Large Loops of Magnetic Current and Confinement in Four Dimensional U(1) Lattice Gauge Theory

John D. Stack
Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

Roy J. Wensley
Department of Mathematical Sciences, Saint Mary's College, Moraga, California 94575
(Received 31 August 1993)

We calculate the heavy quark potential from the magnetic current due to monopoles in four dimensional U(1) lattice gauge theory. The magnetic current is found from link angle configurations generated in a cosine action simulation on a 24^4 lattice. The magnetic current is resolved into large loops which wrap around the lattice and small loops which do not. It is shown that the long range part of the heavy quark potential, in particular the string tension, can be calculated solely from the large loops of magnetic current.

PACS numbers: 11.15.Ha, 12.38.Gc

In this paper, we report new results on confinement via monopoles for U(1) lattice gauge theory in four dimensions. Our main result is that the confining part of the heavy quark potential, in particular the string tension, is determined solely by large loops of magnetic current. It has been established for some time that large loops which extend over the entire lattice are present only in the confined phase of the theory [1,2]. Their presence can now be quantitatively tied to the string tension. Our work is carried out on a 24^4 lattice, near the deconfining transition.

The role of monopoles in U(1) lattice gauge theory is seen most clearly using the Villain [3] or periodic Gaussian form of the U(1) theory. Under a dual transformation, the usual link angle description goes over into one involving an integer-valued magnetic current $m_\mu(x)$, defined on the links of the dual lattice [4]. The link angle path integral becomes a sum over all possible configurations of magnetic current. In this monopole representation, the system can be visualized as a plasma of magnetic monopoles moving on Euclidean world lines, interacting via photon exchange.

In either representation, a Wilson loop calculation is needed to determine the heavy quark potential. In the link angle representation, a Wilson loop is specified by the exponential of a line integral:

$$W(R,T) = \left\langle \exp\left(i \sum_x \theta_\mu(x) J_\mu(x) \right) \right\rangle_\theta , \qquad (1)$$

where the integer-valued electric current J_μ is nonvanishing on the rectangular $R \times T$ loop contour, and $\langle\rangle_\theta$ denotes the expected value taken over configurations of link angles $\theta_\mu(x)$.

In the monopole representation, the determination of a Wilson loop involves the exponential of an area integral over a surface with the loop contour as its boundary [4,5]. The electric current J_μ is first expressed as the curl of a Dirac sheet variable [6]; $J_\mu = \partial_\nu D_{\mu\nu}$, where ∂_ν denotes a discrete derivative. The sheet variable $D_{\mu\nu}$ is nonunique. For $|J_\mu| = 1$, a specific choice is made by setting $D_{\mu\nu} = 1$ on the plaquettes of an (arbitrary) open surface with boundary J_μ, and $D_{\mu\nu} = 0$ on all other plaquettes. The area integral represents the dual flux set up by the magnetic current through this surface. To compute it, we define the magnetic vector potential

$$A_\mu(x) = \sum_y v(x-y) m_\mu(y), \qquad (2)$$

where v satisfies $-\partial \cdot \partial v(x-y) = \delta_{x,y}$. The field strength is given by $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. In terms of $D_{\mu\nu}$ and the dual field strength $F^*_{\mu\nu}(x) = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}(x)$, the monopole representation of a Wilson loop is finally given by

$$W(R,T) = W_{\rm ph}(R,T) \times \left\langle \exp\left(\frac{i2\pi}{2} \sum_x D_{\mu\nu}(x) F^*_{\mu\nu}(x) \right) \right\rangle_m , \qquad (3)$$

where $\langle\rangle_m$ denotes the sum over configurations of magnetic current. The factor 2π which appears in the exponent of Eq. (3) arises from the Dirac condition on the product of electric and magnetic charge, and guarantees that the value of a Wilson loop is independent of the surface chosen to define $D_{\mu\nu}$. The prefactor in Eq. (3) describes one photon exchange between the quark and antiquark:

$$W_{\rm ph}(R,T) = \exp\left(-\frac{e^2}{2} \sum_{x,y} J_\mu(x) v(x-y) J_\mu(y) \right), \qquad (4)$$

where the electric coupling e^2 is related to the coupling β_V in the Villain action by $e^2 = 1/\beta_V$. The factor $W_{\rm ph}(R,T)$ contributes a purely perturbative Coulomb term to the heavy quark potential.

0031-9007/93/72(1)/21(4)$06.00

Fortunately, the numerical evaluation of Wilson loops via Eq. (3) does not require a direct simulation in terms of the magnetic current $m_\mu(x)$. This is impractical owing to the long-range interactions generated by photon exchange between the monopole currents. DeGrand and Toussaint [7] showed how to locate monopoles directly in configurations of link angles. In their procedure, the plaquette angle $\theta_{\mu\nu}(x) = \partial_\mu\theta_\nu - \partial_\nu\theta_\mu$ is resolved into a fluctuating part $\bar{\theta}_{\mu\nu}(x)$, and an integer-valued Dirac sheet variable $m^*_{\mu\nu}(x)$:

$$\theta_{\mu\nu}(x) = \bar{\theta}_{\mu\nu}(x) + 2\pi m^*_{\mu\nu}(x), \tag{5}$$

where $m^*_{\mu\nu}(x)=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}m_{\alpha\beta}(x)$ and $\bar{\theta}_{\mu\nu}(x) \in (-\pi,\pi)$. The magnetic current is then given by $m_\mu(x) = \partial_\nu m_{\mu\nu}(x)$. This procedure allows only values of $m_\mu \in [\pm 2, \pm 1, 0]$, whereas in principle all integer values are allowed. However, at values of the coupling near the deconfining transition, the values $m_\mu = \pm 1$ are overwhelmingly dominant; even $m_\mu = \pm 2$ occurs only a small fraction of the time. Thus negligible error is caused by omitting higher values of m_μ.

The derivation of Eq. (3) as an exact formula is only possible for Villain's form of the U(1) theory. On the other hand, Wilson's cosine form [8] of the U(1) action can be simulated much more efficiently. In our previous work [5,9], we have shown that Villain action results can be extracted from a cosine action simulation, if a simple coupling constant mapping is used. More precisely, a simulation using the cosine action at coupling β_W is equivalent to a Villain action simulation at coupling β_V, with β_V related to β_W by [3,10]

$$1/\beta_V = -2\ln\left(\frac{I_0(\beta_W)}{I_1(\beta_W)}\right), \tag{6}$$

where I_0 and I_1 are modified Bessel functions. Equation (6) determines the value of $1/\beta_V$, and hence e^2, which result from a cosine action simulation at a given value of β_W. The factor $W_{\rm ph}$ in Eq. (3) is then completely determined. The magnetic current is identified from the cosine action link angle configurations and the result used to calculate the second factor of Eq. (3). The Wilson loops calculated in this manner using the cosine action and Eq. (6) differ from pure Villain action results by a harmless perimeter term. The R-dependent terms in the potentials agree within statistical errors [9,11]. In the present work on a 24^4 lattice, we have again checked that potentials deduced directly from link angles and Eq. (1) agree with those obtained from the magnetic current and Eq. (3).

The link angle configurations were generated using a heatbath algorithm [12]. The calculation of $A_\mu(x)$ from $m_\mu(x)$ in Eq. (2) was done using a four dimensional vectorized fast Fourier transform [13,14]. The $R \times T$ rectangle lying in the Wilson loop plane was used as the defining surface for $D_{\mu\nu}$. Magnetic current configurations were saved every 10 sweeps. After Wilson loops were obtained from these configurations using Eq. (3), potentials were extracted using standard methods. The heavy quark potential $V(R)$ was obtained from a straight line fit of $\ln W(R,T)$ vs T, over an interval $T_{\rm min}(R)$ to $T_{\rm max}$, where $T_{\rm min}(R) = R+2$ for $R = 2, 3$, and $R+1$ otherwise, while $T_{\rm max} = 16$. To determine the string tension σ and Coulomb coupling α, the potentials were then fitted to a linear plus Coulomb form, $V(R) = \sigma R - \alpha/R + V_0$, over the interval $R = 2$ to $R = 7$. Errors in physical quantities were estimated using both the jacknife method and binning the data into bins of various size.

It is well established for U(1) that appreciable correlation lengths occur only in the immediate vicinity of the deconfining phase transition. The location of the phase transition moves to larger values of β_W as the lattice size increases, in a manner roughly consistent with finite size scaling theory [2,15]. Since only lattices of size up to 16^4 were available in the published literature when we began our work, it was first necessary for us to locate the transition for a 24^4 lattice. To do this, we performed a series of runs with various initial configurations for $1.0100 < \beta_W < 1.0120$, and monitored the value of the 1×1 Wilson loop, $W(1,1)$. For $\beta_W \geq 1.0114$, the system always reached a state with $W(1,1) \sim 0.65$. For $\beta_W \leq 1.0112$, the system always reached a state with $W(1,1) \sim 0.63$. Subsequent analysis of the heavy quark potential showed these two states to be deconfined and confined, respectively. While we have not precisely located the deconfining phase transition, consistency with our results requires that the transition be in the interval $1.0112 \leq \beta_W < 1.0114$ for a 24^4 lattice.

To avoid problems associated with long autocorrelation times that occur near the transition, we chose to use a run of 20 000 sweeps at $\beta_W = 1.0103$ for the results to be presented below. At this value of β_W, the correlation length is large enough to observe the beginnings of continuum behavior, but small enough to avoid problems with long autocorrelation times. The autocorrelation time τ measured from the 1×1 Wilson loop was approximately 100 sweeps. In Fig. 1, we show the heavy quark potential determined from Eq. (3) for $\beta_W = 1.0103$ using 936 configurations of magnetic current. A linear-plus-Coulomb fit gave a string tension of $\sigma = 0.058(2)$, and a Coulomb coupling of $\alpha = 0.30(2)$. The total number of links carrying magnetic current at this β_W was 98 400(800). For comparison, we also show in Fig. 1, the potential determined from Eq. (3) for the deconfined β_W value, $\beta_W = 1.0114$, where 724 configurations of magnetic current were analyzed. At this value of β_W, the string tension was statistically zero, while the Coulomb coupling was $\alpha = 0.24(2)$. The total number of links carrying magnetic current was 57 940(40).

We now turn to the resolution of the magnetic current into loops. Every 20 lattice sweeps, magnetic current loops were individually identified and catalogued. The

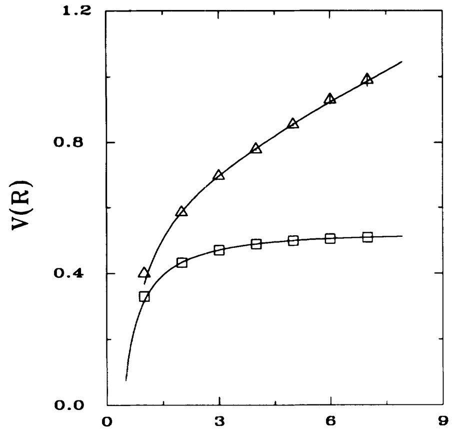

FIG. 1. The quark potential calculated using the magnetic current configurations for $\beta_W = 1.0103$ (triangles) and $\beta_W = 1.0114$ (squares). The solid lines are linear-plus-Coulomb fits to the potentials.

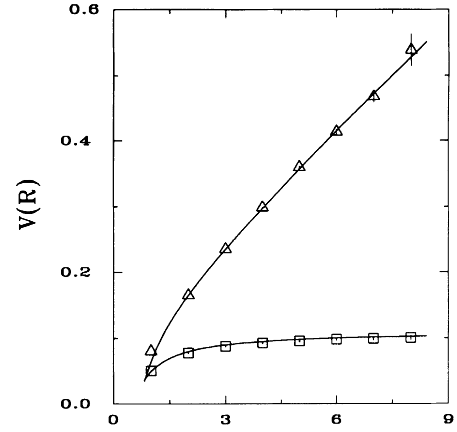

FIG. 2. The potential calculated using only the large monopole loops (triangles) and using only the small monopole loops (squares). The photon contribution from $W_{\rm ph}$ has not been included.

loop-finding algorithm proceeded by choosing a nonzero current link, $m_\mu(x_0)$, and following the current it carried through the lattice until a loop was completed by a return to the site x_0. This process was carried out repeatedly from different starting points and ended when the entire configuration of current had been resolved into loops. The algorithm was deterministic: when looking for an outgoing current link at a particular lattice site, the direction $\mu = 1$ was chosen first, followed by $\mu = 2, 3, 4$. Intersections of loops did occur (i.e., more than one outgoing link associated with a site), so the set of loops identified was not unique. However, self-intersections of loops were relatively rare, occurring with approximately the same probability as self-intersections of a purely random walk in $d = 4$ [16].

The confined state is distinguished by the presence of macroscopic loops containing thousands of links. These do not occur in the deconfined state. Given the finite size of the lattice, it is quite likely that a loop with thousands of links will wrap around the lattice in one or more directions. Periodic boundary conditions were used, permitting topologically nontrivial loops. As a convenient way to distinguish large and small loops, the net current was measured for each loop:

$$\Lambda_\mu = \sum_{x \in \text{loop}} m_\mu(x).$$

Loops which wrap around the lattice have a nonvanishing Λ_μ, with components of Λ_μ which are integer multiples of the lattice size along an axis; $\Lambda_\mu = n_\mu \cdot N$, for a cubic lattice of size N^4. While an individual loop can have a nonvanishing Λ_μ, a net current cannot actually occur on a finite lattice, so the sum of Λ_μ over loops must vanish identically. It was typical for a loop with nonvanishing Λ_μ to be wrapped around the lattice several times in more than one direction. Values of $|n_\mu|$ up to 10 were observed.

At $\beta = 1.0103$, the loop analysis was carried out for 380 configurations. Loops were written into two sets of files, one set containing only those loops with nonvanishing Λ_μ, the other containing only loops for which $\Lambda_\mu \equiv 0$. Of the total of 98 400(800) links carrying magnetic current, 51 000(400) were in loops with finite Λ_μ, the rest in loops with vanishing Λ_μ. For vanishing Λ_μ, the average total number of loops was 6210(6), of which 3507(4) were in the form of elementary loops with four links. Over 90% of the links in this class of loops occurred in loops with 60 links or less. In contrast, for Λ_μ finite, the average number of loops was only 4.6(1), so the typical loop of this type carried tens of thousands of links. Motivated by these results, in what follows loops with finite Λ_μ are simply referred to as "large," and those with vanishing Λ_μ, "small."

Since large loops occur only in the confined phase, it is natural to ask if they can explain the long range, confining part of the heavy quark potential. To investigate this, we computed the heavy quark potential again using Eq. (3), but for each configuration, including only the magnetic current from large loops. The results are shown (omitting the photon factor $W_{\rm ph}$) in Fig. 2. A linear plus Coulomb fit to the resulting potential gave a string tension $\sigma_l = 0.056(2)$, and a Coulomb term $\alpha_l = 0.09(1)$. The string tension is within statistical errors of the value 0.058(2) found earlier from the heavy quark potential calculated using the full magnetic current. Next, we carried out a similar calculation using only the magnetic current from the small loops. This produced the rather flat potential also shown in Fig. 2. A linear plus Coulomb fit to this potential gives zero string tension within statistical errors [$\sigma_s = 0.0000(6)$], and a Coulomb term $\alpha_s = 0.06(1)$. The result of these two fits gives strong evidence that in the long distance region, there is a clean separation between the contributions of the two classes of loops. Only the large loops containing many thousands of links contribute to the confining part of the potential. Although the two classes of loops were found using a topological criterion, we believe loop size is the crucial factor, not topology. Whether this is in fact true could be tested in a simulation on a lattice with boundary conditions which forbid loops with nonzero Λ_μ.

In the fits described above for $\beta_W = 1.0103$, the large loops required a Coulomb term with coupling $\alpha_l = 0.09(1)$, while the small loops required a Coulomb term with coupling $\alpha_s = 0.06(1)$. In addition, there is a Coulomb term coming from the $W_{\rm ph}$ factor in Eq. (3). Using Eq. (6) to evaluate $\beta_V(1.0103)$, gives $\alpha_{\rm ph} = 0.13$ as the Coulomb coupling arising from $W_{\rm ph}$. Simply adding the various terms, we obtain $\alpha_{\rm ph} + \alpha_s + \alpha_l = 0.28(2)$, consistent with our previous result of 0.30(2) obtained with the full magnetic current.

The results on the string tension and Coulomb coupling are consistent with additivity of the potential over the contributions from photons, large loops, and small loops. In Fig. 3, we compare the potential determined from the full magnetic current and $W_{\rm ph}$ (shown previously in Fig. 1), with the potential obtained by summing the contributions from $W_{\rm ph}$, large loops, and small loops, plus a constant. A glance at Fig. 3 shows that the agreement is quite good. Additivity of the potential implies that the contributions from large and small loops factor in the average over configurations. To check factorization, we performed a fit to the "potential" extracted from the ratio of Wilson loops assuming factorization to Wilson loops calculated with the full magnetic current:

$$\langle W_l(R,T)\rangle_m \langle W_s(R,T)\rangle_m / \langle W_l(R,T) W_s(R,T)\rangle_m . \quad (7)$$

The string tension and Coulomb coupling resulting from this were both zero to within statistical errors. This shows that factorization and therefore additivity of the potential is consistent with our data. This is not surprising at large R, where the R dependence comes predominantly from the large loops. However, in the small-R region, both the large and small loops produce Coulomb terms, and additivity is not expected to hold as an exact statement. Nevertheless it appears to be a good approximation and holds within the accuracy of our data.

We have shown that the long range, confining part of the potential in U(1) is produced by the large loops of magnetic current. The small loops contribute only to the Coulombic part of the potential. Still missing is a physical picture of how the large loops of current disorder the vacuum and produce the string tension. The fact that these loops extend over the whole lattice suggests that there are low mass (perhaps massless) magnetically charged excitations present in the confined phase. We plan to report elsewhere on this question as well as how the magnetic current screens itself. The results obtained in our work are likely to have an impact on the monopole approach to confinement in non-Abelian gauge theories.

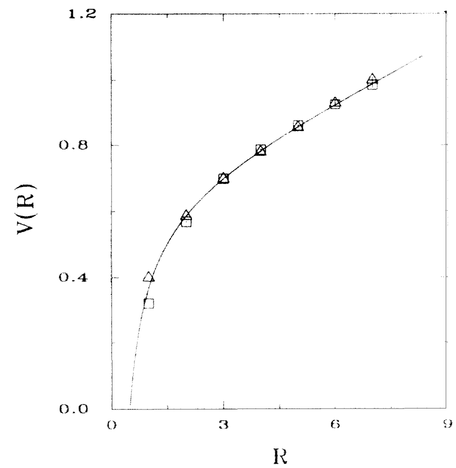

FIG. 3. Comparison of the quark potential calculated using the full magnetic current and photons (triangles) with the potential obtained by summing the potentials determined separately from photons, large loops, and small loops (squares). The solid line is the linear-plus-Coulomb fit from Fig. 1 for $\beta_W = 1.0103$.

This work was supported in part by the National Science Foundation under Grant No. NSF PHY 92-12547. The calculations were carried out on the Cray Y-MP system at the National Center for Supercomputing Applications at the University of Illinois, supported in part by the National Science Foundation under Grant No. NSF PHY920026N. R.J.W. would like to acknowledge support from the Faculty Development Funds of Saint Mary's College of California, and support from Lawrence Livermore National Laboratory.

[1] V. Grösch, K. Jansen, J. Jersák, C. B. Lang, T. Neuhaus, and C. Rebbi, Phys. Lett. **162B**, 171 (1985).
[2] R. Gupta, M. Novotny, and R. Cordery, Phys. Lett. B **172**, 86 (1986).
[3] J. Villain, J. Phys. (Paris) **36**, 581 (1975).
[4] T. Banks, R. Myerson, and J. Kogut, Nucl. Phys. **B129**, 493 (1977).
[5] J. D. Stack and R. J. Wensley, Nucl. Phys. **B371**, 597 (1992).
[6] P. A. M. Dirac, Phys. Rev. **74**, 817 (1948).
[7] T. A. DeGrand and D. Toussaint, Phys. Rev. D **22**, 2478 (1980).
[8] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).
[9] R. J. Wensley and J. D. Stack, Phys. Rev. Lett. **63**, 1764 (1989).
[10] W. Janke and H. Kleinert, Nucl. Phys. **B270**, 135 (1986).
[11] Roy J. Wensley, Ph.D. thesis, University of Illinois, 1989.
[12] M. Creutz, Phys. Rev. D **21**, 2308 (1980).
[13] A. Nobile and V. Roberto, Comput. Phys. Commun. **40**, 189 (1986).
[14] A. Nobile and V. Roberto, Comput. Phys. Commun. **42**, 233 (1986).
[15] V. Azcoiti, G. Di Carlo, and A. F. Grillo, Phys. Lett. B **268**, 101 (1991).
[16] C. Itzykson and J.-M. Drouffe, *Statistical Field Theory* (Cambridge Univ. Press, Cambridge, 1989), Vol. 1, p. 19.