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Quantum Stochastic Resonance

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We demonstrate that stochastic resonance, where an increase in the noise acting on a nonlinear dynamical system increases the signal-to-noise ratio describing the response to periodic driving, can occur in quantum systems as well as classical ones. We show that quantum stochastic resonance can be observed experimentally by measuring conductance fluctuations in mesoscopic metals and describe the experimental parameters for which it occurs.

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Stochastic resonance (SR) is a phenomenon where the response of a nonlinear dynamical system to external driving is enhanced by the presence of noise [1–8]. Here we extend this concept to the quantum regime, defining the conditions for a similar resonance in the dissipative tunneling of a two-state system coupled to its environment. A qualitatively different mechanism leads to SR in this quantum system than in classical ones; in particular, the appearance of a resonance requires an asymmetry in the energies of the two states. We predict that quantum stochastic resonance can be observed experimentally by measuring conductance fluctuations in mesoscopic metals and describe the experimental parameters for which it may be observed.

The canonical example of SR consists of a particle in a double-well potential subject to both random noise (characterized by a temperature T) and periodic forcing, which could consist of a sinusoidal variation of the asymmetry energy ε of the wells with frequency ω_s (Fig. 1). One way of quantifying the response to the drive is the signal-to-noise ratio (SNR) [2,3]; SR occurs when the SNR passes through a maximum as the noise level is increased. Another proposed characterization of SR is the distribution of residence times in each minimum of the potential well [6,7]. An exponential distribution occurs in the absence of driving; in the SR regime a series of harmonic peaks is sometimes observed. We show that both of these characterizations can yield nontrivial SR in the quantum case.

We characterize the system using a rate equation description, which is valid when the dynamics are well described in terms of transition rates W_+ and W_- between two states, and when the drive frequency and the interwell transition rates are much slower than the intrawell relaxation rate [3,9]. The quantum-mechanical dynamics of a two-state system coupled to its environment satisfy both of these conditions over a broad, experi-

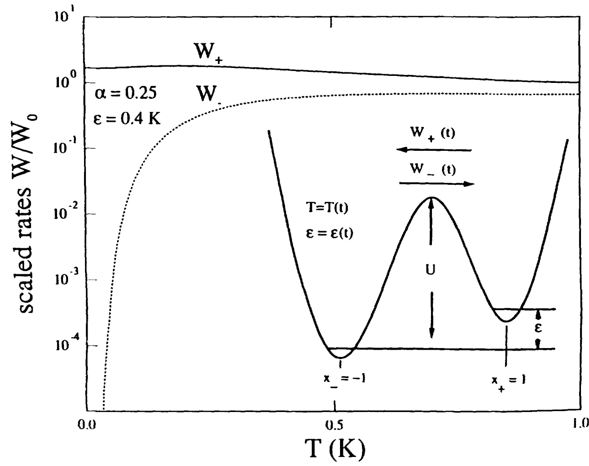

FIG. 1. Transition rates W_+ and W_- of a quantum-mechanical dissipative two-state system versus temperature T [Eq. (7)]. The rates are scaled by $W_0 \equiv W_+(T=1\ \mathrm{K})$. The inset shows the asymmetric potential well; the transition rates are modulated by varying the temperature T or the asymmetry energy ε.

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mentally accessible range [10].

First, following Ref. [3], we calculate the SNR of a two-state system characterized by transition rates [3]. We define the probability of being in the position state $x_+=1$ ($x_-=-1$) as n_+ (n_-), and the transition rate for the system to leave that state $W_+(t)$ ($W_-(t)$), where the time dependence of the W's comes from the external drive. The experimental signature of such a system is the random telegraph signal. The measured correlation function $C(\tau)=\overline{n_{+i}(t)n_{+i}(t+\tau)}$, where each $n_{+i}=0$ or 1 and the overbar indicates an average over many data points i taken at equal time intervals, is described by

$$C(\tau)=\langle n_+(t+\tau|x_+,t)n_+(t|x_0,-\infty)\rangle ,$$

where $n_+(t|x_0,-\infty)$ is the probability of being in the + state at t after being in state x_0 at $t\to-\infty$ and $\langle\rangle$ denotes the average over t. For Markovian transitions, the continuous probability $n_+(t)$ obeys the rate equation:

$$\frac{dn_+}{dt}=W_-(t)-[W_+(t)+W_-(t)]n_+(t) . \qquad (1)$$

The power spectrum $S(\omega)$ is the Fourier transform of $C(\tau)$; it contains a roughly Lorentzian broadband noise background and δ-function peaks at $\omega=0$, the drive frequency ω_s, and its harmonics. The ratio of the coefficient of the fundamental peak and the value of the noise at ω_s is the SNR. Whether the system displays SR (a maximum in the SNR as a function of temperature) depends on the specific form of the transition rates.

To obtain analytic information [11], we expand

$$W_\pm(t)=W_{\pm 0}+w_\pm\cos(\omega_s t)+\cdots ,$$

and solve Eq. (1) and evaluate the correlation function $C(\tau)$ to order $(w_\pm)^2$. For the special case that the transition rates obey detailed balance,

$$W_+/W_-=\exp[\varepsilon_0/k_BT_0+\delta(\varepsilon/k_BT)\cos\omega_s t] ,$$

where k_B is Boltzmann's constant and T is the temperature, the leading order contribution to the SNR is

$$\mathrm{SNR}=\frac{\pi}{4}\frac{W_{+0}}{1+\exp[\varepsilon_0/k_BT_0]}\left[\delta\left(\frac{\varepsilon}{k_BT}\right)\right]^2 ; \qquad (2)$$

to this order the SNR is independent of ω_s. For a sinusoidally modulated asymmetry energy ($\varepsilon=\varepsilon_0+\delta\varepsilon\times\cos\omega_s t$),

$$\delta\left(\frac{\varepsilon}{k_BT}\right)=\frac{\delta\varepsilon}{k_BT_0} ;$$

for temperature modulation ($T=T_0+\delta T\cos\omega_s t$),

$$\delta\left(\frac{\varepsilon}{k_BT}\right)=-\left(\frac{\varepsilon_0}{k_BT_0}\right)\frac{\delta T}{T_0} .$$

The correlation function $\langle x(t+\tau)x(t)\rangle$ can be formed from linear combinations of correlations of $n_+(t)$ and yields the same SNR.

The temperature derivatives of the rates do not enter into Eq. (2). This result can be understood on physical grounds when the modulation is very slow, $\omega_s\ll W_+,W_-$. In this limit the populations of the states follow the drive adiabatically, so that the signal depends only on the ratio of the rates and not on the temperature dependence of each rate separately. The noise is a Lorentzian of width $W_0=W_{-0}+W_{+0}$ and height as $\omega\to 0$ of $1/W_0$, so that the SNR is proportional to the product of W_0 and detailed balance factors.

The expression (2) can be applied both in the classical and quantum regimes. In the classical case $W_{+0}\sim e^{-U/k_BT}$, where U is the height of the energy barrier separating the two states [12,13]. Since $U\gg\varepsilon$, the detailed balance factor plays a minor role; the maximum of the SNR as T is increased results from a competition between the exponentially increasing W_+ and a power law of T that arises because incoherent transitions between the two states destroy the synchronicity. Formally the SNR has a peak when $k_BT\sim U$; however, both the transition rate description of the dynamics and the Kramers form for the rates break down at these temperatures. In the quantum case, where tunneling dominates the transition rates, W_+ is only weakly temperature dependent [Fig. 1; see Eq. (7) below]. For this case the SNR for asymmetry and temperature driving are shown in Fig. 2(a). Quantum SR does not occur in the limit of a symmetric well, $\varepsilon/k_BT\ll 1$, where $W_\pm\propto T^{-(1-2\alpha)}$. Quantum SR does occur for an asymmetric well. When $\varepsilon_0/k_BT_0\gg 1$, the signal is suppressed because the detailed balance factor leads to an exponentially small W_-, i.e., the particle does not leave the lower well. When $k_BT_0\sim\varepsilon_0$ the relative occupation in the upper state depends more sensitively on temperature; when $k_BT_0\gg\varepsilon_0$ the rel-

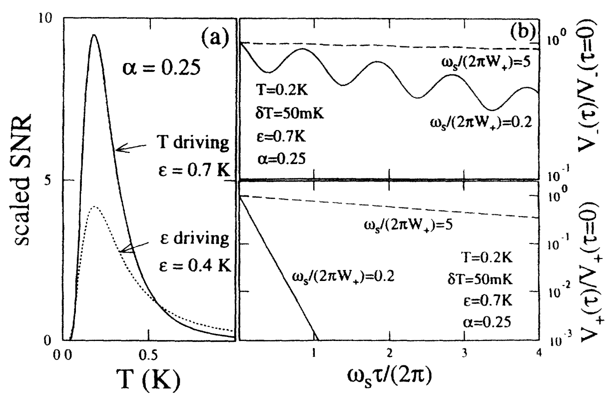

FIG. 2. (a) Signal-to-noise ratio [Eq. (2)] versus temperature. The SNR is scaled by $W_0(\delta h)^2$, where $W_0\equiv W_+(1\text{ K})$ and δh is δT ($\delta\varepsilon$) for temperature (asymmetry) modulation. (b) Distribution of residence times in each well $V_\pm(\tau)$ versus τ (in units of the drive period $2\pi/\omega_s$), obtained by numerical solution of Eqs. (3) and (4). $V_\pm(\tau)$ is normalized to unity at $\tau=0$. For fast modulations $\omega_s/2\pi\gg W_\pm$, both V_+ and V_- are exponential; when $W_-\ll\omega_s/2\pi\ll W_+$, $V_-(\tau)$ oscillates with period $2\pi/\omega_s$. The maximum in the SNR and the structure in $V_-(\tau)$ demonstrate that the quantum-mechanical dissipative two-state system displays SR.

ative occupations are nearly equal and the signal again decreases. The description in terms of transition rates is valid up to temperatures of order the barrier height U [10], so for the quantum system the theory should apply accurately even for temperatures well above the peak in the SNR.

Quite generally, the distribution of residence times, $V_\pm(\tau)$, is the product of an exponential envelope and a function that is periodic with the drive period $2\pi/\omega_s$. This follows because

$$V_\pm(\tau)=N\int_0^{2\pi/\omega_s} dt_0 P_\pm(t_0+\tau|t_0)Y_\pm(t_0),$$

where N is a normalization constant and $P_\pm(t_2|t_1)$ is the probability of first leaving state $\pm$ at time t_2 given that the state was entered at time t_1,

$$P_\pm(t_2|t_1)=W_\pm(t_2)\exp\left[-\int_{t_1}^{t_2} dt^* W_\pm(t^*)\right].$$

$Y_\pm(t)$ is the probability that the state was entered at a time t^* satisfying $t^*=t\bmod(2\pi/\omega_s)$; by definition, $Y_\pm(t)=Y_\pm(t+2\pi/\omega_s)$. Defining $W_\pm(t)=\langle W_\pm\rangle+\delta W_\pm(t)$,

$$V_\pm(\tau)=Ne^{-\langle W_\pm\rangle\tau}G_\pm(\tau),$$

$$G_\pm(\tau)=\int_0^{2\pi/\omega_s} dt_0 W_\pm(t_0+\tau)\times\exp\left[-\int_{t_0}^{t_0+\tau} dt^*\delta W_\pm(t^*)\right]Y_\pm(t_0). \tag{3}$$

Since $\int_{t_1}^{t_1+2\pi/\omega_s}dt^*\delta W_\pm(t^*)=0$ and $W_\pm(t+2\pi/\omega_s)=W_\pm(t)$, $G_\pm(\tau)=G_\pm(\tau+2\pi/\omega_s)$.

To calculate $G_\pm(\tau)$ one must determine $Y_\pm(t_0)$. At long times the probability of arrival relative to the phase of the drive tends to a fixed distribution, so $Y_\pm$ satisfies

$$Y_\pm(t)=\int_{-\infty}^{t} dt_1 P_\mp(t|t_1)Y_\mp(t_1). \tag{4}$$

One can solve the self-consistency Eqs. (4) numerically and then use Eq. (3) to obtain $V_\pm(\tau)$ [see Fig. 2(b)]. Analytically, we have calculated $G_\pm(\tau)=G_{\pm 0}+\delta G_\pm(\tau)$ perturbatively in $\delta W_\pm(t)=w_\pm\cos\omega_s t$ [11]. If $w_\pm=0$, then the normalization of $V_\pm(\tau)$ implies that $G_{\pm 0}(\tau)=W_\pm$. The leading correction $\delta G_\pm(\tau)$ is at the drive frequency ω_s. Two interesting limiting cases are $\omega_s/2\pi\gg W_+, W_-$, where to lowest order

$$\frac{\delta G_\pm(\tau)}{G_{\pm 0}}=\frac{1}{2}\frac{w_-}{W_{-0}}\frac{w_+}{W_{+0}}\cos\omega_s\tau, \tag{5a}$$

and $W_-\ll\omega_s/2\pi\ll W_+$, $\delta W_+\to 0$, where $G_+(\tau)=W_+$, and

$$\frac{\delta G_-(\tau)}{G_{-0}}=\frac{1}{2}\frac{w_-^2}{W_{-0}^2}\cos\omega_s\tau. \tag{5b}$$

In the slow modulation limit $\omega_s/2\pi\ll W_\pm$ the exponential falloff in (3) dominates and no interesting structure in $V(\tau)$ is observed.

The classical, symmetric well has $w_+=-w_-$; Eq. (5a) implies that $G(\tau)$ is a maximum and $V(\tau)$ displays peaks at values $\tau=(2n+1)\pi/\omega_s$ when $\omega_s/2\pi\gg W_\pm$ [6,7,14]. Figure 2(b) shows $V_\pm(\tau)$ for the quantum case evaluated at the peak of the SNR curve for T driving shown in Fig. 2(a) for two frequencies. The fast modulation limit [Eq. (5a)] yields very small harmonic content because w_+ is very small. However, in the limit $W_-\ll\omega_s/2\pi\ll W_+$ [Eq. (5b)] $G_-(\tau)$ is a maximum and $V_-(\tau)$ displays peaks at values $\tau=2n\pi/\omega_s$ [15]. We have also calculated $V_\pm(\tau)$ analytically for $\varepsilon/k_BT\to\infty$ [11]; the structure of "odd" harmonics for the classical case and "even" harmonics for the quantum case occurs in this limit also. Note that $V_\pm(\tau)$ depends explicitly on the temperature dependence of the rates, so that it differs fundamentally from the SNR. In the fast modulation limit $\omega_s/2\pi\gg W_\pm$ the quantum system dislays a maximum in the SNR but very little structure in $V_\pm(\tau)$. Therefore, these two quantities probe different aspects of the dynamics.

Dissipative tunneling of a two-state system can be described using a model Hamiltonian which couples it to an ensemble of harmonic oscillators [10]:

$$H=\tfrac{1}{2}\varepsilon\sigma_z-\tfrac{1}{2}\hbar\Delta\sigma_x+\sigma_z\sum_\eta V_\eta(b_\eta^\dagger+b_\eta)+\hbar\sum_\eta\omega_\eta, \tag{6}$$

where ε is the asymmetry energy, Δ is the tunneling matrix element, the σ_i are Pauli matrices, and $b_\eta^\dagger$ is a harmonic oscillator creation operator with frequency ω_η. All necessary information about the effects of the environment is contained in the spectral density $J(\omega)=(\pi/2)\times\sum_\eta V_\eta^2\delta(\omega-\omega_\eta)$. Tunneling in metals [16] is described by Ohmic dissipation, $J(\omega)=\alpha(2\pi\hbar\omega)$ for $\omega\ll\Omega_c$, where Ω_c is a cutoff frequency large compared to Δ [17].

A parameter determining the dynamics is the renormalized tunneling matrix element Δ_r, related to Δ by $\Delta_r=\Delta(\Delta/\Omega_c)^{\alpha/(1-\alpha)}$, where Ω_c is the bath cutoff frequency [18]. If either ε or αT is much greater than $\hbar\Delta_r$, then the rapid fluctuations of the bath act to dephase the tunneling particle so that the probability of a transition between $x_\pm$ and $x_\mp$ is independent of the system's previous history. The two transition rates obey detailed balance, and the fast rate has the form [19]

$$W_+=\frac{\Delta_r^{2-2\alpha}}{2}\left(\frac{2\pi k_BT}{\hbar}\right)^{2\alpha-1}\frac{e^{\varepsilon/2k_BT}}{\Gamma(2\alpha)}\left|\Gamma\left(\alpha+\frac{i\varepsilon}{2\pi k_BT}\right)\right|^2, \tag{7}$$

where Γ is the (complex) gamma function. Figure 1 shows $W_\pm(T)$ below 1 K for $\alpha=0.25$, $\varepsilon=0.4$ K.

Recent experiments on submicron Bi wires have measured transition rates of two-state systems coupled to conduction electrons that below 1 K are well described by Eq. (7) with values $\alpha\sim 0.2$–0.25 and $\Delta_r\sim 1$–5×10^{-7} [20,21]. The asymmetry energy ε in these experiments depends on magnetic field [22]; changes in ε of 0.05 K have been induced by changes in the magnetic field as small as 0.01 T [21]. Therefore, modulation of the asymmetry energy as well as the temperature is possible. Recently, coherent signals in response to both temperature modulation and to electric field modulation of the asymmetry have been observed in this system [23].

The SNR has dimensions of frequency; to resolve the variation of the SNR as the temperature is varied, the measurement time l must be long enough that SNR $\gg 2\pi/l$. The maximum SNR attainable increases as the lowest experimentally available temperature $T_{\min}$ decreases. For reasonable experimental parameters [21], $T_{\min}\sim 0.1$ K, $\varepsilon\sim 0.4$ K, $\delta\varepsilon\sim 0.05$ K, and $W_+\sim 10$ Hz, a maximum SNR of 10 (at 0.2 K) requires $l\sim 1100$ s. For temperature driving with $\delta T\sim 0.05$ K and $\varepsilon\sim 0.7$ K, resolving a SNR of 10 takes 360 s. These times are reasonable since typical measurement intervals are 200–2000 s. For the parameters of Fig. 2(b) with $W_+\sim 10$ Hz, the structure in $V_-(\tau)$ is well resolved in $\lesssim 1000$ s.

Another candidate system for the observation of quantum SR is the tunneling of flux in a SQUID, which is also described by Eq. (6) [24]. The asymmetry of the potential well depends on the externally applied dc magnetic flux in the loop. A peak in the SNR as a function of temperature for ε driving occurs only if $\alpha < 3/2$; otherwise the SNR monotonically increases as T increases. For temperature modulation, a peak in the SNR occurs for $\alpha < 5/2$. Published experimental data [24] have $\alpha = 1.44$, so that a "resonance" in the SNR should be observable.

Although we have considered the particular transition rates that describe the dissipative tunneling of a particle coupled to an Ohmic bath, the results (2)–(5) remain completely general (within the confines of the adiabatic limit). Even if a theoretical expression such as Eq. (7) for the transition rates is not known, experimental data provide all the necessary input to predict the SNR and $V_\pm(\tau)$ with no adjustable parameters. Deviations from the theory may probe departures from Markovian dynamics.

In summary, we have extended the concept of stochastic resonance to quantum systems and shown that modulating the tunneling of a defect in a mesoscopic metal should provide an experimental realization of quantum SR. Previous experimental realizations of SR are limited to the classical case [1,2,7,8], so the dissipative dynamics of a two-state defect provide a fundamentally different embodiment of SR.

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