# Is Photon Angular Momentum Conserved in a Dielectric Medium?

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We report the first measurement of photon angular momentum inside a dielectric medium. In contrast to the corresponding photon linear momentum, which is proportional to the refractive index $n$, we find the angular momentum to be independent of $n$.

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The ratio of the momentum $P$ to the energy $E$ carried by an electromagnetic wave in a dielectric medium has been controversial for a long time. The problem is related to the choice and interpretation of the energy-momentum tensor in material media. According to the formulation by Minkowski the ratio $P/E$ is $n/c$, where $c$ is the velocity of light. This was contradicted by Abraham who claimed it to be $1/nc$. This so-called Abraham-Minkowski controversy has been the subject of an extended and confusing theoretical discussion [1–6]. The controversy is rooted in the fact that the constitutive Maxwell equations are usually of a *phenomenological* nature, i.e., they are not based on a microscopic theory. Reviews of the field have been given by Brevik [4], Peierls [5], and Nelson [6].

Since the theoretical arguments in this field are usually long and complicated and since simple heuristic arguments have often led to wrong results [4,5,7], it is important to perform independent experimental tests of the theory. Such tests are scarce. In this context we should mention the experiments addressing the mechanical effect of the linear momentum of photons interacting with a macroscopic material body inside a dielectric liquid [8–11]. From the experimental results it was concluded that the linear photon momentum at optical frequencies is proportional to the refractive index, $n$, of the dielectric. This experimental result favored the Minkowski formulation. Later experiments at low frequency [12] seemed to favor the Abraham formulation. For a while the confusion was total. In a rough sense this is due to the fact that we deal with two systems, namely, field and matter. The momentum can be divided between these two systems. An elegant way to take this into account is to introduce the concept of pseudomomentum [5]. Whereas momentum is the conserved quantity corresponding to the translational symmetry of free space, pseudomomentum is the conserved quantity corresponding to the translational homogeneity of the dielectric medium involved. In practical cases the conservation law for pseudomomentum is much more useful than that for momentum [5]. The reason for this is that acoustic transients may occur when the electromagnetic radiation enters the dielectric medium via an interface. These transients do affect the momentum balance but, due to their long wavelength, they hardly affect the pseudomomentum balance. Although the introduction of pseudomomentum has appreciably clarified the situation, it seems that several questions remain [4,6].

In contrast to the case of linear photon momentum, the mechanical influence of *angular* photon momentum has never been measured in a dielectric medium. A suggestion to perform such an experiment in a dielectric medium was made by Brevik [4] as part of his treatment of the Abraham-Minkowski controversy. Brevik predicted that the mechanical effect of photon angular momentum on a macroscopic body would be the same in a dielectric medium as in vacuum (contrary to the case of linear momentum). We find it important to verify this prediction experimentally in view of all the confusion in the past in the case of linear momentum. In this Letter we report results from experiments on mechanical detection of angular photon momentum in a dielectric liquid which confirm the prediction by Brevik. We also give a short discussion in terms of the angular pseudomomentum.

In our experiment we have used microwave radiation in a waveguide. The advantage of using microwaves is that the angular momentum is inversely proportional to the electromagnetic angular frequency $\omega$ (for constant power) and therefore much larger than for optical radiation. In the latter case the mechanical effects are extremely small [13]. The only relevant photon angular momentum in a waveguide is the component of the total angular momentum along the direction of propagation, $J_z$. In an empty circular waveguide this angular momentum is conserved due to the rotational symmetry of Maxwell's equations and of the boundary conditions.

Our experimental setup is shown schematically in Fig. 1. Microwaves from an $X$-band sweep generator are amplified in a traveling-wave tube (TWT) and launched in a rectangular waveguide where they pass through an isolator to prevent influence from reflections. We have inserted directional couplers to measure the power transmitted to and reflected from the experiment. With a commercial adapter the microwaves are then transferred to a circular waveguide. This adapter launches the microwaves in the $TE_{11}$ mode. The electric field pattern of this mode is shown as an inset in the figure. We performed the experiments at 9.18 GHz ($\lambda = 32.7$ mm) where the $TE_{11}$ mode is the only allowed mode in the $D = 23.2$ mm diam circular waveguide. Photons in the

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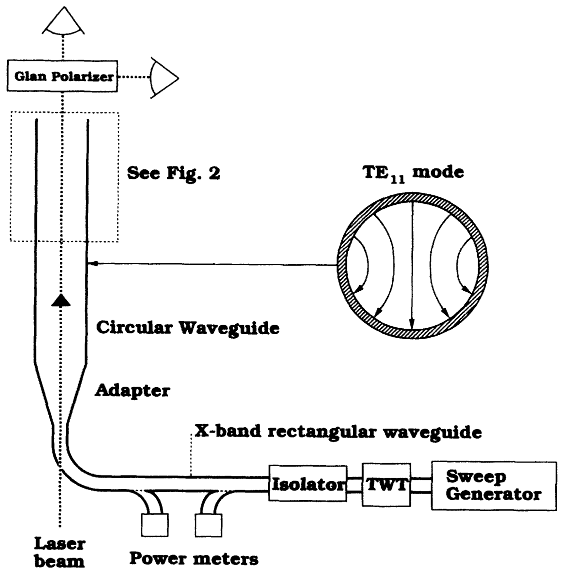


FIG. 1. Schematic diagram of the setup for measurement of the photon angular momentum in a dielectric material. The electric field pattern for the TE$_{11}$ mode in the circular waveguide is inserted for illustration. The pattern rotates at the microwave frequency.

TE$_{11}$ mode each possess one unit of angular momentum ($J_z = \pm \hbar$). In this respect such photons are analogous to circularly polarized photons in free space.

The angular momentum of the microwaves along the direction of propagation is detected with an oversized dipole antenna of length $L$ [$D > L > \lambda/2 \gg$ (antenna thickness)]. The antenna is suspended in the waveguide using a thin tungsten wire, thus forming a torsion pendulum. At equilibrium the torque from the microwaves equals the torque from the tungsten wire. The latter torque can be calculated from the deflection angle, using the bulk modulus of tungsten [14]. The microwave power at the position of the antenna can be determined from measurements on the system without antenna. In order to measure the deflection angle we have attached a small disk of polaroid to the antenna. We send the beam from a HeNe laser through this polarizer and measure the transmitted polarization direction with a balanced bridge setup consisting of a double exit Glan polarizer and two photodiodes. This setup is mounted on top of the circular waveguide.

In a previous publication we have shown experimentally that such an antenna detects the total angular momentum of the TE$_{11}$ mode [15]. For an antenna with $L = 18$ mm in the $D = 23.2$ mm waveguide we measured transfer of $(1.01 \pm 0.05)\hbar$/photon from the TE$_{11}$ mode at 9.18 GHz. This observation is in good agreement with a simple model of the interaction between the antenna and the microwave field. Because of the symmetry of the boundary condition imposed at the antenna surface, an oversized antenna large enough to make contact with the wall of the waveguide transmits and reflects the linear combinations of modes TE$_{11} \pm$ TE$_{-11}$ where a minus in front of the first mode index indicates that the sign of $J_z$ has been reversed. These two combinations of modes resemble linear polarizations in free space parallel and perpendicular to the antenna. They each contain equal amounts of the two TE modes with opposite $J_z$ and they carry no angular momentum. Therefore the incident angular momentum is expected to be transferred to the antenna. This simple picture does not take into account that close to the antenna also evanescent (i.e., nonpropagating) modes are generated [16]. It follows from the experimental results [15] that apparently the evanescent modes do not carry angular momentum. A rigorous electromagnetic calculation to prove this point is still lacking.

In a practical experiment the antenna length has to be somewhat shorter than the waveguide diameter. We found experimentally that this does not affect the detection efficiency of the antenna. Specifically, by measuring the torque as a function of the microwave frequency for constant $L$ and $D$ it was found that the transfer of angular momentum per photon decreased by only $(3 \pm 5)\%$ when $\lambda/2$ increased from $0.8L$ to $L$. For $\lambda/2$ larger than $L$ the torque decreased rapidly. This is similar to the $\lambda$ dependence of the scattering cross section of a linear antenna in free space [17].

In order to be completely sure we decided not to depend on absolute measurements in the determination of the photon angular momentum in a dielectric liquid. Instead we compared a measurement of the photon angular momentum in a waveguide filled with a dielectric liquid with a similar measurement in an empty waveguide. The key point is that in our experiment, as shown in Fig. 2, the two waveguides and antennas have the same effective size, i.e., $L$ and $D$ are $n$ times smaller in the filled waveguide than in the empty waveguide.

For the experiment in dielectric liquid [Fig. 2(a)] the main difficulty is to prevent false signals from flow currents, thermal expansion, and turbulence in the liquid. These are an unavoidable consequence from microwave heating of the liquid. Flow currents and turbulence are most easily prevented by confining the liquid in as narrow a cylinder as possible, because these effects increase with a high power of $D$ [18]. The minimum value of $D$ is given by the cutoff for microwave propagation. Mechanical requirements also define some lower limit for the size below which the setup becomes inconvenient to work with. We found that a reasonable compromise was to use $D = 15.5$ mm. This is still well above cutoff for the TE$_{11}$ mode when the refractive index of the liquid is taken into account. It is necessary to use a nonpolar liquid in order to minimize microwave absorption. Most nonpolar liquids have refractive indices around $n = 1.4$. We have chosen the nonpolar liquid $n$-hexane (refractive index 1.37, giving $\lambda/2 = 11.9$ mm in the liquid) because of its low viscosity which allows the pendulum to be only weakly damped.

The microwaves enter and leave the short section of

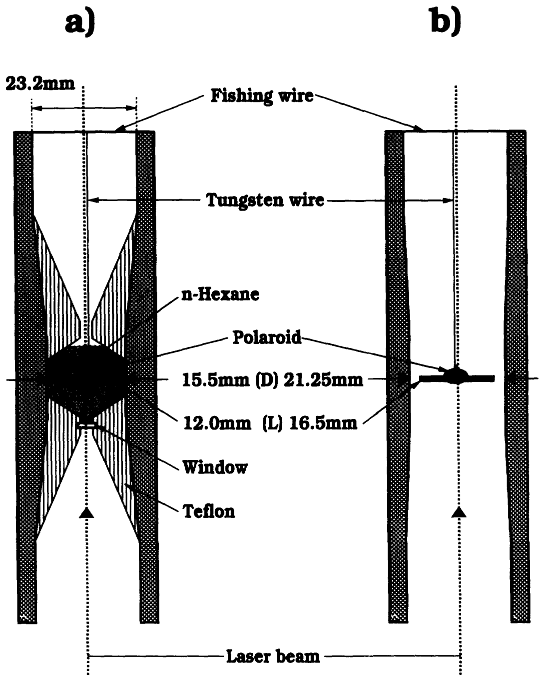


FIG. 2. Detailed view of the dipole antenna placed in the liquid (a) or in the empty waveguide with the same effective diameter (b). In both cases the antenna is suspended from a 13.3 $\mu$m diam tungsten wire. The tungsten wire is attached to a piece of 100 $\mu$m diam nylon fishing wire which spans the open end of the circular waveguide.

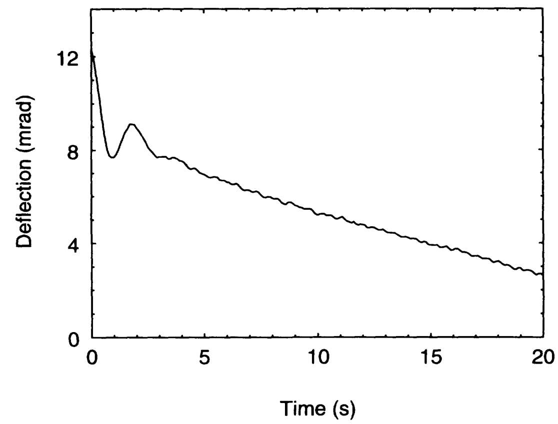


FIG. 3. Total response of the torsion pendulum for 0.5 W microwave power in a dielectric liquid illustrating the different time scales for thermal effects and effects due to transfer of photon angular momentum.

$D=15.5$ mm waveguide filled with liquid through tapered transitions made of (nonabsorbing) Teflon. The bottom of the filled section consists of a thin glass window. Special care was taken to keep the system rotationally symmetric through the whole transition to prevent excitation of modes with a different angular momentum. In the previous experiment we have shown that a rotationally symmetric tapered transition does not change the angular momentum of the microwaves [15]. The symmetric design also reduces the tendency to excite rotational flow currents in the liquid, for instance, by rotationally asymmetric heating. The antenna ($L=12.0$ mm) is suspended in *n*-hexane from a 7.75 cm long and 13.3 $\mu$m diam tungsten wire.

The liquid level was set in the conical taper slightly *below* the "bottleneck" to reduce the influence of surface tension which generated lensing effects distorting the laser beam. This precaution was also important to prevent the thermal expansion due to residual heating by the microwaves from generating large changes in the liquid level. Such large changes could again produce deflections and distortions of the laser beam leading to systematic errors in the measurements.

The setup without dielectric liquid is shown in Fig. 2(b). We used $D=21.25$ mm and $L=16.5$ mm which gives the same effective dimensions as in the liquid. The length of the waveguide piece with reduced diameter is also approximately $n$ times longer, but the taper regions are relatively shorter in order to fit the entire piece into the setup. In this case we measure transfer of $(1.03 \pm 0.05)\hbar$/photon, in good agreement with the previous result [15].

Figure 3 shows the response of the pendulum with dielectric liquid present after the microwaves are switched off. The curve shows two different time scales for the relaxation. At first there is a fast oscillatory response due to stopping of the transfer of photon angular momentum; a similar response is observed without liquid. This is followed by a slower thermal response due to stopping of the residual microwave heating of the system. The details of the thermal response are not important for the measurement of the photon angular momentum as long as the two time scales are very different. The important point is that the thermal response is small and virtually linear during the fast relaxation of the pendulum immediately after the microwaves are switched off. When this is the case it is possible to overcome thermal effects by modulating the microwaves "on" and "off" with a time constant a few times longer than the relaxation time for the pendulum. Furthermore, we subtracted the curve recorded during the on period from that obtained during the off period and removed the small remaining linear slope. The result is shown in Fig. 4. We observe a fast pendulum response converging to a stationary state of deflection due to transfer of angular photon momentum. For 0.5 W microwave power we find a stationary deflection of the pendulum which corresponds to transfer of $(0.96 \pm 0.06)\hbar$/photon. The results with and without dielectric liquid agree with each other within the uncertainty indicating that the photon angular momentum is the same in vacuum (or air) as in a dielectric liquid. The ratio of the two results is $0.93 \pm 0.08$. This excludes possibilities such as $J_z$ proportional to $n$ at the $5.5\sigma$ level or $J_z$ proportional to $1/n$ at the $2.5\sigma$ level. If we use the ab-

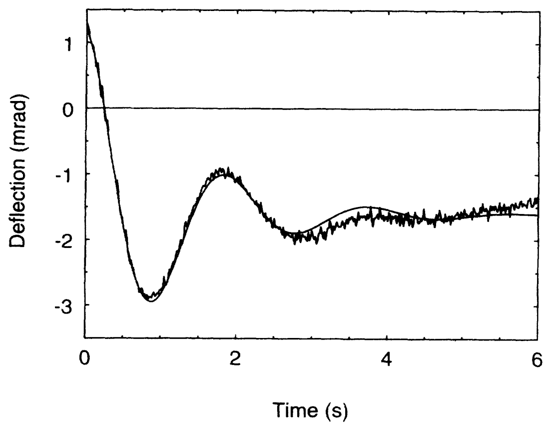


FIG. 4. Modulated response of the pendulum for 0.5 W microwave power incident on the pendulum without the linear thermal background. The theoretically predicted stationary deflection corresponding to $1\hbar$/photon is 1.7 mrad. The smooth curve is a least squares fit to the data.

solute value measured in the liquid directly we can exclude the $1/n$ possibility at the $4\sigma$ level.

We would like to discuss our result in terms of the angular pseudomomentum. The definition of the angular pseudomomentum, $K_z$, along the direction of propagation is analogous to the case of the linear pseudomomentum [5]:

$$K_z = \int_V [\mathbf{r}\times(\mathbf{D}\times\mathbf{B})]_z \, dv \,. \tag{1}$$

Here $\mathbf{D}=n^2\varepsilon_0\mathbf{E}$ is the dielectric displacement and $\mathbf{B}$ is the magnetic field. From this expression it is possible to show that $K_z$ assumes the value $\hbar$ per photon in a dielectric [16]. This shows that the angular pseudomomentum inside the dielectric is equal to the angular momentum in vacuum. This result is contrary to the case of linear pseudomomentum which is proportional to the refractive index $n$. For an immersed body in a liquid the sum of the angular momentum of the body and the angular pseudomomentum of the field may be expected to be conserved because of the analogy with the linear pseudomomentum [5]. If this is true, the radiation torque on our antenna should be the same in the liquid as in vacuum. This is indeed what we have observed experimentally.

As is usual in discussions of radiation pressure we have used in this Letter the word *photon* as a shorthand notation for $E/\hbar\omega$. We do not imply quantum aspects. This could be different for "dressed" photons in condensed matter systems. Also in that case momentum is divided between field and matter. One intriguing example is the question whether an exciton polariton in a crystalline semiconductor [19] can be said to possess a well defined angular momentum, and, if so, what is experimental consequences are [20].

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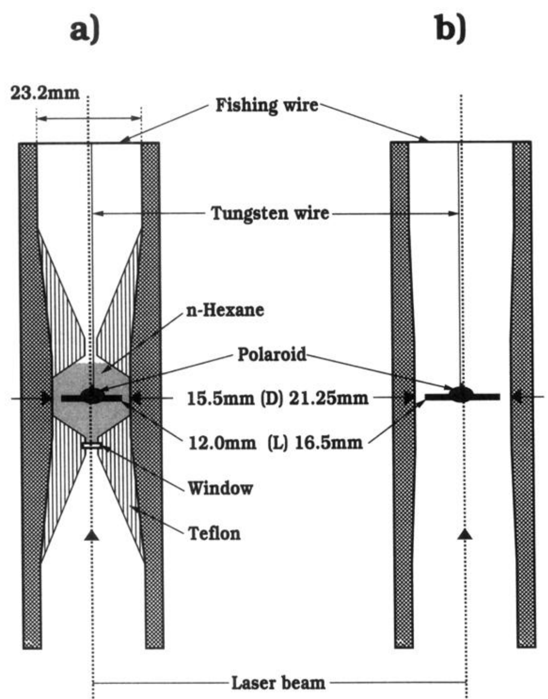


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