Experimental Synchronization of Chaotic Lasers

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We report the observation of synchronization of the chaotic intensity fluctuations of two Nd:YAG lasers when one or both the lasers are driven chaotic by periodic modulation of their pump beams.

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The synchronization of chaotic nonlinear oscillators has attracted much attention in recent years, motivated by the possibility of practical applications of this fundamental phenomenon. Several papers [1,2] have shown that such synchronization may be achieved in electronic oscillator circuits, with applications in the transmission of information signals masked in a background of chaos, followed by real-time recovery of signals at the receiver. A theory of synchronization of coupled, chaotic, nonlinear oscillators has been developed independently by Rabinovich and co-workers [3] in the context of turbulence in fluids. It has also been known for several years that lasers can exhibit chaotic intensity fluctuations under different circumstances. Winful and Rahman have theoretically investigated the possibility of synchronization of chaotic lasers, and some evidence of such behavior has been found in semiconductor laser arrays [4]. However, a direct test of their predictions on an experimental system where the coupling between the lasers is systematically varied has yet to be performed.

The scheme for chaotic synchronization developed by Pecora and Carroll [1,2] requires that a chaotic system exists, from which one can separate a stable subsystem with only negative Lyapunov exponents. When a chaotic system and a stable response subsystem are linked with a common drive signal, the two may display synchronized chaos. An example of this construction is given by the Lorenz system $dx/dt=\sigma(y-x)$, $dy/dt=rx-y-xz$, $dz/dt=xy-bz$ and the response system $dx'/dt=\sigma(y'-x')$, $dy'/dt=rx-y'-xz'$, $dz'/dt=xy'-bz'$ [2]. While this ingenious scheme has been practically implemented with electronic oscillators, it appears impossible to separate the elements of a chaotic laser and obtain a stable subsystem in precisely this manner.

In this paper we report the observation of synchronization of two coupled, chaotic, Nd:YAG (neodymium doped yttrium aluminum garnet) lasers. The equations that describe the lasers are of the form

$$\frac{d}{dt}\begin{pmatrix}E_1\\N_1\end{pmatrix}=\bar{f}(E_1,N_1,\kappa E_2)$$

and

$$\frac{d}{dt}\begin{pmatrix}E_2\\N_2\end{pmatrix}=\bar{g}(E_2,N_2,\kappa E_1)\,,$$

where the E's and N's are the complex electric fields and the population inversions for the two lasers and κ is a parameter that provides a measure of the mutual coupling between the lasers [4,5]. One or both the lasers may be driven chaotic through a periodic modulation of their pump excitation, and synchronized, chaotic intensity fluctuations may be observed in both cases when the lasers are sufficiently coupled. Thus it is possible, by a somewhat less restrictive procedure than that of Refs. [1] and [2], to obtain synchronized chaotic intensity fluctuations for two lasers. In particular, we show that a master-slave relationship is not necessary to obtain synchronization of chaotic oscillators. Mutual coupling can be used to obtain chaotic synchronization even for the case where the two uncoupled oscillators are both chaotic.

The laser system [Fig. 1(a)] consists of two Nd:YAG lasers of wavelength 1.06 μm generated in the same crystal by two almost equal intensity 514.5 nm pump beams obtained from an argon laser [5]. The spatial separation d of the parallel pump beams may be varied and is much larger (>0.5 mm) than their radius (about 20 μm) within the crystal. There is thus virtually no coupling through overlap of the population inversions of the two lasers; instead, the coupling between the lasers is provided by overlap of the intracavity laser fields of approximate radius 200 μm. The laser cavity consists of a high reflectivity coating (at 1.06 μm) on one end of the 5 mm long laser rod and a flat output mirror with 2% transmittance. The cavity length is 1.5 cm, and the output power of the lasers was measured to be about 5 mW each with a pump excitation of about twice above threshold.

A simple measure of the coupling is obtained from the overlap integral of the two fields, normalized such that the coupling coefficient $|\kappa|=\exp(-d^2/4\sigma^2)$ is unity for $d=0$ and where σ is related to the $1/e^2$ radius r of the intensity profile by $r=\sigma\sqrt{2}$ [5]. At the smallest separation of the beams in this study ($d\sim0.6$ mm) the overlap of the fields is $|\kappa|\sim1.1\times10^{-2}$, while at the largest separation ($d\sim1.5$ mm) $|\kappa|\sim6.1\times10^{-13}$. The coupling between the lasers is varied experimentally by translating the beam combiner in Fig. 1(a), which changes the separation of the pump beams.

The far field of the output from the lasers is observed on a video camera. When the lasers are phase locked (this occurs with a π-phase difference), the far field is a double-lobed pattern with a node in the center. When the lasers are mutually incoherent, the far-field pattern is

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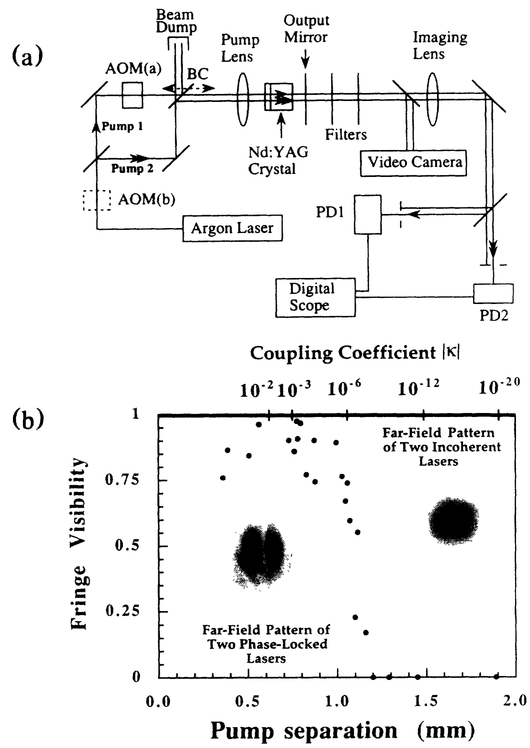

FIG. 1. (a) Experimental system for generating two spatially coupled chaotic lasers and monitoring their outputs. An acousto-optic modulator (AOM) can be placed in position (a) to modulate only laser 1, or in position (b) to modulate both lasers simultaneously. Beam splitters divide the argon laser output into two beams, each of which pumps a spatially separate region in the Nd:YAG crystal. The separation of the beams can be varied by moving the beam combiner, BC. The video camera is used to monitor the beam profiles. A lens is used to image the lasers so that the individual beams can be resolved and monitored by the photodiodes PD1 and PD2. The time traces of the two lasers can be displayed and stored by the digital oscilloscope. (b) Visibility of interference fringes for the superposed laser fields vs pump separation (from Ref. [5]). Also shown are two representative far-field intensity patterns for pump beam separations of $d=0.6$ mm and $d=1.5$ mm.

Gaussian. The lasers are found to be phase locked for separations of less than about 1 mm ($|\kappa|\sim 3.7\times 10^{-6}$), and display a very sharp transition from incoherence to coherence with decreasing separation of the pump beams. A plot of the visibility of the interference fringes formed by the superposition of the beams from the two lasers versus the separation of the pump beams is shown in Fig. 1(b), along with representative far field intensity patterns as described above. An imaging lens allows us to examine the near fields of the lasers as well, and two fast photodetectors connected to a digital oscilloscope display the temporal fluctuations of the individual laser intensities. A detailed description of this system of two coupled lasers is contained in Ref. [5], where the phase locking of the lasers through spatial overlap of their electric fields was investigated.

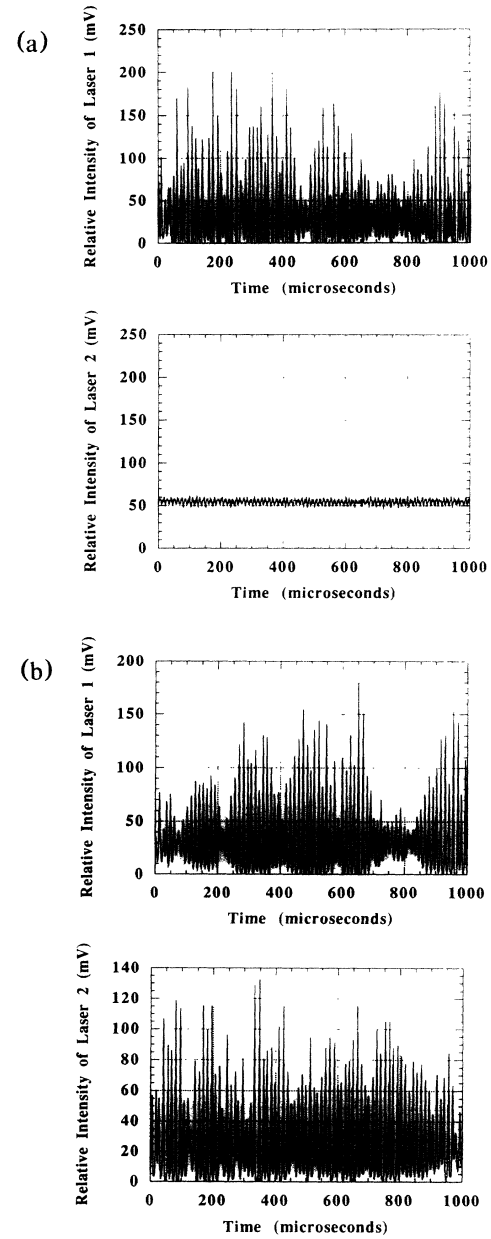

FIG. 2. (a) Relative intensity of the uncoupled lasers, for $d\sim 1.5$ mm. Only laser 1 is modulated, and there is no appreciable interaction between the two lasers. (b) Relative intensities of lasers 1 and 2, with the AOM in position (b). Notice the asynchronous fluctuations of the two intensities, typical of the uncoupled case, even though both lasers experience the same pump modulation.

In the Nd:YAG lasers studied here, the decay time of the upper lasing level is $\tau_f\sim 240$ μsec, while the round-trip time of light in the cavity is $\tau_c\sim 0.12$ nsec. This leads to relaxation oscillations at a frequency $\nu_{\rm rel} = (1/2\pi)[\delta(\gamma/\gamma_{\rm th}-1)/\tau_f\tau_c]^{1/2}$, where $\gamma_{\rm th}$ and γ are the threshold and actual pump rates, δ is the fractional cavity loss ($\sim$2%) per pass [6]. At a pump excitation of twice the threshold value, $\nu_{\rm rel}\sim 130$ kHz. This is in very good agreement with measured relaxation oscillation frequencies for our laser system. An acousto-optic modulator (AOM) may be used to sinusoidally amplitude modulate a single pump beam in position (a), or to modulate both pump beams simultaneously in position (b), indicated by the dotted lines in Fig. 1(a). If the pump beam of one of the lasers is modulated at close to the relaxation oscillation frequency, the laser intensity may be driven into chaotic fluctuations, as seen in Fig. 2(a). A wide variety of periodic wave forms, including period-doubled oscillations, may be observed for different frequencies and amplitudes of modulation. Similar behavior in other modulated laser systems has been extensively studied in the past [7].

With the AOM in position (a), and for a large separation $d \sim 1.5$ mm of the pump beams within the crystal, the two lasers are effectively *uncoupled*. We see in Fig. 2(a) that laser 1 shows chaotic intensity fluctuations when the frequency and amplitude of modulation of the pump beam are appropriately adjusted. In these experiments, the maximum depth of modulation was adjusted to be about 50%. For a modulation frequency close to the relaxation oscillation frequency of the laser, the intensity fluctuations increase markedly in amplitude and become chaotic. Laser 2 is unaffected by the fluctuations of laser 1 and shows a steady intensity.

If the pump beams of the two uncoupled lasers are both modulated with the AOM in the position shown by the dotted lines, both lasers display chaotic intensity fluctuations that are not synchronized, as shown in Fig. 2(b). This is to be expected, since the lasers are not coupled to any appreciable extent at this large separation, as may be verified from the Gaussian far-field pattern of the laser intensities monitored by the video camera system.

For *intermediate coupling* at a somewhat smaller separation ($d \sim 1$ mm), close to the phase-locking threshold, the two laser intensities are both found to be chaotic and irregular when only the pump beam for laser 1 is modu-

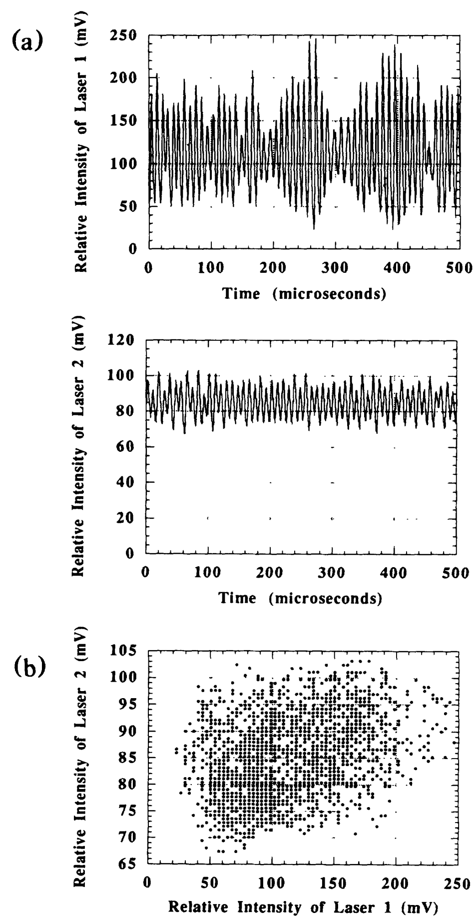

FIG. 3. (a) Relative intensities of two intermediately coupled lasers ($d \sim 1.0$ mm) when only the pump beam for laser 1 is modulated. Although still asynchronous, the modulation of one laser now affects the other appreciably. (b) X-Y plot of the two laser intensities shown in (a). The dispersion of the points indicates that they are not synchronized.

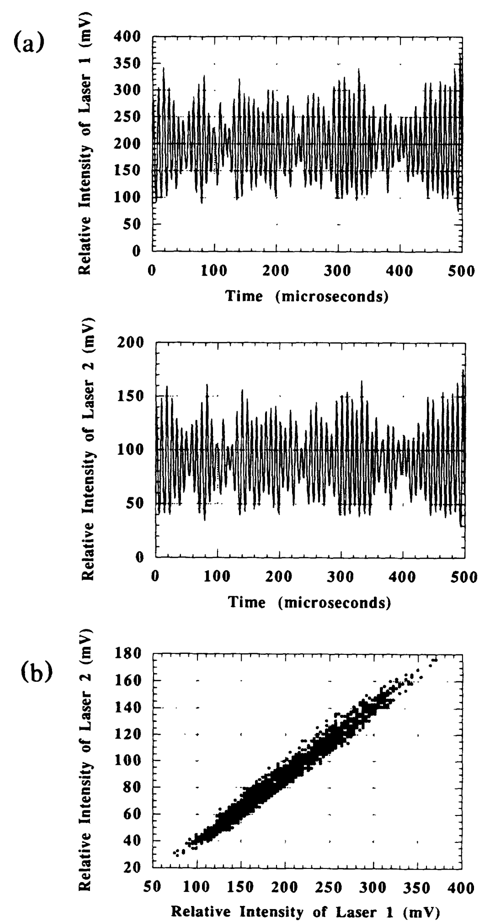

FIG. 4. (a) Relative intensities of two strongly coupled lasers ($d \sim 0.75$ mm) with the pump beam for laser 1 modulated by the AOM. Note the strong synchronization of the two laser intensities. (b) X-Y plot of the two laser intensities shown in (a). Note the strong linearity of this figure, indicating the synchronized nature of the time traces. Compare this figure to Fig. 3(b).

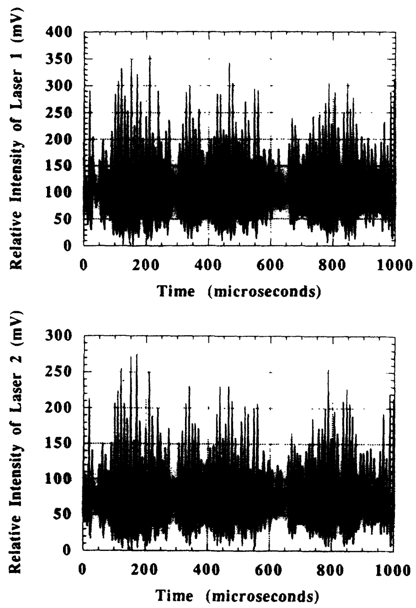

FIG. 5. Relative intensities of two strongly coupled lasers ($d \sim 0.60$ mm), with the AOM in position (b). Notice that, in contrast to the uncoupled case shown in Fig. 2(b), the lasers now fluctuate synchronously.

lated, as shown in Fig. 3(a). Clearly, the chaotic behavior of laser 1 now has a significant influence on its neighbor and destabilizes the intensity of laser 2. However, the plot of the fluctuations of laser 1 vs those of laser 2 displayed in Fig. 3(b) show a random set of points, indicating that the fluctuations are unsynchronized.

For *strong coupling* at a smaller separation ($d \sim 0.75$ mm) of the pump beams, the far-field pattern is distinctly double lobed; this reveals that the lasers are phase locked. Even in this "strong coupling" regime, the overlap of the two laser intensity profiles is extremely small, $|\kappa| \sim 8.8 \times 10^{-4}$. Modulation of the pump beam for laser 1 now leads to well synchronized chaotic fluctuations of the two laser intensities, as shown in the time traces of Fig. 4(a). A plot of the intensity of laser 1 vs the intensity of laser 2 [Fig. 4(b)] is now remarkably different from the random scatter of points shown in Fig. 3(b), and the synchronized nature of the chaotic lasers is evident. Synchronization of the chaotic lasers persists stably over periods of tens of minutes, as long as the temperature and other environmental conditions are maintained constant.

Figure 5 shows the result of modulating both the pump beams simultaneously for strongly coupled lasers with the pump beams separated by 0.6 mm. In contrast to Fig. 2(b), where both the uncoupled lasers are independently chaotic (and thus would have positive Lyapunov exponents), we now see that the two mutually coupled chaotic lasers are well synchronized. The scheme for synchronization is more general than those developed earlier [1,2] where a stable subsystem with negative Lyapunov exponents is necessary.

The laser outputs, though adjusted to be roughly equal in intensity, produce different voltages due to the difference in sensitivity of the photodetectors and differences in apertures, beam splitters, etc. The lasers themselves are of course not identical, due to imperfections in the crystal and mirror, or nonparallelism of the pump beams. One laser may thus have a somewhat higher pump threshold than the other, and therefore a relaxation oscillation frequency that differs by as much as 10% from that of the other laser. Synchronization is achieved despite these differences, and is remarkably robust in nature. We also note that the coupled lasers remain phase locked with a π-phase difference even when their intensities exhibit synchronized chaotic fluctuations.

In conclusion, we have reported the direct observation of the synchronization of intensity fluctuations of two chaotic lasers. It should be possible to extend these observations to large arrays of coupled nonlinear oscillators, including linear and two-dimensional arrays of lasers, Josephson junctions, and other physical and chemical systems.

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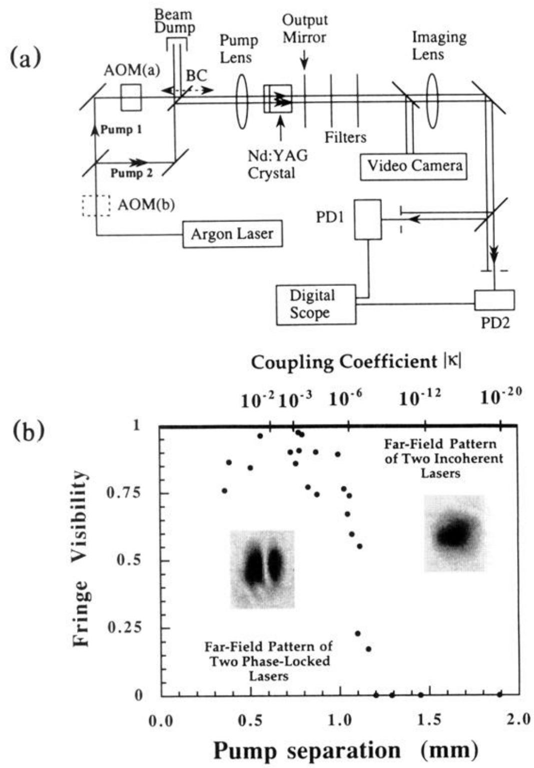

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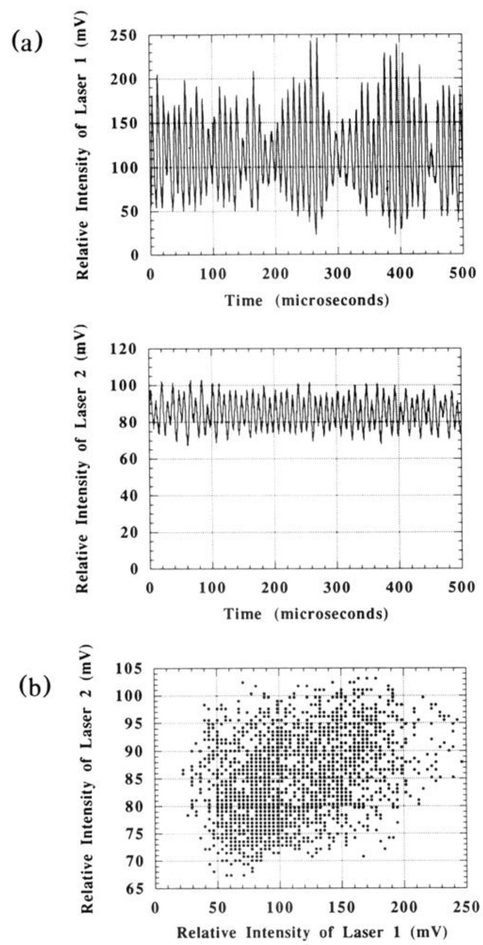

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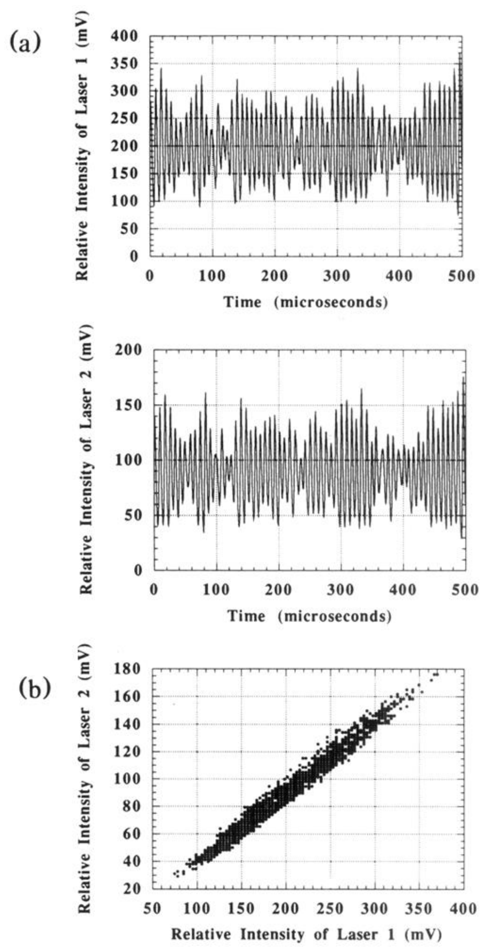

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