Ziff Replies: I welcome the comments of Aharony and Hovi (AH) [1] concerning my Letter [2] on the behavior of the spanning function in site percolation. The following are my replies to the four main points they raise.

(1) AH are correct that my conclusions about the applicability of the renormalization group (RG) method to percolation refer more precisely to the position-space or Monte Carlo RG method [3] rather than the RG method in general. In the position-space RG method, a sequence of larger and larger cells is used to systematically improve the RG estimate p^* of the critical point, p_c. Although one might expect that $R_L(p_c)$ should approach p_c as $L\rightarrow\infty$ (so that the infinite cell exactly satisfies renormalization), there is indeed no requirement from RG theory that this must be the case. As a consequence of this not occurring for site percolation on a square lattice, I showed that the RG estimate p^* converges to p_c more slowly than many other estimates for p_c (such as the mean value of p at spanning, $\bar{p}$). It also follows that these other estimates, which are frequently used in percolation studies, do not actually represent the RG fixed point.

(2) AH are also correct that my statement of universality as given in [2] is not worded precisely in that a scale factor is omitted from the variable x. (Clearly, the argument of f must be dimensionless, which x is not.) This omission, however, does not affect any of the conclusions in [2].

In more recent work [4], I propose choosing this scale factor such that $x\sim(L/\xi)^{1/\nu}$ for large L, because the ratio of L to the correlation length ξ is a universal measure of the criticality of the system. In that work, I also analyze the moments ratios μ_4/μ_2^2 and μ_6/μ_2^3 for both site and bond percolation, where $\mu_n\equiv\int(p-p_c)^n R_L'(p)dp$, and find that these two quantities approach the limiting values of 3.15 ± 0.05 and 18.5 ± 0.5, respectively, as L gets large, supporting universality in a form similar to that suggested by AH. These results imply $f_1(x)$ is not quite a Gaussian (for which the moment ratios are 3 and 15, respectively) as has been frequently assumed (e.g., [5,6]).

(3) AH propose that the leading correction term to $\mathcal{R}_L(p)$ should be the usual correction-to-scaling term $L^{-\vartheta_1}$ with $\vartheta_1\approx0.85$ rather than the L^{-1} behavior that I found. A reexamination of my data shows that the $L^{-\vartheta_1}$ term alone is not sufficient to fit the data, but by taking both an L^{-1} term and an $L^{-\vartheta_1}$ term, the data can be fit well. However, over this large range in L, the L^{-1} term is dominant, and the fit using both terms is only marginally improved over that using the L^{-1} correction alone.

(4) AH predict that $f_2(x)$ is an even function of x by virtue of their finding from RG theory that the correction-to-scaling functions are universal along with $f_1(x)$. The systems for which I have exact results $(L\leq7)$ are not sufficiently large to test this prediction, so further Monte Carlo simulations are now being carried out to study this question. The results of these simulations will be reported elsewhere.

I acknowledge support from NSF DMR-9122341, and stimulating correspondence with A. Aharony and J.-P. Hovi.

Robert M. Ziff
Departments of Physics and Chemical Engineering
University of Michigan
Ann Arbor, Michigan 48109-2136

Received 5 August 1993;
revised manuscript received 14 February 1994
PACS numbers: 64.60.Ak, 05.70.Jk

[1] A. Aharony and J-P. Hovi, preceding Comment, Phys. Rev. Lett. **72**, 1941 (1994).
[2] R. M. Ziff, Phys. Rev. Lett. **69**, 2670 (1992).
[3] P. J. Reynolds, H. E. Stanley, and W. Klein, J. Phys. A **11**, L199 (1978); Phys. Rev. B **21**, 1223 (1980).
[4] R. M. Ziff (to be published).
[5] M. E. Levinshtein, B. I. Shklovskii, M. S. Sur, and A. L. Efros, Sov. Phys. JETP **42**, 197 (1976).
[6] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor and Francis, London, 1992), pp. 73 and 84.