}{d^2p_tdy}=\frac{dn}{dy}\frac{\exp(-p_t/T_t)}{2\pi T_t^2}.$$

With this parametrization, the average phase space density is given by

$$\langle f\rangle_{dy}=\left(\frac{\pi}{2}\right)^{1/2}\frac{1}{R_LR_sR_0T_t^3}.$$

Reference [8] quotes the following numbers for S+Pb $\rightarrow\pi^++X$ at midrapidity: $\lambda\approx0.4$, $R_t\approx6.0$ fm, $R_L\approx6.0$ fm, and $dn/dy\approx40$. From their Fig. 7 can be deduced $T_t\approx187$ MeV/c. Taking $R_s=R_0=R_t$ and inserting these numbers in Eq. (3), I obtain

$$\langle f\rangle_{dy}\approx0.07\text{–}0.16.$$

The range is obtained from the quoted experimental errors combined quadratically. I note that the NA35 experiment found a somewhat smaller source size; the lack of agreement between experiments is a caution not to draw strong conclusions from the present data.

One should also be reminded that the assumptions going into the interferometric formula, Eq. (2), may not be well satisfied. The most critical assumption is that the one-particle distribution function is not affected by the Bose symmetrization, which is only satisfied for low phase space densities.

Given these caveats, what does one conclude? The extracted phase space density is consistent with local statistical equilibrium for a chemical freezeout temperature in the 100–200 MeV range. Some slight enhancement might be expected from the chemical freezeout of pions occurring before the kinetic freezeout [10]. Also, the analysis includes pions from long-lived resonance decays such as $\omega\rightarrow3\pi$ and $\eta\rightarrow3\pi$. If these could be subtracted out, the phase space density would be somewhat higher. Thus there may be an excess of pions produced in the source, and the possibility of coherent pion effects should not be ruled out.

The extracted phase space density appears high enough to make unlikely that long-lived intermediates such as droplets of quark-gluon plasma are produced abundantly. This is consistent with the present theoretical expectation of no strong first-order phase transition in the quark-gluon plasma [11].

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