Nonlinear Conductivity of a Wigner Crystal in a Strong Magnetic Field

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We investigate the sliding state of a two-dimensional Wigner crystal in a strong magnetic field in the presence of randomness and free carriers, using a high-velocity perturbation theory. We find that the Hall resistivity retains its classical value within the second order Born approximation, and that disorder induces predominantly transverse distortions. We calculate the interference effects that arise in combined ac+dc fields for the present system. We examine recent experiments that measure the nonlinear conductivity for the insulating phases in the fractional quantum Hall effect regime.

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The possibility of observing a Wigner crystal in a two-dimensional (2D) electron gas in a strong magnetic field continues to draw attention [1–6]. Around magnetic filling factor $\nu = 1/5$, reentrant insulating phases have been established in the cleanest samples currently available [2–6]. Nonlinearities in the conductivity, attributed to the collective sliding motion of the Wigner crystal, have been reported by several groups [3–6]. Unfortunately, at present, these experiments differ in many important ways, making a consistent interpretation difficult [7].

In this work we provide some theoretical basis for a more rigorous and quantitative interpretation of these interesting experiments. We calculate the nonlinear conductivity and various correlation functions in the sliding state of a Wigner crystal in the presence of strong magnetic field, random disorder potential, and free carriers. We model the Wigner crystal as an elastic medium and use a perturbative treatment of disorder effects, valid at large sliding velocities. We estimate the regime of validity of the elastic theory. We discuss the current experiments in light of our results, and suggest some future experiments.

We view the Wigner crystal as a deformable classical elastic medium and adopt the Fukuyama-Lee-Rice (FLR) model for pinning, originally devised for charge-density wave (CDW) systems [8–14]. We first ignore the free carriers that are thermally excited, and assume that in the ground state there are no free carriers to carry transport current. The equation of motion in real space is then

$$\rho_m \ddot{\mathbf{u}} + \lambda \rho_m \dot{\mathbf{u}} + \rho_m \omega_c \mathbf{e}_z \times \dot{\mathbf{u}} + \int D(\mathbf{r}-\mathbf{r}') \cdot \mathbf{u}(\mathbf{r}',t) d^2\mathbf{r}' = \rho_c \mathbf{E}^{\rm ext} - \rho(\mathbf{r}) \nabla_{\mathbf{r}} V_{\rm dis}(\mathbf{r} + \mathbf{u}(\mathbf{r},t)), \quad (1)$$

where the Wigner crystal is taken to be in the (x, y) plane, $\mathbf{e}_z$ is a unit vector in the direction of the magnetic field, $\mathbf{u}(\mathbf{r}, t)$ is the 2D displacement at $(\mathbf{r}, t)$, ρ_m and ρ_c are respectively the average mass and charge density, λ is a phenomenological damping constant that includes all other sources of dissipation except those of disorder and free carriers, $\omega_c = \rho_c B/\rho_m c$ is the cyclotron frequency, and $D(\mathbf{r} - \mathbf{r}')$ is the dynamic matrix. $\rho(\mathbf{r}) = \sum_{\mathbf{G}} \rho(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}$ on the right-hand side is the spatially varying charge density of the original Wigner crystal with $\mathbf{G}$ being a reciprocal lattice vector, and $\mathbf{E}^{\rm ext}$ is the external driving field. In the presence of a strong magnetic field, as long as the relevant frequency is much less than $\min(\lambda, \omega_c)$, we can always ignore the inertial term [7]. This condition is invariably satisfied in actual experiments and we will neglect the inertial term in Eq. (1) henceforth [7].

Related models with $B = 0$ have been used for the CDW systems [11–13]. They exhibit a pinned phase below a threshold field $E_{\rm Th}$, and a sliding phase above it. Roughly, $E_{\rm Th} \sim \rho_m c^T a / 2\pi \rho_c \xi_T^2$ [8], where a is the lattice constant, ξ_T is the correlation length, and c^T the shear modulus. For $B \neq 0$, the low energy dynamics is determined by a mode with dispersion $\omega^2 \omega_c^2 = c^L c^T (k/a)^3$. Here $c^T = \alpha \Omega^2 a^2$ and $c^L = \Omega^2 [2\pi a + (1+\alpha) a^2 k]$, with $\alpha \sim 0.02$, and $\Omega^2 = \rho_c^2 / \rho_m a \epsilon$ [8]. The present model differs from the previous CDW cases in that it involves a large magnetic field, different dispersions of the collective modes, and charge-density order parameter $\rho(\mathbf{G})$ that are nonzero for $\mathbf{G}$'s on a two-dimensional grid.

We study in this paper the sliding phase of Eq. (1) in a large external dc field $\mathbf{E}^{\rm ext}$. It is convenient to write

$$\frac{\rho_c}{\rho_m} \mathbf{E}^{\rm ext} = \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \mathbf{v}_0 \quad (2)$$

and

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{v}_0 t + \mathbf{x}(\mathbf{r}, t). \quad (3)$$

$\mathbf{v}_0 t$ solves Eq. (1) if $V_{\rm dis} = 0$; $\mathbf{x}(\mathbf{r}, t)$, represents the additional displacement due to $V_{\rm dis}$. Formally, the Fourier transformed $\mathbf{x}$ can be written in terms of the tensor Green's function G as

$$\mathbf{x}(\mathbf{k}, \omega) = \mathsf{G}(\mathbf{k}, \omega) \cdot \mathbf{F}(\mathbf{k}, \omega; \{\mathbf{x}\}), \quad (4)$$

where $\mathbf{F}(\mathbf{k}, \omega; \{\mathbf{x}\})$ is the Fourier transform of $-\rho(\mathbf{r}) \times \nabla_{\mathbf{r}} V_{\rm dis}[\mathbf{r} + \mathbf{v}_0 t + \mathbf{x}(\mathbf{r}, t)]$. G is

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$$\mathsf{G}(\mathbf{k},\omega) = P^{-1}(\mathbf{k},\omega)\left[c^L k\left(1-\frac{\mathbf{kk}}{k^2}\right) + c^T k^2\frac{\mathbf{kk}}{k^2} - i\omega\begin{pmatrix}\lambda & \omega_c\\ -\omega_c & \lambda\end{pmatrix}\right], \tag{5}$$

where we have left out the cyclotron resonance in G, and taken the long wavelength, elastic medium theory limit which implies $\omega \ll \Omega$. (Throughout this paper, G denotes the 2×2 Green's function, and $\mathbf{G}$ denotes the reciprocal lattice vectors.) The poles in G then determine the collective modes at $\omega \ll \omega_c$, given by

$$P(\mathbf{k},\omega) = 2\pi\Omega^4\alpha\rho_m[(ka)^3 - \kappa_\omega^3 - i(ka)\kappa_\lambda^2] = 0. \tag{6}$$

In the above equation, we have defined two dimensionless quantities:

$$\kappa_\omega^3 = \frac{\omega^2(\omega_c^2+\lambda^2)}{2\pi\Omega^4\alpha}, \quad \kappa_\lambda^2 = \frac{\omega\lambda}{\Omega^2\alpha}. \tag{7}$$

κ_ω/a is the wave vector at which the $k^{3/2}$ mode defined before has an energy ω, and κ_λ measures the strength of dissipation at the frequency ω. Equation (6) gives the response function of the system: For $\kappa_\omega \ll \kappa_\lambda$, the response given by Eq. (6) is overdamped; but in the opposite limit, it is underdamped despite the fact that $\omega \ll \lambda$, owing to the presence of a large magnetic field.

We generate a perturbation series for $\mathbf{x}(\mathbf{r},t)$ in powers of the random potential by expanding $\mathbf{F}(\{\mathbf{x}\})$ in Eq. (4) in powers of $\mathbf{x}$ and solving it iteratively [7,12,13]. We obtain for the displacement to first order in $V_{\rm dis}$

$$\mathbf{x}(\mathbf{r},t) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}+i(\mathbf{q}\cdot\mathbf{v}_0)t}[-\rho(\mathbf{k}-\mathbf{q})][\mathsf{G}(\mathbf{k},-\mathbf{q}\cdot\mathbf{v}_0)\cdot(i\mathbf{q})]V_{\rm dis}(\mathbf{q}), \tag{8}$$

and the velocity to second order

$$\rho_m\begin{pmatrix}\lambda & -\omega_c\\ \omega_c & \lambda\end{pmatrix}\cdot\langle\dot{\mathbf{x}}\rangle = \sum_{\mathbf{G}}|\rho(\mathbf{G})|^2\int\frac{d^2\mathbf{q}}{(2\pi)^2}\ \mathbf{q}\Gamma(\mathbf{q})\{\mathbf{q}\cdot[-{\rm Im}\mathsf{G}(-\mathbf{q}+\mathbf{G},\mathbf{q}\cdot\mathbf{v}_0)]\cdot\mathbf{q}\}, \tag{9}$$

where $V_{\rm dis}(\mathbf{q}) = \int d^2\mathbf{r}\, e^{-i\mathbf{q}\cdot\mathbf{r}}V_{\rm dis}(\mathbf{r})$, and $\langle V_{\rm dis}(\mathbf{q})V_{\rm dis}(\mathbf{q}')\rangle = \delta(\mathbf{q}+\mathbf{q}')\Gamma(\mathbf{q})$, where $\langle\cdots\rangle$ indicates averaging over disorder. In the above two equations, integration over $\mathbf{k}$ is restricted within the first Brillouin zone and that over $\mathbf{q}$ is over the entire reciprocal lattice space.

We now focus on Eq. (9), the integral in which is dominated by two infrared terms where the elastic medium theory is valid. They are respectively related to κ_ω and κ_λ in Eqs. (6) and (7) with $\omega = \omega_v = |\mathbf{G}\cdot\mathbf{v}_0|$. Both κ_ω and κ_λ are much less than unity at this frequency [7], and their ratio $\kappa_\omega/\kappa_\lambda = (\omega_v\omega_c^4\alpha/4\pi^2\lambda^3\Omega^2)^{1/6}$ depends on the sliding velocity $\mathbf{v}_0$. We find that in general

$$\begin{aligned}\begin{pmatrix}\lambda & -\omega_c\\ \omega_c & \lambda\end{pmatrix}\cdot\langle\dot{\mathbf{x}}\rangle &= -\alpha_0\frac{1}{4\pi\rho_m^2 c^T}\sum_{\mathbf{G}}|\rho(\mathbf{G})|^2\Gamma(\mathbf{G})|\mathbf{G}|^2\mathbf{G}\ {\rm sgn}(\mathbf{G}\cdot\mathbf{v}_0)\\ &= -(E_0\rho_c/\rho_m)\frac{\mathbf{v}_0}{v_0},\end{aligned} \tag{10}$$

where the constant α_0 is $\pi/3$ if $\kappa_\omega \gg \kappa_\lambda$ and is $\pi/4$ if $\kappa_\omega \ll \kappa_\lambda$, and the second equality, where it is assumed that sliding occurs along the direction of one of the reciprocal lattice vectors $\mathbf{G}$, defines the quantity E_0. E_0 is uncertain up to a numerical prefactor of order unity that depends on the direction of $\mathbf{v}$ relative to the lattice orientation.

It is then straightforward to obtain the sliding current which is given by the total velocity $\mathbf{v}_{\rm tot} = \mathbf{v}_0 + \langle\dot{\mathbf{x}}\rangle$, for a given external field $\mathbf{E}^{\rm ext}$ in Eq. (2). For later convenience, we here give results in the equivalent form of resistivity: Taking the direction of the current flow to be along $\mathbf{x}$ ($j_{\rm tot} = j_x$), we find

$$E_y = \frac{\omega_c\rho_m}{\rho_c^2}j_x, \tag{11}$$

$$E_x = \frac{\lambda\rho_m}{\rho_c^2}j_x + E_0. \tag{12}$$

E_0 is an estimate of the threshold field, which differs only by a numerical prefactor from that estimated from the static properties [8]. Notice that only c^T, the shear modulus, appears in the above equations. In a recent work [15], a perturbation theory on the static lattice is carried out. Our expression for E_0, the approximate depinning threshold, is similar to the expression for the square of the pinning gap obtained for a static Wigner crystal in Eqs. (3) and (4) of Ref. [15]. However, unlike Ref. [15], we focus in the sliding state on the infrared singular contribution, which depends only on the shear modulus. We will present a detailed discussion of the relation between calculations for the static and sliding crystals in a subsequent paper [7].

We see from Eq. (11) that ρ_{xy} is not modified to leading order E_0. We believe this result holds everywhere in the sliding phase, provided the pinning is isotropic on average, because if the whole Wigner crystal slides with an average velocity v, then there is an average Lorentz force per unit area $vB = jB/\rho_c$ (j is the sliding current) which must be balanced by a transverse electric field, implying $\rho_{xy} = B/\rho_c$. Also, in Eq. (12) the correction term E_0 does not depend on the sliding velocity itself, as was previously found for 2D CDWs [12,13]: The differences between the CDW and the present system outlined in the introduction do not change this result.

We now consider the effects of thermally excited free carriers [9], which become important at somewhat higher temperatures ($\sim$ 50 mK and above). Under the assumption that there is no interconversion between the free carriers and the sliding lattice, and that the coupling between the Wigner crystal and the free carriers is electrostatic, the effects amount to changing the longitudinal propagator to [8]

$$c^L k = \Omega^2 \left[\frac{2\pi ka}{1 - (ika\sigma_{xx}^F/\sigma_0)/(\omega/\Omega)} + (1+\alpha)(ka)^2 \right], \tag{13}$$

where σ_{xx}^F is the longitudinal conductivity of the free carriers which is the measured conductivity below the sliding threshold [8]. The conductivity scale, $\sigma_0 = a\epsilon\Omega/2\pi$, is slightly smaller than e^2/h for a typical GaAs/AlGaAs modulation doped FQHE sample. The quantity $M(k,\omega) = \frac{ka\sigma_{xx}^F/\sigma_0}{\omega/\Omega}$ determines the importance of free carriers at wave vector k and frequency ω. For $M \gg 1$, $c^L k = i\omega\lambda_1 + \Omega^2(ka)^2$, with $\lambda_1 = 2\pi\Omega\sigma_0/\sigma_{xx}^F$, in which limit the free carriers simply provide an effective damping term.

By repeating the perturbative analysis, we find [7] that the resulting nonlinear conductivity is the same as Eqs. (11) and (12), because λ does not enter E_0 in Eq. (10). This is a special feature of the second order perturbation theory in two dimensions. However, the enhanced dissipation must enter at fields close enough to the threshold where longitudinal distortions are large; therefore we expect a crossover from a high-velocity regime where the differential conductivity is σ_{xx}^F independent, to a lower velocity regime where it depends on σ_{xx}^F, and consequently on temperature.

We now turn to the recent experiments. Detailed nonlinear I-V features from the experiments vary from sample to sample, and within one sample from cooldown to cooldown. According to the behavior of the conductivity at fields above threshold, the current experiments fall into two categories:

(1) The *differential* conductivity is voltage- and temperature-independent above threshold. This is reported in Ref. [4] which finds a very large threshold field ($\sim$ 50 mV) at the lowest temperatures $\sim$ 36 mK, and in Ref. [5] which finds a much smaller threshold field ($\sim$ 0.2 mV) and is done at very low temperatures $\sim$ 20 mK.

(2) The differential conductivity above the threshold field continues to depend on voltage and temperature, as reported in Refs. [3] and [6]. The threshold field is found to be small ($\sim$ 0.2 mV) [3,6]. The temperature dependence of the differential resistivity is found in Ref. [6] to be thermally activated with an activation gap identical to that below threshold, for temperatures between $\sim$ 80 mK and $\sim$ 200 mK. However, it is also reported in Ref. [6] that there exists a second "upper threshold" field at $\sim$ 30 mV above which the differential resistivity is no longer field or temperature dependent, and its value is consistent with those in Refs. [4,5].

We first compare our theory with experiments done at the lowest temperatures where thermally activated free carriers are unimportant, i.e., the experiments in the first category described above. The predicted I-V curve from Eq. (12) agrees very well with the low temperature experiments (Fig. 2 in Ref. [5] and Fig. 2 in Ref. [4]) —that is, the differential dc resistivity does not depend on the driving field, or equivalently on the resulting current above the sliding threshold. In addition, the intercept of the I-V curve from the high-velocity perturbation theory also gives a measure of the sliding threshold field. From Eqs. (11) and (12), one sees that this statement can only be true when, in plotting the I-V, the field V is measured along the direction of the current flow [7].

We now consider the large field ($>$ 30 mV), therefore high-velocity data, reported in Ref. [6] in which electron heating was invoked to explain the data. We wish to propose an alternative possibility. The differential conductivity in the $V >$ 30 mV regime is field and temperature independent, and has roughly the same value ($\sim 10^5\ \Omega$) as the lowest temperature data in Ref. [5]. This experimental fact [16] is consistent with our result that the large-velocity differential resistivity is σ_{xx}^F independent. However, as we pointed out above, as the velocity decreases, a crossover is expected to a regime where σ_{xx}^F affects the differential conductivity, and where our second order perturbation theory is no longer valid. This crossover may correspond to the upper threshold in Ref. [6].

To provide some qualitative feel for the sliding state and to check the adequacy of the elastic medium approximation, we have also calculated the velocity-velocity and the strain-strain correlation functions [7]. The latter is defined as (averaged over disorder)

$$E_L^s = [\nabla \cdot \mathbf{x}(\mathbf{r},t)]^2, \quad E_T^s = [\nabla \times \mathbf{x}(\mathbf{r},t)]^2. \tag{14}$$

They are respectively proportional to the *elastic* energies associated with the bulk and shear moduli if there were no long range Coulomb interaction. We find

$$E_T^s = \beta \frac{E_0\rho_c/\rho_m}{\Omega^2 \alpha a} \ln \frac{1}{\max(\kappa_\lambda, \kappa_\omega)} \sim \left(\frac{a}{\xi_T}\right)^2 \ln \frac{\Omega^2\alpha}{\omega_v \lambda}. \tag{15}$$

Here β is a dimensionless constant of the order of unity. At small velocities, it diverges very weakly as $\ln \frac{E-E_0}{E_0}$. Although the functional form will likely be modified by higher order terms in perturbation theory, this indicates the breakdown of the elastic approximation close enough to threshold [14]. The divergence is so weak and the prefactor so small that we expect in practice the elastic approximation to be valid throughout the sliding regime. Using parameters typical of those in the experiments [8], E_T^s is about 10% when the field is a few percent greater than the threshold field according to Eq. (15). Similar analysis shows that in all cases $E_L^s/E_T^s < (\alpha/2\pi)^2 \sim 10^{-5}$.

We have also explored the combined ac + dc effects in the present system. A full account of the results will be given later [7]. For convenience, we define

$$A(\mathbf{G}) = \frac{1}{4\pi\rho_m c^T}|\rho(\mathbf{G})|^2\Gamma(\mathbf{G})|\mathbf{G}|^3. \tag{16}$$

For a dc current $\mathbf{j}_{\rm dc} = \rho_c \mathbf{v}$ in the presence of an ac current $\mathbf{j}_{\rm ac} = \rho_c \mathbf{v}^{\rm ac}\cos\omega_0 t$, the dc field (Shapiro anomaly) is found to be

$$\rho_c\mathbf{E} = \rho_m \begin{pmatrix} \lambda & -\omega_c \\ \omega_c & \lambda \end{pmatrix} \cdot \mathbf{v} + \rho_c E_0 \mathbf{v}/v + \alpha_0 \sum_{\mathbf{G},n\neq 0} A(\mathbf{G})\mathrm{sgn}(\mathbf{G}\cdot\mathbf{v} - n\omega_0) J_n^2(\mathbf{G}\cdot\mathbf{v}^{\rm ac}/\omega_0)\mathbf{v}/v. \tag{17}$$

$J_n(x)$ is the nth Bessel function. The inclusion of higher order $\mathbf{G}$'s introduces more resonances which become progressively weaker due to the $\mathbf{G}$ dependence of $A(\mathbf{G})$. However, the decay is slow if the interparticle distance is much greater than the magnetic length.

The linear ac response depends on the relative direction of the ac and the dc current flow [7]. Here we assume that they are parallel. For an ac current given above, the in-phase component of the external ac field parallel to the current (i.e., Reρ_{xx}) is

$$(\rho_c\mathbf{E})_{\rm in} = \rho_m \lambda \mathbf{v}^{\rm ac} + \alpha_0 \sum_{\mathbf{G}} A(\mathbf{G}) \frac{\mathbf{v}^{\rm ac}}{a\omega_0}\mathrm{sgn}(\omega_0 - \mathbf{G}\cdot\mathbf{v}), \tag{18}$$

and the out-of-phase component (Imρ_{xx}) is

$$(\rho_c\mathbf{E})_{\rm out} = \sum_{\mathbf{G}} A(\mathbf{G}) \frac{\mathbf{v}^{\rm ac}}{a\omega_0} \ln\frac{|\omega_0 - \mathbf{G}\cdot\mathbf{v}|}{\mathbf{G}\cdot\mathbf{v}}. \tag{19}$$

The out-of-phase part of the longitudinal ac resistivity should show an inductive to capacitive transition as ω_0 goes from below $\sim \mathbf{G}\cdot\mathbf{v}$ to above. The in-phase diagonal resistivity changes from roughly $\frac{\rho_m}{\rho_c^2}[\lambda - E_0\rho_c/(\rho_m a\omega_0)]$ to $\frac{\rho_m}{\rho_c^2}[\lambda + E_0\rho_c/(\rho_m a\omega_0)]$ as ω_0 increases through $\mathbf{G}\cdot\mathbf{v}$. The Hall resistivity for both the Shapiro anomaly and the ac response (with parallel dc and ac currents) shows no interference effects. Preliminary experimental data have indeed observed an inductive behavior in the low-frequency ac response of a sliding Wigner crystal, which when pinned exhibits a capacitive response [17].

To summarize, we have studied the sliding state of the Wigner crystal in the FQHE regime with a high-velocity perturbation theory. The theoretical nonlinear conductivity agrees with experimental results obtained at the lowest temperatures or large external fields. In addition, we predict interference phenomena under a combined ac and dc field.

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