Nucleon-Nucleon Potential from an Effective Chiral Lagrangian

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The nucleon-nucleon potential obtained from the most general effective chiral Lagrangian involving low momentum pions, nonrelativistic nucleons, and Δ isobars was considered to third order in the chiral expansion. The parameters of the potential were adjusted to reproduce the low energy nucleon-nucleon scattering phase shifts as well as the properties of the deuteron. Reasonable fits are obtained in this first application of a nucleon-nucleon potential based on chiral symmetry.

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Theoretical efforts to account for the nucleon-nucleon (*NN*) interaction have been underway for many decades. Early field theoretical approaches [1] in the 1950's were generally unsuccessful. These eventually gave way to more phenomenological treatments [2] which provided a pragmatic way to describe the abundant *NN* scattering and bound state (deuteron) data. Beginning in the 1970's theoretical models emerged which were more successful than in earlier attempts. These were based on one-pion exchange (OPE), heavy meson exchange, and multi-meson exchange plus short-range phenomenology [3–5]. The Paris [3], Nijmegen [4], and Bonn [5] models are representative of these more recent efforts. The Paris model includes OPE and single ω-meson exchange along with two-pion exchange (TPE) contributions calculated using πN phase shifts, $\pi\pi$ interactions, and dispersion relations, plus short-range phenomenological potentials. Regge pole theory was used by the Nijmegen group to obtain a *NN* interaction model which includes numerous one-boson exchange terms with exponential form factors and repulsive central Gaussian potentials arising from Pomeron and tensor trajectories. The Bonn group included a number of one-boson exchange terms plus two-meson exchanges (2π, $\pi\rho$, and $\pi\omega$), correlated TPE in the form of the exchange of an effective scalar meson, effective three-pion exchange, and intermediate Δ isobars, all in the context of "old fashioned" perturbation theory. Each of these models contains many parameters and each provides impressive agreement with the deuteron data and with *NN* scattering data up to 350 MeV laboratory kinetic energy.

Despite this considerable effort the connection between the *NN* interaction and the underlying theory of the strong interaction, assumed to be quantum chromodynamics (QCD), remains mysterious. In the above models it is not possible to distinguish terms which are "put in by hand" from those which are required by QCD. In particular, any effective mesonic theory should at least share the same symmetries of QCD, like (approximate) chiral symmetry. Furthermore, meson exchange models generically lack a consistent and systematic approximation scheme which justifies neglecting or keeping some contributions.

Partly to overcome these deficiencies Weinberg [6,7] recently applied the method of phenomenological Lagrangians [8] to a system of low momentum pions and nonrelativistic nucleons. The starting point is the most general Lagrangian incorporating the approximate chiral symmetry of QCD, written in terms of the pion and nucleon fields and their covariant derivatives. Higher mass mesons and baryons are integrated out (except for the Δ as in the present work), their effects being contained in undetermined coefficients that multiply nucleon contact terms as well as terms with more derivatives. The use of old fashioned perturbation theory allows a ready separation of the connected diagrams which form the potential from the disconnected diagrams that are summed by solving the Lippmann-Schwinger or Schrödinger equations. Weinberg showed that a systematic expansion of the nuclear potential exists in powers of (Q/M) where Q is the typical momentum involved and M is a characteristic QCD mass scale. He also considered the leading terms. Subsequent work by two of us [9] extended the expansion of the Lagrangian in Refs. [6] and [7] to second order and extended the potential to order $(Q/M)^3$ including all TPE, one-loop terms.

What distinguishes the present approach to the two-nucleon problem from others is the use of a general effective chiral Lagrangian and the development of a systematic perturbative expansion of the potential based on powers of (Q/M). This approach is *a priori* model independent and is completely consistent with the symmetries of QCD and with low energy $\pi\pi$ and πN scattering [10].

In the present work the second order chiral Lagrangian of Ref. [9] was extended to include the Δ (1232 MeV) isobar. TPE one-loop diagrams with one or two intermediate Δ's were added to the *NN* potential. Complete details will be presented in a later publication [11]. The main purpose of this Letter is to report the results of numerical calculations which demonstrate the capability of the chiral symmetry potential for describing low energy

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NN data. It is not our intention to present a NN interaction model which is phenomenologically superior to the many others [2–5] which already provide excellent descriptions of NN data. Rather we intend that this work and that in Refs. [6], [7], and [9] establish a link between QCD and models of the NN force, a link which is made credible by the ability of the model to fit NN data as demonstrated here.

In principle the undetermined coefficients of the potential can be calculated from QCD. Here, however, we obtained values for these coefficients by freely adjusting them such that the deuteron properties and low energy NN scattering phase shifts are well described. Given that the form of the NN potential is fully determined by the effective chiral Lagrangian and the perturbation expansion, the ability of this chiral potential to describe NN data is a nontrivial result.

In Refs. [6] and [7] Weinberg obtained the lowest order NN potential consisting of the usual static OPE part plus spin-dependent NN contact terms. This lowest order potential is of course too simple to account for the known properties of the NN force [2–5] (e.g., intermediate-range attraction, short-range repulsion, and spin-orbit term). Extension of the effective chiral potential to order $(Q/M)^3$ in Ref. [9] and the inclusion of intermediate Δ isobars here is sufficient to generate the relevant features of the NN potential with respect to spin structure and range.

The present chiral symmetry potential includes the usual long-range static OPE term with axial coupling constant g_A and pion decay constant F_π plus corrections to the OPE potential that can be thought of as arising from expansions of the $\pi \bar{N} N$ form factor through second order in momentum, resulting in two undetermined coefficients. Nonstatic corrections to the OPE terms arising from the recoil of the nucleon upon pion emission are also included through second order in momentum. In momentum space this correction is given by an energy-dependent term,

$$V_{\rm NS}(E,k,q)=\left(\frac{g_A}{F_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+m_\pi^2} \times \frac{E-(1/4m_N)(4\vec{k}^{\,2}+\vec{q}^{\,2})}{\sqrt{\vec{q}^{\,2}+m_\pi^2}}\,, \tag{1}$$

where $\vec{k}$ and $\vec{q}$ are the average NN momentum and momentum transfer, respectively, the nucleon spin (isospin) operator is denoted by $\vec{\sigma}(\boldsymbol{\tau})$, the pion (nucleon) mass is m_π (m_N), and E is $\sqrt{s}-2m_N$. The recoil correction in Eq. (1) replaces that erroneously given in Eq. (6) of Ref. [9].

Intermediate-range contributions are included via a number of TPE terms with either zero, one, or two intermediate Δ isobars where the $\pi N\Delta$ coupling constant h_A was allowed to vary but the Δ mass was fixed to 1232 MeV. Contributions of box and crossed box diagrams are standard [1]. Iterated OPE graphs (up to one loop) with one or two intermediate Δ isobars are included in the potential rather than being included via coupled channels [12]. We emphasize that there also exist in our potential TPE contributions that are less common, some of which are fixed by chiral symmetry in terms of g_A and F_π but others result from first-order corrections to the Lagrangian which introduce three more undetermined coefficients. One of these TPE diagrams gives rise to a scalar-isoscalar interaction which is reminiscent of the σ or σ' exchange in meson exchange models [5]. We stress that these new contributions from the nonlinear πN coupling are a consequence of chiral symmetry and are usually not included in meson exchange potentials. (An exception is the work of Ref. [13], where, however, only those contributions from the lowest order Lagrangian without the isobar have been considered.) These terms represent the only form of "correlated" TPE in our potential.

Finally the short-range parts are given in terms of contact terms. These contain the effects of exchange of higher energy modes and are not constrained by chiral symmetry, being only required to satisfy parity and time-reversal invariance. Including terms through second order in momentum results in 9 additional parameters for each NN isospin state. In all, our potential contains 26 parameters.

In order to do the integrals over momentum associated with the one-loop terms as well as the Fourier transforms which were used to convert the momentum space potential into coordinate space (for the numerical calculations), we found it convenient, following the Nijmegen group [4,14], to use a Gaussian cutoff function, $\exp(-l^2/\Lambda^2)$, where l is either the loop integration variable or the momentum transfer and parameter Λ is of order M. Other regularization schemes are possible; however, the cutoff method is appropriate given the noncovariant nature of the potential.

The potential in coordinate space has the form

$$V=\sum_{p=1}^{20} V_p\left(r,\frac{\partial}{\partial r},\frac{\partial^2}{\partial r^2},E\right)\mathcal{O}^p\,, \tag{2}$$

where

$$V_p\left(r,\frac{\partial}{\partial r},\frac{\partial^2}{\partial r^2},E\right)=V_p^0(r,E)+V_p^1(r,E)\frac{\partial}{\partial r}+V_p^2(r,E)\frac{\partial^2}{\partial r^2}\,. \tag{3}$$

The 20 spin/isospin operators $\mathcal{O}^p$ are given by

$$\mathcal{O}^{p=1,\ldots,20}=1,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,\vec{\sigma}_1\cdot\vec{\sigma}_2,\vec{\sigma}_1\cdot\vec{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,S_{12},S_{12}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,\vec{L}\cdot\vec{S},\vec{L}\cdot\vec{S}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,\vec{L}^2,\vec{L}^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,\vec{L}^2\vec{\sigma}_1\cdot\vec{\sigma}_2,\vec{L}^2\vec{\sigma}_1\cdot\vec{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$
$$(\vec{L}\cdot\vec{S})^2,(\vec{L}\cdot\vec{S})^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,S_{12}\vec{L}\cdot\vec{S},S_{12}\vec{L}\cdot\vec{S}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,S_{12}\vec{L}^2,S_{12}\vec{L}^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,S_{12}(\vec{L}\cdot\vec{S})^2,S_{12}(\vec{L}\cdot\vec{S})^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2, \tag{4}$$

where S_{12}, $\vec{S}$, $\vec{L}$, and $\vec{L}\cdot\vec{S}$ are the NN tensor, total spin, orbital angular momentum, and spin-orbit operators, respectively. The first 8 operators are standard while the next 6 complete the set used in the phenomenological Urbana v_{14} potential [15]. The novel features of the present potential include the V_p^1 and V_p^2 terms for operators $p=1$–8, and all of the $p=15$–20 terms. The 60 radial functions V_p^i in Eqs. (2) and (3) (some vanish) are given in Ref. [11].

The Schrödinger equation with the nonlocal potential in Eqs. (2)–(4) was solved by standard partial wave expansion methods where the total NN wave function was expanded in spin-angle basis functions assuming L-S coupling. For spin singlets and uncoupled spin triplet partial wave channels the first and second order derivative terms in the potential were included by factoring the radial wave function according to $R_\alpha(r)\equiv K_\alpha(r)\phi_\alpha(r)$ (subscript α indicates partial wave quantum numbers) and defining $K_\alpha(r)$ such that the sum of all first order derivative terms of $\phi_\alpha(R)$ vanishes. For the spin triplet coupled partial waves an analogous procedure was followed except that two sets of pairs of coupled, first order differential equations for auxiliary functions $K_{\alpha\alpha'}(r)$ were solved using the fourth order Runge-Kutta method. Following the evaluation of $K(r)$ the usual second order, homogeneous differential equation for $\phi(r)$ was solved and the wave function determined by the usual scattering wave boundary conditions. The resulting S matrix was parametrized in terms of phase shifts and mixing angles as in Ref. [12].

The 26 parameters of the model were varied to optimize the fit to the isospin 0 (np) and 1 (pp) phase shifts of Ref. [16] at 10, 25, 50, and 100 MeV laboratory kinetic energy for all states with total angular momentum $J\leq 2$. The phase shifts for higher partial waves in this energy range are determined by the OPE potential and were not varied in Ref. [16]. The cutoff parameter Λ was taken at 3.9 fm^{-1} (the ρ mass; variation of the cutoff from 3.6 to 4.4 fm^{-1} did not produce significant changes in the fitting.) In addition the $I=0$ ($^3S_1$-$^3D_1$) bound state in the model was fitted to the deuteron binding energy, magnetic moment, and electric quadrupole moment. A grid search of all parameters was initially conducted to roughly locate a minimum in chi square. The fits were optimized using the downhill simplex method of chi-square minimization [17].

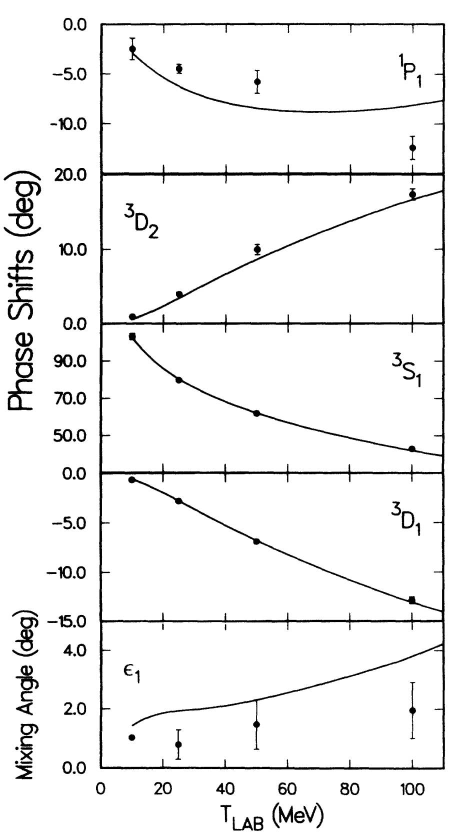

FIG. 1. Best fits (solid curves) to the $I=0$ np phase shifts and the mixing angle ϵ_1 from Ref. [16].

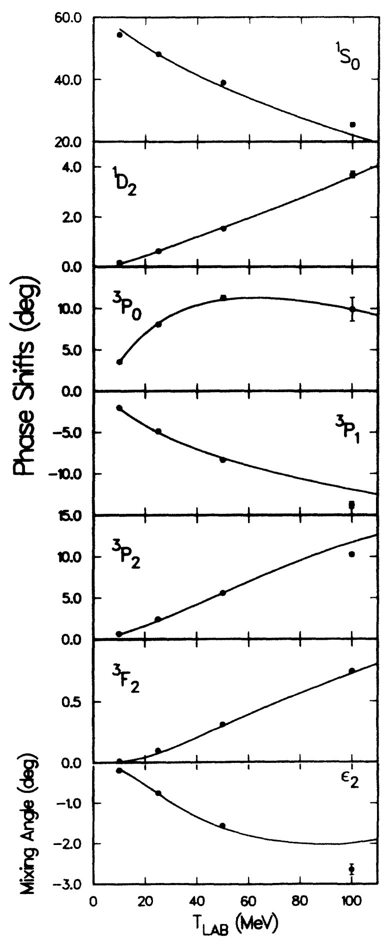

FIG. 2. Best fits (solid curves) to the $I=1$ pp phase shifts and the mixing angle ϵ_2 from Ref. [16].

The best fits are shown in Figs. 1 and 2 where quantitative descriptions of the phase shifts were obtained for most cases. The deuteron binding energy, magnetic moment, quadrupole moment, and asymptotic d-state to s-state wave function ratio obtained are 2.18 MeV, 0.851 nuclear magnetons (μ_N), 0.231 fm^2, and 0.0239, respectively, which compare well with the experimental values of 2.224579(9) MeV, 0.857406(1)μ_N, 0.2859(3) fm^2, and 0.0271(4) (Ref. [18]). The calculated d-state fraction is 4.97%. The $L=0$ singlet and triplet scattering lengths predicted by the model (these were not used to constrain the fit) are -15.0 and 5.46 fm, respectively, in comparison with the measured values of $-16.4(1.9)$ fm (Ref. [19]) and 5.396(11) fm (Ref. [20]). The resulting values of the parameters of the model will be provided in Ref. [11] along with a complete description of the potential. The values of g_A, h_A, and F_π from our fit are 1.33, 2.03, and 192 MeV, respectively.

In conclusion, the numerical results reported here show that the general features of the NN potential can be understood on the basis of the symmetries of QCD. While our method is not intended to provide descriptions of NN data competitive with purely phenomenological or meson exchange models, the numerical results do encourage the use of this potential in calculations of nuclear structure, nuclear matter, and few-nucleon systems at low energy. The effective chiral Lagrangian approach may also prove useful for analyses of π-deuteron scattering and NN pion production.

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