Coulomb Effects in Transport Properties of Quantum Wires

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The Coulomb $1/r$ interaction plays an important role in quantum wires. We study the interplay between this long-range interaction and impurity. For a single band quantum wire, we find that the transport properties are strongly modified. The linear conductance G is found to vanish with temperature as $G \sim \exp[-\nu \ln^{3/2}(1/T)]$ and the current voltage characteristics acquires a thresholdlike behavior. The possible extension of the theory to the case of many-band quantum wires is discussed.

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The recent progress in the submicron-size technology made it possible to fabricate very clean quantum wires, where only a few (even one) electron channels are populated, and where only several impurities or defects are effective [1]. In this situation Anderson's localization does not occur. Nevertheless, there is theoretical evidence that, even in the presence of *one single* impurity, repulsive electron-electron interaction leads to a conductance that vanishes with temperature [2–4]. Previous studies have dealt with short-range interaction, which was shown to lead to a conductance $G \sim T^{\gamma}$, where γ is a nonuniversal exponent. The assumption of a short-range repulsion is, however, justified only if Coulomb interaction is efficiently screened, e.g., by the vicinity of metallic environments, but this is not the case for many experimental setups (see, e.g., Ref. [5]). The role of unscreened Coulomb repulsion was recently explored in a clean 1D system by Schulz [6], who discussed how the Wigner crystallization arises from long-range repulsion.

In this paper, we address the following question: Does Coulomb repulsion modify the transport properties in presence of an impurity, and how? We show that the conductance is indeed drastically affected by a long-range repulsion, and vanishes with temperature faster than any power.

We study a 1D electron gas with Coulomb $1/r$ repulsion. It is known that the low-energy physical properties of the 1D electron gas are quite different from those in higher dimensions [7]. There are no coherent single-particle excitations but only collective charge and spin sound modes, which are dynamically independent (so-called spin-charge separation). These sounds were recently measured by light scattering in quantum wires [5]. A helpful analogy is to imagine these density fluctuations just as sound waves in an elastic string. The method of connecting the original description of the system in terms of electron creation and annihilation operators and the description in terms of the only existing coherent excitations (charge and spin sounds) is known as bosonization [7,8]. According to this method, the electron annihilation operator can be written as

$$\Psi_\sigma(x) \sim \sum_{n=\pm 1} \exp\left\{in\left[k_F x + \sqrt{\pi}\phi_\sigma(x)\right]\right\} \times \exp\left[i\sqrt{\pi}\theta_\sigma(x)\right], \quad (1)$$

where $\sigma = \uparrow, \downarrow$ labels the spin. The bosonic phase fields are related to the low-momentum electron densities by $\rho_\sigma(x) = \sqrt{1/\pi}\,\nabla\phi_\sigma(x)$ and $\nabla\theta_\sigma(x) = \Pi_\sigma(x)$, where Π_σ is the momentum conjugate to ϕ_σ. The phase fields, corresponding to the charge and spin density excitations, are defined by

$$\phi_{c/s} = \frac{1}{\sqrt{2}}\left(\phi_\uparrow \pm \phi_\downarrow\right).$$

In terms of bosonic fields, the charge excitations are described by the Hamiltonian [6,8]

$$H_c = \frac{v_F}{2}\int dx\left[(1+g_1)\Pi_c^2 + (1-g_1)(\nabla\phi_c)^2\right] + \frac{1}{\pi}\int dx\,dx'\,U(x-x')\nabla\phi_c(x)\nabla\phi_c(x'), \quad (2)$$

where v_F is the Fermi velocity, g_1 is the strength of the interaction at $2k_F$ transferred momentum. The momentum Π_c, conjugate to ϕ_c, is related to the current density by $I(x) = ev_F\sqrt{2/\pi}\,(1+g_1)\Pi_c(x)$. The potential $U(x)$ is the matrix element of the three-dimensional Coulomb interaction $e^2/\kappa r$ with respect to the transverse eigenfunction of the lowest band in the confining potential; κ is the dielectric constant. The potential behaves as $U(x) \sim 1/|x|$ at large x while it approaches a finite value for $|x|$ smaller than the width l of the confining potential, so that at small transferred momentum q, its Fourier transform is $U(q) = (2e^2/\kappa)\ln(1/|q|l)$. The singularity at small q of the potential leads to a nonlinear dispersion relation of the charge sound mode (*plasmon*)

$$\omega_c(q) = v_F|q|\sqrt{(1+g_1)[1-g_1+2U(q)/\pi v_F]} \quad (3)$$

$$\to v_F|q|\sqrt{\frac{4\alpha}{\pi}(1+g_1)\ln\left(\frac{1}{|q|l}\right)}, \quad \text{for } q \to 0,$$

where $\alpha = e^2/\kappa v_F$ is the dimensionless Coulomb coupling constant. For repulsive interaction, the spin excitations are described by a Hamiltonian H_s of a form similar to Eq. (2) but without the long-range part of the Coulomb interaction. Therefore the spin sound mode $\omega_s(q)$ maintains a linear dispersion relation at small q.

In the presence of impurities, the total Hamiltonian contains an additional term

$$H_{\text{imp}} = \int dx\, V(x) \sum_\sigma \Psi_\sigma^\dagger(x)\Psi_\sigma(x). \tag{4}$$

We consider the case of a single impurity which is represented by a short-range potential $V(x)$. By means of Eq. (1), we express the potential scattering in terms of the bosonic fields [2]:

$$H_{\text{imp}}(\phi_{c0}, \phi_{s0}) = \sum_{n,m} V_{n,m} \cos(\sqrt{2\pi} n\phi_{c0}) \cos(\sqrt{2\pi} m\phi_{s0}). \tag{5}$$

Here $\phi_{c(s)0}$ is the charge (spin) field at the impurity site, and the integers n and m are such that $n+m$ is even. The term $V_{1,1}$ is the single-electron backscattering, Eq. (4). The terms with $n \neq 1$ or $m \neq 1$ describe multielectron scattering processes at the impurity site. They are not actually present in (4), but, since they are generated by perturbation theory, they should be included in the effective Hamiltonian as explained in Ref. [2].

Following Refs. [2,3], we express the partition function $Z = Sp\exp(-H/T)$, with $H = H_c + H_s + H_{\text{imp}}$, as an imaginary-time path integral over the bosonic fields. Then we integrate out all the degrees of freedom, except those at the impurity site. After this integration, the free part of the action as a functional of the fields $\phi_{c0}(\tau)$ and $\phi_{s0}(\tau)$ at the impurity site $[0 \leq \tau \leq 1/T]$ takes the form

$$S_0[\phi_{c0}, \phi_{s0}] = T \sum_n \sum_{\nu=c,s} J_\nu(\omega_n) |\phi_{\nu 0}(\omega_n)|^2, \tag{6}$$

where $\omega_n = 2\pi n T$, and the spectral density J_ν is defined as

$$J_\nu^{-1}(\omega_n) = \int \frac{dq}{\pi} \langle \phi_\nu(q,\omega_n)\phi_\nu(-q,-\omega_n)\rangle = \frac{K_\nu(\omega_n)}{|\omega_n|}. \tag{7}$$

For short-range interaction, $K_{c(s)}(\omega_n \to 0)$ is a finite constant which determines the low-energy behavior of the charge (spin) $2k_F$ density-density correlation function. As a result of the divergent charge sound velocity for long-range interaction [see Eq. (3)], $K_c(\omega_n)$ vanishes with $\omega_n \to 0$ and the spectral density behaves as

$$J_c(\omega_n) \sim |\omega_n| \sqrt{-\ln|\omega_n|},$$

while $K_s(\omega_n \to 0)$ is finite.

The total action [9], which we obtain by integrating out all the degrees of freedom away from the impurity site, can be written as

$$S[\phi_{c0}, \phi_{s0}] = S_0[\phi_{c0}, \phi_{s0}] + \int_0^{1/T} d\tau H_{\text{imp}}(\phi_{c0}(\tau), \phi_{s0}(\tau)). \tag{8}$$

In order to analyze the low-temperature behavior of the model, we make use of standard renormalization group (RG) methods. In the *weak* barrier limit, it is straightforward to obtain first order scaling equations for the effective potentials:

$$\frac{dV_{n,m}(\xi)}{d\xi} = V_{n,m}(\xi)\left[1 - \frac{3n^2\nu}{4\sqrt{\xi}} - \frac{m^2 K_s}{2}\right], \tag{9}$$

where $\xi = \ln(1/T) \to \infty$ and $\nu = (2/3)\sqrt{(1+g_1)\pi/4\alpha}$. It is important to realize that $V_{n,0}$ grows under RG scaling process, independently of the precise values of the parameters entering the equations, in contrast to the case of short-range interaction studied in Refs. [2,3].

As the impurity potential flows towards strong coupling, a perturbation expansion of the conductance in powers of the potential is not justified, since the impurity corrections diverge [10]. Therefore we have to resort to nonperturbative methods. One possibility has been recently devised for the short-range interaction case by Matveev, Yue, and Glazman [4], but it relies on weak interaction. This is, however, usually not the case for real systems where Coulomb interaction α is of the order of unity [1]. Therefore it is more appropriate to start from the opposite limit [2,3] of a strong barrier. In this limit we have to minimize the impurity potential Eq. (5) first. It has an infinite number of equivalent minima, which are related by a shift (n,m) of the fields $(\phi_{c0}, \phi_{s0}) \to (\phi_{c0}+n\sqrt{\pi/2}, \phi_{s0}+m\sqrt{\pi/2})$, for $n+m$ even. The kinetic term allows the fields to tunnel. The tunneling events, which produce the shifts $(\pm 1, \pm 1)$, have the physical meaning of transferring a single electron through the barrier. Similarly the shifts $(\pm 2, 0)$ represent the transfer of a singlet pair of electrons. The generic shift (n,m) corresponds to multielectron tunneling process.

The probability $P(n,m)$ of tunneling between two minima, connected by the shift (n,m), can be calculated in a standard way by the Fermi golden rule. We will not write the full expression for $P(n,m)$ since it is quite cumbersome, but only its leading low temperature behavior $P(n,m) \sim \exp[-\nu n^2 \ln^{3/2}(1/T)]$. An applied electric field lifts the degeneracy between those minima in the direction of ϕ_{c0}. The tunneling processes which transfer one charge unit [shifts $(1,m)$] give the dominant contribution to the electric current. Without presenting the detailed calculations, we simply quote the results for the linear conductance and current voltage characteristics (I-V):

$$G(T) \sim \exp[-\nu \ln^{3/2}(T_0/T)],$$
$$I(V) \sim \exp[-\nu \ln^{3/2}(T_0/eV)]. \tag{10}$$

The cutoff T_0 is typically of the order of the bandwidth, and Eq. (10) holds for $\ln(T_0/T), \ln(T_0/eV) \gg 1$ [11].

It is worth noticing that already the clean system static conductance G_0 is strongly affected by Coulomb forces. It is well known that G_0 is proportional to the product of the charge sound velocity and the compressibility. In the limit of infinite wire length L, the charge sound velocity diverges, but the compressibility vanishes faster (the system is almost *incompressible*). As a result ($\hbar = 1$ in the chosen units)

$$G_0 = \lim_{\omega\to 0} \frac{e^2}{\pi} K_c(\omega) = \frac{3e^2\nu}{2\pi\sqrt{\ln(l/L)}} \tag{11}$$

is logarithmically suppressed for long wires.

These results are not just limited to the case of one single band but can be extended to the situation where M transverse bands with different Fermi velocities v_{Fn} are populated. The system is described by the charge (spin) fields $\phi_{n,c(s)}$, $n = 1, \ldots, M$, which are coupled together by the interaction. As before, the Coulomb repulsion affects only the charge fields giving rise (for small transferred momentum) to a singular coupling of the form

$$\sum_{n,m} \frac{1}{\pi} \int \frac{dq}{2\pi}\, q^2\, \frac{2e^2}{\kappa} \ln\left(\frac{1}{|q|l}\right) \phi_{n,c}(q)\phi_{m,c}(-q)\ . \tag{12}$$

The large transferred momentum part of the interaction cannot be written simply as a quadratic form of the fields and in general couples together charge and spin fields. However, as a first step we consider just the singular coupling Eq. (12). Then the Hamiltonian (without impurity) can straightforwardly be diagonalized. In this approximation the spin sound modes maintain their bare velocities v_{Fn}. Among the M charge sound modes only one, the plasmon, is strongly affected by the interaction

$$\phi_p(q\to 0) = \left[\frac{4e^2\ln(1/|q|l)}{\pi\kappa M\bar{v}}\right]^{1/4} \sum_n \phi_{n,c}(q),$$

with the average Fermi velocity $\bar{v} = (1/M)\sum_n v_{Fn}$.

We analyze the effect of the impurity potential $H_{\rm imp}(\{\phi_{n,\nu 0}\})$ analogously to the single band case. In the weak barrier limit, we can show that the impurity is again a relevant perturbation. However, contrary to the single band case, if we included the high transferred momentum interactions, this result could change, as it depends on the low-energy properties of *all* modes, and not simply on the existence of a plasmon mode with a singular dispersion relation. The crucial difference is that the term $\cos\left(\sqrt{8\pi}\sum\phi_{n,c0}\right)$ does not occur in the bare impurity potential $H_{\rm imp}$ for $M > 1$ according to standard bosonization rules [8], although it is generated in Mth order of perturbation theory. Because of this complication, we cannot make a definite statement concerning the relevance of the impurity potential in the general case [12]. On the contrary, in the strong barrier limit the situation is completely analogous to the one-band case. The single plasmon mode completely determines the low-temperature behavior of the electric current. The results are similar to those given by Eqs. (10) with the replacement $\nu \to \nu/\sqrt{M}$, and they are valid in a narrow low temperature range shrinking with $M \to \infty$ at least as $\sim 1/M$.

In conclusion, we have shown that a long-range interaction does modify the transport properties of a single band 1D electron gas, leading to the results Eq. (10), independently of the details of the model. At low temperature, such an anomalous behavior of the conductance, should also persist in the case of many populated bands, at least for strong impurity potential. Because of the Coulomb forces, already the conductance of the pure system is logarithmically suppressed, Eq. (11). The latter would be difficult to observe, since it would require the comparison of measurements on samples of different lengths. On the contrary, we expect that the peculiar temperature dependence of the conductance, arising from the interplay of the Coulomb interaction and the impurity, could more likely be observed experimentally. As regards to the I-V characteristics, the strong vanishing of the current at small voltage found above, Eq. (10), is consistent with the threshold behavior reported in Ref. [13].

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