How to Make Large Domains of Disoriented Chiral Condensate

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Rajagopal and Wilczek have proposed that relativistic nuclear collisions can generate domains in which the chiral condensate is disoriented. If sufficiently large (i.e., nucleus sized), such domains can yield measurable fluctuations in the number of neutral and charged pions. However, by numerical simulation of the zero-temperature two-flavor linear sigma model, we find that domains are essentially *pion* sized. Nevertheless, we show that large domains can occur if the effective mesons masses are much lighter.

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In high multiplicity hadronic collisions, it is natural to ask if some of the observed pions are produced by the coherent decay of a (semi-) classical pion field [1,2]. Bjorken, Kowalski, and Taylor have speculated that events can occur in which the pion field is oriented along a single direction in isospin space throughout a large fraction of the collision volume [3,4]. They refer to such domains as "disoriented chiral condensates" (DCC)—disoriented because the pion field vanishes in the true vacuum. DCC formation can produce a spectacular event structure with some regions of the detector dominated by charged pions and others by neutral pions. Hints of this behavior may have been observed in cosmic ray collisions [5].

Rajagopal and Wilczek (RW) [6] have proposed that the nonequilibrium dynamics of the chiral transition in relativistic heavy ion collisions can yield large DCC domains. They use a linear sigma model to describe the collective chiral behavior, and make the drastic assumption that the expansion of the collision fragments "quenches" the field. The initial state is taken to be chirally symmetric, as appropriate at high temperature, but its time evolution follows the classical equations of motions at *zero* temperature. RW argue that this quench can amplify the long wavelength pion modes leading to large DCC domains.

In this Letter we extend the analysis of RW to estimate the domain size and the experimentally relevant fraction of neutral pions $\mathcal{R}_3 \equiv n_{\pi^0}/(n_{\pi^+}+n_{\pi^0}+n_{\pi^-})$. We take all of the assumptions of RW for granted; an analysis incorporating a more realistic description of heavy ion dynamics will be presented elsewhere [7]. While our results, like those of RW, are the product of numerical simulations, much of our understanding follows from the analysis of Boyanovsky *et al.* [8], who study domain growth in the context of inflationary cosmology.

To identify a nuclear collision in which pion production is dominated by domain formation in experiments, one can compare the measured spectra of neutral and charged pions on an event-by-event basis [9]. The probability for a domain to yield a particular fraction of neutral pions $\mathcal{R}_3$ is then $P(\mathcal{R}_3) = 1/2\sqrt{\mathcal{R}_3}$ [10], provided that all isospin orientations are equally likely [3,4,9,11]. In contrast, typical hadronic collisions produce a binomial distribution of $\mathcal{R}_3$ peaked at the isospin symmetric value of 1/3 (modulo effects related to the isospin of the target and projectile). These distributions are very different, particularly at small $\mathcal{R}_3$. The size of domains is of central experimental interest. Nuclear collisions dominated by a single, large domain can produce a domainlike $P(\mathcal{R}_3)$. On the other hand, events that produce many small, randomly oriented domains result in a binomial distribution.

To describe the evolution of the classical pion field, we use a linear sigma model that characterizes the chiral order parameter of QCD as an O(4) vector field $\Phi = (\sigma, \boldsymbol{\pi})$, where σ is an isosinglet $J^P = 0^+$ field. The field interacts through a potential $\lambda(\boldsymbol{\pi}^2 + \sigma^2 - v^2)^2/4 - H\sigma$; at zero temperature the vacuum state is $\langle\Phi\rangle = (f_\pi, \mathbf{0})$. The parameters of the model are the coupling λ, an external field, H, which breaks the O(4) symmetry and the vacuum expectation value of the sigma field in the absence of the external field, v. Our primary interest is in the physically relevant case of strong coupling, as studied by RW. One takes $\lambda = 20$, $v = 87.4$ MeV, and $H = (119\,\mathrm{MeV})^3$, so that the pion decay constant is $f_\pi = 92.5$ MeV and the meson masses are $m_\pi = 135$ MeV and $m_\sigma = 600$ MeV. To establish the role of the m_σ in domain formation, we compare these results to an extremely weakly coupled case of $\lambda = 10^{-4}$, $v = 87.4$ MeV, and $H = (2.03\,\mathrm{MeV})^3$. The pion and sigma fields are very light in this regime, $m_\pi = 0.3$ MeV and $m_\sigma = 1.8$ MeV, but f_π retains the measured value.

Approximating the evolution of the system by a quench, one assumes that the field is initially chirally symmetric with $\langle\Phi\rangle = 0$, and follows its time evolution using the zero-temperature classical equations of motion. To simulate the effect of fluctuations in the high temperature initial state, we distribute the fields as Gaussian random variables with $\langle\Phi\rangle = 0$, $\langle\Phi^2\rangle = v^2/4$, and $\langle\dot{\Phi}^2\rangle = v^2/1\,\mathrm{fm}^2$ following RW [6]. Pion domains can form in a quench because the chirally symmetric initial state, $\langle\Phi\rangle = (0, \mathbf{0})$, is unstable against small fluctuations in the $T = 0$ potential [6,8]. In essence, the system "rolls down" from the unstable local maximum of $V(\Phi)$ towards the nearly stable values $\Phi^2 = f_\pi^2$. This process is known in condensed matter physics as spinodal decomposition [8]. Long-lived DCC field configurations with $\boldsymbol{\pi} \neq 0$ can develop during the rolldown period. The field will eventually settle into stable oscillations about the unique $H \neq 0$ vacuum, $\langle\Phi\rangle = (f_\pi, \mathbf{0})$. Oscillations will then continue until interactions eventually damp the motion. In the heavy-ion system a domain can radiate pions preferentially according to its isospin content.

To be more concrete, we write the equations of motion for the Fourier components of the pion field in the Hartree approximation [6,8]:

$$\frac{d^2}{dt^2}\boldsymbol{\pi}_{\mathbf{k}} = [\lambda(v^2 - \langle\Phi^2\rangle) - k^2]\boldsymbol{\pi}_{\mathbf{k}} . \qquad (1)$$

Field configurations with $\langle\Phi\rangle = 0$, $\langle\Phi^2\rangle < v^2$, and momentum $k < \sqrt{\lambda}v$ are unstable in that they grow exponentially; modes with higher momentum do not grow. The $k = 0$ mode grows the fastest, with a time scale

$$\tau_{\rm sp} = \{\lambda(v^2 - \langle\Phi^2\rangle)\}^{-1/2} \sim \sqrt{2}/m_\sigma, \qquad (2)$$

of order 0.5 fm for $\lambda = 20$. The exponential growth of the unstable modes continues until $\langle\Phi^2\rangle$ reaches v^2, when it begins to oscillate about the stable vacuum. Rajagopal and Wilczek found that the power $\propto \pi_{\mathbf{k}}^a\pi_{-\mathbf{k}}^a$ in the low momentum pion modes indeed grows when the exact classical equations of motion are integrated for $\lambda = 20$. What is not clear, however, is whether this observed amplification of power is sufficient to create large domains.

To get a qualitative idea of the domain size, we follow Boyanovsky *et al.* [8] and estimate the contribution of the unstable modes to the pion correlation function. For small times, we treat $\langle\Phi^2\rangle = v^2/4$ as a constant, so that $\boldsymbol{\pi}_{\mathbf{k}}(t) = \mathbf{A}_{\mathbf{k}}e^{t/\tau_k} + \mathbf{B}_{\mathbf{k}}e^{-t/\tau_k}$, with $\tau_k^{-1} = \sqrt{3\lambda v^2/4 + k^2}$. The averages $\langle\langle A_kA_{k'}\rangle\rangle, \langle\langle A_kB_{k'}\rangle\rangle$ over the initial conditions can be computed explicitly [7]. For $r^2 \gg \tau_{\rm sp}t$, we find

$$\langle\langle\pi(\mathbf{x},t)\pi(\mathbf{0},t)\rangle\rangle \approx \langle\langle\pi(\mathbf{0},t)^2\rangle\rangle \exp\{-x^2/8\tau_{\rm sp}t\} , \qquad (3)$$

suggesting that domains grow roughly as $\xi_D \sim \sqrt{8\tau_{\rm sp}t}$. We therefore expect typical domains to reach a size $\sim \sqrt{8}\tau_{\rm sp}$, since they can only grow until the time $t \sim \tau_{\rm sp}$ when the field reaches the minimum of the potential. If the coupling were weak and the sigma meson light, it would take a long time to roll down to the bottom of the potential, so that domains would have plenty of time to grow. However, for the more realistic strongly coupled case, the rolldown is very rapid and the domains are small, perhaps ~ 1.4 fm, which is comparable to the Compton wavelength of the pion.

We confirm this intuition through numerical simulations. Specifically, we simulate the model on a three dimensional $10^2 \times 40$ lattice with a spacing $a = 1$ fm, computing its evolution using a simple leap-frog algorithm. To extract domain-size information more easily, we employed an asymmetric lattice and studied the pion field averaged over the short dimensions, $\boldsymbol{\pi}_L(z,t) = \sum_{x,y}\boldsymbol{\pi}(x,y,z,t)/(10)^2$. Our simulations qualitatively reproduce the time evolution of the momentum space quantity $\pi_{\mathbf{k}}^a\pi_{-\mathbf{k}}^a$ reported by RW for a symmetric lattice at $\lambda = 20$. In configuration space, π_L starts out at zero with small Gaussian fluctuations. Figure 1 shows the profile of $\boldsymbol{\pi}_L$ at a time $t = 100$ fm in weak coupling, $\lambda = 10^{-4}$, and the same profile at a time $t = 30$ fm in strong coupling, $\lambda = 20$. For weak coupling, regions in which the field is slowly varying about some nonzero value—domains—are evident. On the other hand, in strong coupling the pion field is oscillating with small amplitude about zero. We have also measured the pion correlation function: it is long ranged in weak coupling, but drops to zero within

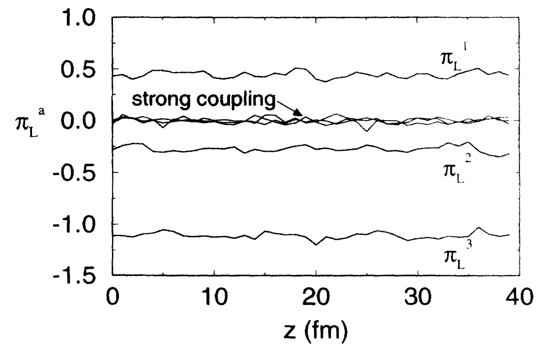

FIG. 1. Snapshot of the pion field $\boldsymbol{\pi}_L$ on a $10^2 \times 40$ lattice taken in weak coupling ($\lambda = 10^{-4}$) at $t = 100$ fm and at $t = 30$ fm at strong ($\lambda = 20$) coupling. Note how at strong coupling the three components of $\boldsymbol{\pi}_L$ fluctuate around zero. The labeled curves show $\boldsymbol{\pi}_L$ at weak coupling. All three components are nonzero and essentially constant throughout the length of the box.

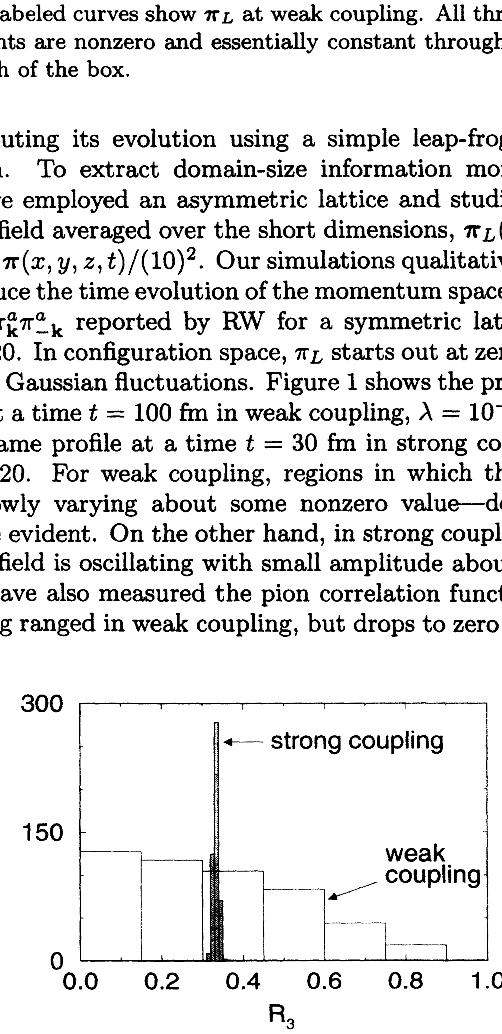

FIG. 2. The distribution of the fraction of neutral pions $\mathcal{R}_3$ on a $10^2 \times 40$ lattice in weak coupling ($\lambda = 10^{-4}$) at $t = 100$ fm and at $t = 30$ fm at strong ($\lambda = 20$) coupling. In each case 500 different initial configurations were used to generate the histograms. At strong coupling the distribution is narrowly peaked around $\mathcal{R}_3 = 1/3$ whereas at weak coupling it is broad and clearly gives more weight to small values of $\mathcal{R}_3$.

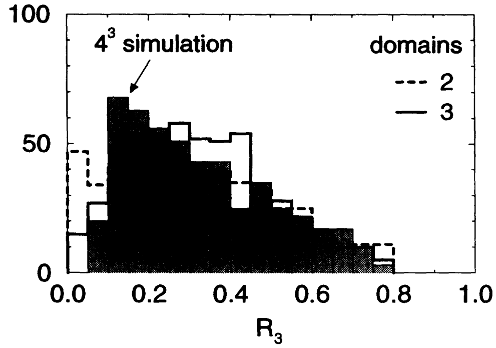

FIG. 3. The distribution of $\mathcal{R}_3$ on a small-volume 4^3 lattice in strong coupling (gray histogram) compared to Monte Carlo distributions obtained assuming two (dashed) and three (solid) uncorrelated DCC domains.

a few lattice spacings in strong coupling [7]. We also varied the time at which correlations were measured, and were never able to find large domains in strong coupling.

We also studied the distribution of the fraction of neutral pions $\mathcal{R}_3 = \langle(\pi^3)^2\rangle / \sum\langle(\pi^a)^2\rangle$. Histograms of $\mathcal{R}_3$ were obtained by evolving 500 independent configurations in time to $t = 150$ and $t = 30$ fm at weak and strong coupling, respectively. These are shown in Fig. 2. In weak coupling the distribution is far from binomial and favors small $\mathcal{R}_3$ like the DCC distribution $P(\mathcal{R}_3) = 1/2\sqrt{\mathcal{R}_3}$. In strong coupling, however, the distribution is clearly binomial, peaked about the expected value of $1/3$.

DCC-produced fluctuations manifest the $P(\mathcal{R}_3) = 1/2\sqrt{\mathcal{R}_3}$ behavior only if the domain size is comparable to (or larger than) the system size. To explicitly demonstrate this size dependence, we show $P(\mathcal{R}_3)$ computed for a small-volume 4^3 lattice, in which the effective linear dimension is 4 fm modulo the periodic boundary conditions. For a 1.4 fm domain size, one would expect from two to four domains in the system. In Fig. 3 we compare the simulated distribution with "Monte Carlo" distributions assuming two and three uncorrelated domains. The distribution for each domain is obtained by generating random numbers distributed according to $P(\mathcal{R}_3) \sim 1/\sqrt{\mathcal{R}_3}$. The three domain distribution agrees best with the simulation.

We remark that even small domains produce a characteristic event structure that can perhaps be identified, e.g., by binning neutral and charged pions in rapidity and azimuthal angle. Pions radiated from a domain of size ξ have transverse momenta $p_T \sim \xi^{-1}$, so that, in principle, one can enhance the signal relative to the background by selecting low-p_T pions. Strategies for detecting low-p_T π^0's at the BNL Relativistic Heavy Ion Collider are currently under investigation [12].

In summary, we find that distributions of the fraction of neutral pions do not reflect domain formation for the two-flavor linear sigma model assuming the RW quench. The $\mathcal{R}_3$ distribution can reflect domain formation only when the domain size ξ_D exceeds the system size R. We find that DCC-like behavior disappears for couplings exceeding $\lambda \sim 10^{-2}$ in our $10^2 \times 40$ lattice, in accord with (3) [7]. However, a system the size of a gold nucleus can exhibit domainlike behavior for $\lambda < 8/(vR)^2 \sim 1$.

A necessary condition for large domains of DCC's to arise is that there is a light particle about—the dynamics of a quench does not give us a large distance scale for free. However, heavy ion collisions can create a high temperature state that cools over time scales $\gg \tau_{\rm sp} \sim 0.5$ fm. The quench might therefore not be a a very good approximation. *If* the equilibrium thermodynamics of the QCD phase transition is near a critical point, then near the transition temperature there is automatically a light field about, providing a natural mechanism for the growth of large domains. In forthcoming papers we will show how such a critical point is present in the QCD phase diagram, and analyze the phenomenology of the DCC domains produced thereby [7].

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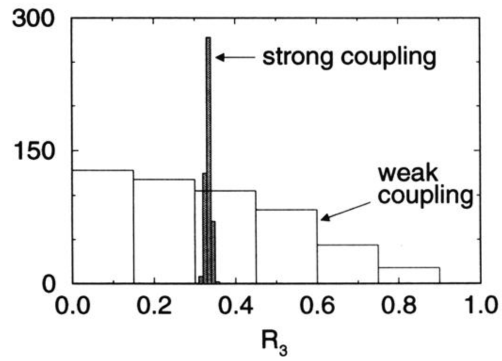

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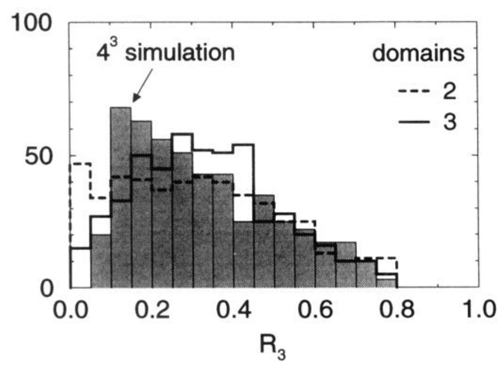

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