Theory of Spin-Split Cyclotron Resonance in the Extreme Quantum Limit

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We present an interpretation of recent cyclotron resonance experiments on the two-dimensional electron gas in GaAs/AlGaAs heterostructures. We show that the observed dependence of the resonance spectrum on Landau level occupancy and temperature arises from the interplay of three factors: spin splitting of the cyclotron frequency, thermal population of the two spin states, and coupling of the resonances for each spin orientation by Coulomb interactions. In addition, we derive an f-sum rule which allows spin polarization to be determined directly from resonance spectra.

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Since the discovery of the quantum Hall effect, there has been intense interest in the properties of a two-dimensional electron gas in a strong magnetic field. Although it was the dramatic transport phenomenon that originally drew attention to these systems, increasing use is being made of other experimental probes. At very low filling fraction, transport measurements are problematic and optical techniques such as photoluminescence and cyclotron resonance are particularly valuable. The optical spectra show many interesting features, and are still far from understood.

Recently, two groups have reported intriguing cyclotron resonance measurements in GaAs inversion layers at low Landau level filling fraction, ν [1,2]. For $\nu > 1/6$, a single resonance peak is observed, which shows no anomalous behavior at any of the fractional quantum Hall states. Below $\nu \approx 1/6$, a second resonance peak appears, and the absorption spectrum is strongly dependent on both filling fraction and temperature. These observations were interpreted initially as a signature of Wigner crystallization [1]. Subsequently, it was demonstrated [1,2] that the frequency splitting of the two resonances at the lowest filling fractions is very similar to the spin splitting measured for cyclotron resonance in bulk GaAs, which arises because the effective g factor is energy dependent [3]. It has since been suggested that the observations may indicate a spin-ordering transition, or localization of electrons in the extreme quantum limit [2].

In this paper we offer a detailed theoretical interpretation of these experimental results. We show that the filling-factor dependence of the cyclotron resonance spectrum is not due to any change in the positional or magnetic correlations of the two-dimensional electron gas. Instead, it is the result of a crossover from independent, spin-split resonances at low ν to a single mode dominated by Coulomb interactions at high ν (a possibility which has been noted in Ref. [4]). Moreover, we are able to account for the temperature dependence of the spectrum in terms of the thermal populations of the two spin states, calculated using the known single-particle g factor and without invoking exchange interactions.

Kohn's theorem [5] provides a natural point of departure for theories of cyclotron line shape: In a translationally invariant, one-component system, cyclotron resonance couples only to the center of mass, and the resonance is unaffected by interactions or correlations. Spectral structure may be observed if translational invariance is broken, either by disorder [6] or by nonparabolicity of the conduction band [7]. Alternatively, it may arise if there are carriers of more than one type, with different cyclotron frequencies, so that center of mass motion does not separate from internal motion of the electron gas. This happens in strained Si inversion channels, in which separate pockets of the conduction band are occupied, and has been studied experimentally [8] and theoretically [9] at weak magnetic field. There is a clear indication that the same mechanism is at work in the experiments we are concerned with, from the fact that the frequency splitting of the resonance at low ν matches the bulk spin splitting [2].

Motivated by this, we develop in the following a theory of cyclotron resonance for a two-component system in the strong field limit, the two components representing spins of each orientation. A complete treatment of many-body correlations at finite temperature would clearly be very difficult. Fortunately, the experimental conditions suggest several simplifications. First, the filling fractions of interest are small, so exchange energies are negligible [10], and the electrons may be treated as distinguishable particles, each with a conserved spin orientation. Second, the cyclotron energy is much larger than all other energy scales, so that only the two spin states of the lowest Landau level are populated, and cyclotron resonance involves transitions only into the corresponding spin states of the next level. Since higher Landau levels do not contribute, the two spin orientations may each be treated as if they have a parabolic dispersion relation, with slightly different effective masses. The relative concentrations of these two types of particle are fixed simply by their Boltzmann weights, in terms of the Zeeman splitting of the lowest Landau level. The favored spin state has the higher cyclotron frequency, so the minority spins can be thought of as randomly distributed "heavy" impurities in a sea of the majority spin state.

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At low density, the natural coordinates are the guiding center and orbital coordinates of the electrons

$$\boldsymbol{R}_i = \boldsymbol{r}_i - \boldsymbol{\rho}_i\ ,\ a_i = \frac{\rho_i^x - i\rho_i^y}{\sqrt{2}l}\ ,\ a_i^\dagger = \frac{\rho_i^x + i\rho_i^y}{\sqrt{2}l}, \quad (1)$$

where $\boldsymbol{\rho}_i = (\boldsymbol{p}_i + e\boldsymbol{A}_i) \times \hat{\boldsymbol{z}}/eB$, and $l = \sqrt{\hbar/eB}$ is the magnetic length. The only nonzero commutators are

$$[a_i, a_j^\dagger] = \delta_{ij}, \quad [R_i^x, R_j^y] = il^2\delta_{ij}. \quad (2)$$

Cyclotron resonance measures the real part of the dynamical conductivity, which is proportional to [11]

$$S(\omega) = \frac{1}{\omega}\int_{-\infty}^{\infty}\sum_{i,j}\langle a_i(t)a_j^\dagger(0)\rangle e^{i\omega t}\,dt. \quad (3)$$

In view of the low filling fraction, we expand the interactions in powers of the orbital coordinates. At leading order in the interactions, the equations of motion are

$$\dot{\boldsymbol{R}}_i = -\frac{1}{eB}\hat{\boldsymbol{z}} \times \frac{\partial U}{\partial \boldsymbol{r}_i}\bigg|_{\{R_N\}}, \quad (4)$$

$$\dot{a}_i = -i\omega_i a_i - \frac{i}{2eB}\sum_j a_j\left(\frac{\partial^2 U}{\partial x_i \partial x_j} + \frac{\partial^2 U}{\partial y_i \partial y_j}\right)\bigg|_{\{R_N\}}, \quad (5)$$

where ω_i is the cyclotron frequency of electron i, $U(\{r_N\})$ is the Coulomb energy, rapidly oscillating terms are omitted, and an additive constant has been absorbed in a_i.

Thus, from Eq. (4), the guiding centers drift along contours of the local potential gradient (independently of the orbital motion), while, from Eq. (5), the orbital coordinates perform coupled oscillations, with the coupling determined by the instantaneous positions of the guiding centers. For Coulomb interactions the coupling matrix is

$$M_{ij} = \begin{cases} \sum_{k\neq i}|\boldsymbol{R}_i - \boldsymbol{R}_k|^{-3}, & i = j, \\ -|\boldsymbol{R}_i - \boldsymbol{R}_j|^{-3}, & i \neq j. \end{cases} \quad (6)$$

At the lowest experimental temperatures and densities, the system is expected to form a Wigner crystal, with the guiding centers located at the sites of a triangular lattice. In these circumstances, the coupling matrix is time independent and the resonances are normal modes satisfying

$$\omega a_i = \delta\omega_i a_i + I\sum_j M_{ij}a_j, \quad (7)$$

where we measure frequency, ω, relative to the cyclotron frequency, ω_c, of the majority spins, in units of the splitting, $\delta\omega_c$, so that $\delta\omega_i = 0, -1$ for the majority and minority spins, respectively. The dynamical matrix (6) is defined by the triangular lattice vectors, in units of the lattice constant, $a = [(\sqrt{3}/2)\,n]^{-1/2}$, and I is the dimensionless interaction strength

$$I = \frac{e^2l^2}{8\pi\epsilon\epsilon_0 a^3}\frac{1}{\hbar\delta\omega_c}. \quad (8)$$

Hence we have established a model for cyclotron resonance in the Wigner crystal which has just two parameters: the interaction strength, I, controlled by the electron density, and the concentration of the minority spin state, p, set by the ratio of temperature to Zeeman splitting. It is similar to models for the band structure of uncorrelated binary alloys, and various approximate methods of solution are available. To facilitate a direct comparison with experiment, we choose, however, to solve Eq. (7) by numerical diagonalization.

Before presenting the results of our calculations, it is appropriate to review the experimental data, the most striking aspect of which is the temperature dependence of the spectrum. At low density, two resonance peaks are observed at the positions of the separate spin transitions [Fig. 1(a)]. At a slightly higher density, the behavior is much more complicated: Both peaks shift continuously with temperature and eventually collapse together [Fig. 1(b)]. Above $\nu \approx 1/6$, there is a single peak (not shown), whose position depends weakly on temperature.

These three types of behavior correspond to the weak, intermediate, and strong coupling regimes of our model. We have calculated the absorption spectra in each regime for a system of 225 particles, using periodic boundary conditions and Ewald summation for the dipole forces [12], and averaging over 400 realizations of the spin distribution. Longer studies of up to 900 particles show that finite size and statistical errors are small under these conditions. We display our results in Fig. 2. At weak coupling, the two spins are probed almost independently [Fig. 2(a)]. At intermediate coupling, the resonance structure becomes strongly dependent on the spin polarization, controlled by temperature [Fig. 2(b)]. In the strong coupling regime (not shown), we obtain a single resonance at the average cyclotron frequency, which

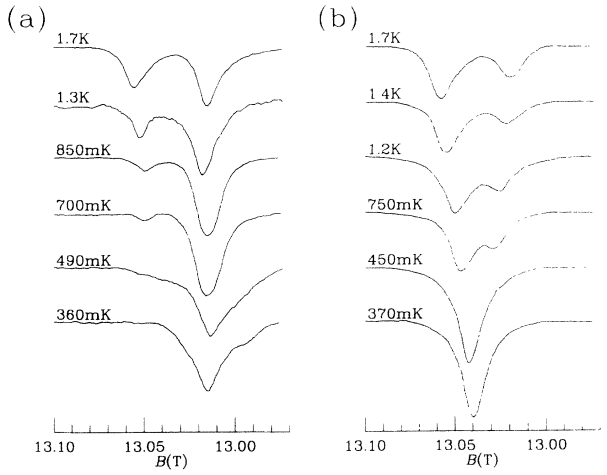

FIG. 1. Temperature variation of the experimental resonance spectrum, reproduced from [2]. The electron densities are (a) 2.6×10^{10} cm^{-2} and (b) 3.4×10^{10} cm^{-2}.

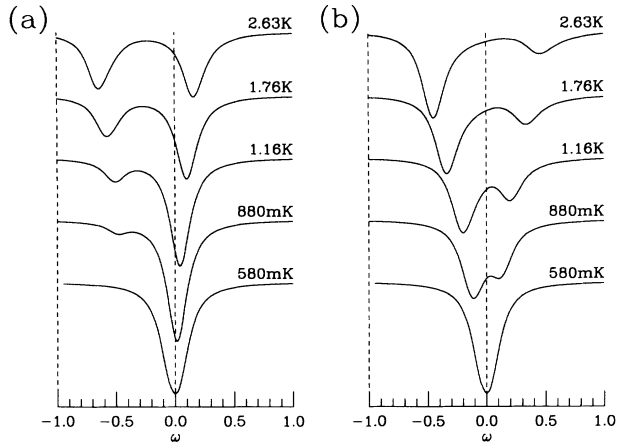

FIG. 2. Predicted resonance spectrum of the Wigner crystal: (a) $I = 0.05$ and (b) $I = 0.1$.

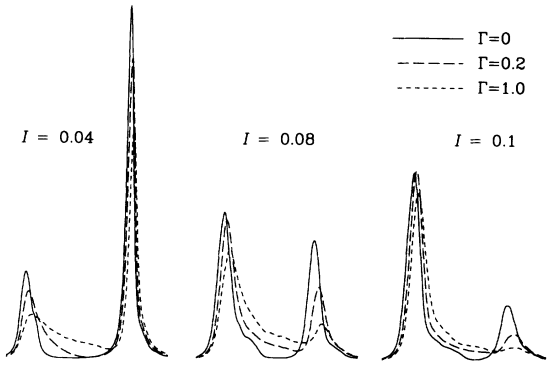

FIG. 3. Comparison of the resonance spectrum in the Wigner crystal ($\Gamma = 0$) and in the liquid at two temperatures ($\Gamma = 0.2, 1.0$) for $p = 0.1$.

shifts with temperature to follow the changing spin population.

The successful reproduction in the calculated spectrum (Fig. 2) of most of the observed features (Fig. 1) constitutes our main result. We consider it persuasive evidence that our interpretation of the experiments incorporates the relevant physical ingredients. In making this comparison, the only scope for adjustment lies in the conversion between experimental electron densities and the interaction strength, I, of the model. The experimental densities of Fig. 1 lead to values for the interaction strength of $I = 0.1, 0.17$, while we find the best fit with smaller values, $I = 0.05, 0.1$. We attribute the discrepancy to the finite thickness of the electron gas, which softens the Coulomb repulsion at short distances; comparable behavior has been noted previously in calculations on the fractional quantum Hall effect [13]. The spin populations are determined by the Zeeman splitting, using the value [14] $g = -0.4$ for the electron g factor, and the theoretical spectrum is convolved with a Lorentzian to match the experimental resolution.

It is initially surprising that the results of our calculations represent so well the observations, since the theory assumes that guiding center coordinates lie on a lattice, while most of the measurements are for temperatures well above the predicted melting point of the Wigner crystal. The explanation, we believe, is that the crossover from single-particle to collective cyclotron resonance is relatively insensitive to interparticle correlations, because the interactions are long ranged. In principle, one could investigate resonance in the liquid phase by using molecular dynamics to follow guiding center motion, simultaneously integrating Eqs. (4) and (5). The data presently available do not justify such an approach. Instead, we have examined the importance of interparticle correlations by diagonalizing Eq. (7) for *fixed* liquid configurations of guiding centers, generated using a Monte Carlo method. This introduces a third parameter to the model, $\Gamma = kT 4\pi\epsilon\epsilon_0 a/e^2$. In Fig. 3 we compare the results, at temperatures corresponding to 4 and 20 K for an electron density of 3×10^{10} cm^{-2}, to those for the Wigner crystal. The line shape changes by less than the experimental resolution, even at these high temperatures.

Further insight comes from studying a simplified model (suggested by the long-range character of interactions) in which each electron is coupled equally to all others, with a coupling constant inversely proportional to the total number of particles. This model is obviously insensitive to the positions of guiding centers. Using this symmetry, it is straightforward to diagonalize the equation of motion, (7). We find that there are only two optically active modes, the in-phase and out-of-phase oscillations of the two spin populations, with frequencies and oscillator strengths given by

$$\omega_{\text{in}} = [\alpha - 1 - \sqrt{(1-\alpha)^2 + 4\alpha p}]/2, \tag{9}$$

$$\omega_{\text{out}} = [\alpha - 1 + \sqrt{(1-\alpha)^2 + 4\alpha p}]/2, \tag{10}$$

$$S_{\text{in}} = 1 - S_{\text{out}} = \frac{\omega_{\text{out}} + p}{\omega_{\text{out}} - \omega_{\text{in}}}, \tag{11}$$

where p is the concentration of the minority spins. For comparison with our previous results, we choose the coupling constant, α, so that the sum of the interaction matrix over all neighbors is the same as that obtained with Coulomb interactions in the Wigner crystal, which requires that $\alpha = 11.034I$. The frequencies and oscillator strengths are displayed in Fig. 4: As the coupling is varied, there is a crossover from independent resonances of the two spin populations at weak coupling to a single, in-phase mode which will lie at the average cyclotron frequency, $-p$, at strong coupling. The crossover is most sharply defined for a single minority spin, $p = 0$, when it marks the transition from a bound state on the impurity spin (with relative oscillator strength inversely proportional to total particle number) to an extended state which carries all the oscillator strength. This transition

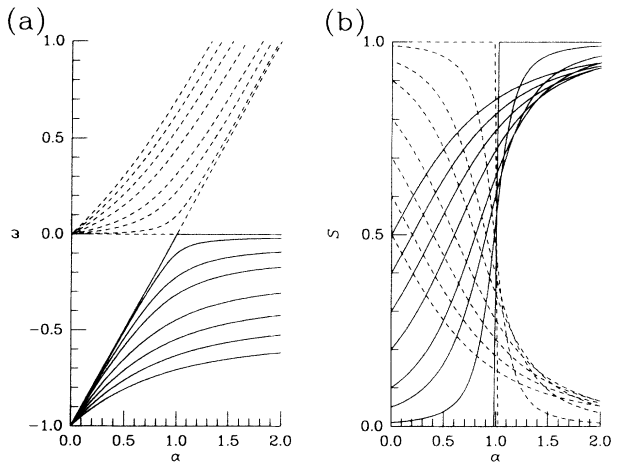

FIG. 4. Crossover of the mean field resonances, for $p = 0, 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5$: (a) frequencies and (b) oscillator strengths.

would not be present in a crystal with short-range interactions, since an arbitrarily weak attractive potential always has a bound state in two dimensions when the dispersion relation is quadratic at small wave vector. It does, however, occur in the two-dimensional Wigner crystal with Coulomb interactions, because the magnetoplasmon dispersion relation is *linear* at small wave vector. We find the critical interaction strength for the appearance of a bound state to be $I_c = 0.104$ in this case, while for infinite range interactions $\alpha_c = 1$, implying $I_c = 0.091$.

It is also interesting to consider briefly a similar approach to the influence of Coulomb interactions on split cyclotron resonances in bulk, nondegenerate semiconductors. We expect coupling to be much weaker in this case, because the spherical average (as opposed to the planar average) of the dipolar interaction, Eq. (5), is zero. Indeed, a single-particle theory appears to account well for measurements on bulk GaAs [15].

Although spin-split cyclotron resonance is, as we have shown, very insensitive to spatial correlations between electrons, it does provide a direct measure of spin polarization. We demonstrate this by considering the first moment of the spectrum. To first order in $\delta\omega_c/\omega_c$,

$$\langle w\rangle = \int_{-\infty}^{\infty} \omega S(\omega)\, d\omega \Big/ \int_{-\infty}^{\infty} S(\omega)\, d\omega, \tag{12}$$

$$\simeq \sum_{i,j} \langle \omega_i a_i a_j^\dagger + (l/\hbar)\partial_i U a_j^\dagger\rangle \Big/ \sum_{i,j} \langle a_i a_j^\dagger\rangle, \tag{13}$$

where $\partial_i = \partial/\partial x_i + i\partial/\partial y_i$ and U is the full interaction. For pairwise interactions, $\sum_i \partial_i U = 0$, and in the absence of Landau level mixing, $\langle a_i a_j^\dagger\rangle = \delta_{i,j}$, so we find

$$\langle w\rangle = \frac{1}{N}\sum_{i=1}^{N} \omega_i. \tag{14}$$

Thus the mean absorption frequency is simply the average cyclotron frequency of the particles. Given the cyclotron frequencies for each spin orientation, it provides a direct measure of the relative populations.

In conclusion, we have presented a model for spin-split cyclotron resonance at low filling fraction. Within this model we have been able to account fully for recent cyclotron resonance measurements on GaAs heterostructures. We find that the technique is rather insensitive to the liquid-solid phase transition, but provides a direct measure of spin polarization.

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