Measurements of the Effective Mass and Scattering Times of Composite Fermions from Magnetotransport Analysis

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An experimental analysis of the fractional quantum Hall effect is described in which oscillations in ρ_{xx} are treated as Shubnikov–de Haas oscillations of the composite fermions (CF) formed by a flux attachment transformation. We find a CF effective mass of $0.5m_e$ at zero effective field which increases weakly with effective magnetic field at a rate independent of carrier density. The CF quantum scattering times are the same as for the underlying single particle electron states.

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The fractional quantum Hall effect (FQHE) [1–4] may be observed in high mobility two dimensional electron gasses as a rich spectrum of resistivity minima in ρ_{xx} at filling factors $\nu = n_e\Phi_0/B = p/q$, for integral p, q where q is odd, accompanied by plateaux in ρ_{xy} quantized at values of $(q/p)h/e^2$. The flux quantum $\Phi_0 = h/e$ and n_e is the sheet electron density. Traditionally the FQHE has been explained in terms of strong electron-electron interactions and a hierarchical model in which daughter states evolve from the principal ($p = 1$) correlated ground states (e.g., 1/3) [5] by adding fractionally charged quasiparticles. Although the hierarchical model correctly predicts the positions of the fractions observed, it fails to account for the order and strength in which they appear in the experiments. The theory also requires the higher order fractions to be generated from very large densities of quasiparticles of the lower order states.

In an alternative approach introduced recently, Jain has used the idea of a "composite fermion" (CF) [6], in which the electrons interact with a Chern-Simons gauge field, equivalent to attaching an even number ($2m$) of flux quanta to each electron. Halperin, Lee, and Reed (HLR) [7] have shown that the average of the fictitious magnetic field exactly balances the external field at $\nu = \frac{1}{2}m$. Thus the system of interacting electrons in high magnetic field may be replaced by one of independent composite Fermions in zero magnetic field acting as a metallic state with a well defined Fermi energy, an idea consistent with the observations of pronounced features around occupancies of 1/2 and 1/4 in surface acoustic wave measurements [8]. In this approach the FQHE now appears as simply the Shubnikov–de Haas (SdH) and quantum Hall effect of the composite fermions. As the field moves away from $B_{1/2m}$ $(= 2m\Phi_0 n_e)$, by an amount $\Delta B = B - B_{1/2m}$, there is Landau quantization of the CF energy levels and energy gaps open up. The CF filling factor ν^* is then simply related to ν for electrons by

$$\nu = \frac{\nu^*}{2m\nu^* + 1}. \quad (1)$$

For $m = 1$, the series of positive (negative) integer ν^* generates the dominant series of oscillations $\nu = 1/3, 2/5, 3/7, \ldots (1, 2/3, 3/5, \ldots)$, both of which converge to the low field limit of $\nu = 1/2$. Similarly with $m = 2$, attaching 4 flux quanta to each electron, gives the series $1/5, 2/9, 3/13, \ldots (1/3, 2/7, 3/11, \ldots)$ converging on $\nu = 1/4$. These are precisely the families of fractions at which the strongest minima are observed in Fig. 1.

Inspection of the data for both $\Delta B > 0$ and $\Delta B < 0$ shows the oscillations have the form of an exponentially

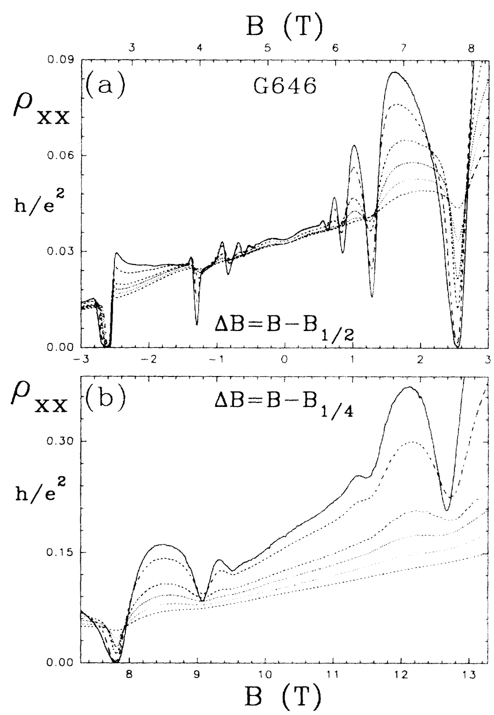

FIG. 1. ρ_{xx} plotted as a function of ΔB, at temperatures of 320, 409, 570, 740, 870, and 1100 mK, showing how the FQHE may be regarded as SdH oscillations of composite fermions in the region of (a) $\nu = 1/2$ and (b) $\nu = 1/4$.

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damped harmonic series typical of Shubnikov–de Haas oscillations. In this paper we provide strong support for the CF picture by showing quantitatively that the FQHE structure in ρ_{xx} corresponds very well to the form expected for SdH oscillations. We present an analysis of the oscillatory structure which allows us to measure such important parameters as the effective mass of the renormalized composite fermions M^* and their scattering times. The advantage of this method is that the results are independent of the presence of disorder and allow us to examine such factors as the field or carrier density dependence of the mass. We find the somewhat surprising result that the effective mass scales only with ΔB, and not the external field. A previous analysis of the activated conductivity minima by Du *et al.* [9] has suggested that the mass may be independent of field, a result contradicted by our study. We also find that the CF quantum lifetime $\mathcal{T}_q$ is identical to that of electrons at low field, which suggests the same scattering mechanisms dominate in the two cases.

The measurements were performed on 500 μm wide Hall bars of three high quality GaAs-GaAlAs heterojunctions grown by molecular beam epitaxy at Philips Research Laboratories, with undoped spacer layer widths of 1200, 1600, and 2400 Å for samples G640, G641, and G646 [10]. After illumination by room lights via a quartz fiber the respective electron densities were 1.2×10^{15}, 0.9×10^{15}, and 0.6×10^{15} m^{-2} with mobilities of 680, 400, and 200 m^2/V s at 320 mK.

The resistivity components ρ_{xx} and ρ_{xy} were measured between 320 mK and 1.6 K using a ^{3}He cryostat in magnetic fields up to 14 T. Typical results for sample G646 appear in Fig. 1, where the oscillations in ρ_{xx} are shown as a function of the effective field ΔB for both the $\nu = 1/2$ and $\nu = 1/4$ series, using $B_{1/2}$ and $B_{1/4}$ as the respective origins for ΔB. The oscillations were somewhat better resolved in the higher density samples, but only the features around $\nu = 1/2$ could be seen in our field range. Equivalent quantum Hall plateaus are also seen.

All three samples show a temperature independent resistivity at $\nu = 1/2$, indicative of metallic behavior with a well defined Fermi surface, together with an oscillatory series periodic in $1/\Delta B$. The oscillations were analyzed using the conventional Ando formula [11]:

$$\Delta\rho_{xx} \propto \frac{X}{\sinh X} \exp\left(-\frac{\pi}{\omega_c \tau_q}\right) \cos 2\pi(\nu - 1/2), \qquad (2)$$

where $X = 2\pi^2 k_B T/\hbar\omega_c$ and $\omega_c = eB/m^*$ is the cyclotron frequency. For composite fermions we replace B by ΔB, ν by ν^*, m^* by M^*, and τ_q by $\mathcal{T}_q$. This is equivalent to the well known Dingle formula, and is the first term in a harmonic series containing peak sharpening terms that account for the increased resolution of the oscillations at low temperatures. By 320 mK the oscillations take on the same cusplike shape seen for the integer oscillations at low field. Analysis of the high temperature dependence of the amplitudes thus yields the CF effective mass. An advantage of this analysis is that it includes the level broadening, and assuming that $\mathcal{T}_q$ does not change with temperature over the region of interest (the resistance measurements show that neither τ_t [12] at $B = 0$ nor $\mathcal{T}_t$ at $B_{1/2}$ change) we can use the temperature dependence to obtain a value of M^* for each oscillation. In addition the exponential damping term allows us to use the field dependence of the oscillations to find $\mathcal{T}_q$.

Equation (2) was found to give a very good description of the temperature and magnetic field dependence of the oscillation amplitudes, in the range of conditions corresponding to $\Delta\rho/\rho < 0.5$. All the oscillations at small ΔB are in this range where they take a predominately cosine form, but the analysis was restricted to the high temperature limit for peaks at larger ΔB (e.g., 1/3). A similar analysis of the single particle SdH at low fields with this condition gives $m^* = 0.070 m_e$.

The single exception is in the region around $\nu = 2/3$, where a very sharp minimum was observed at low temperatures. This is related to two factors. First, the normal assumption of the CF theory is that the original electron system is spin polarized, allowing the new particles also to be considered as a system of spinless fermions. However, it is well known both theoretically [4,13] and experimentally [14] that the 2/3 state may become unpolarized with equal populations of each spin state when it occurs at low fields. Second, a further feature is seen in the region around $\nu = 3/4$, corresponding to $\nu^* = 3/2$, with additional minima at $\nu^* = 4/3$ and 5/3 ($\nu = 4/5$ and 5/7). This is evidence for the appearance of a new CF generation arising from the higher order renormalization that must occur when ν^* is fractional, and may be regarded as the attachment of 2 more flux quanta to those $m = 1$ CFs in the partially occupied level. The data from the region between $\nu = 1$ and 2/3 was therefore treated as an anomaly and not included in our analysis, although the low temperature dependence of the 2/3 minimum gave a mass value in agreement with those found at other fields.

Figure 2 shows the temperature dependence of the amplitude of the oscillations in ρ_{xx} for sample G646 for both the maxima and minima between $\nu = 1/2$ and 1/3 (i.e., integral and half integral ν^* from 1 to 4), together with fits to Eq. (2). This shows excellent agreement with the predicted systematic behavior. Values of the $m = 1$ CF effective mass deduced from these fits for each sample are shown in Fig. 3, plotted as a function of the effective field ΔB for both positive and negative values of ν^*.

The remarkable result of the analysis is that the composite fermions display a consistent value of M^*, dependent only on the single parameter ΔB, and independent of both the sign of this field and n_e, within the limits of experimental error. While the error associated with an individual data point is up to $\pm 0.05 m_e$ the overall trend from the three samples is very clear. The field depen-

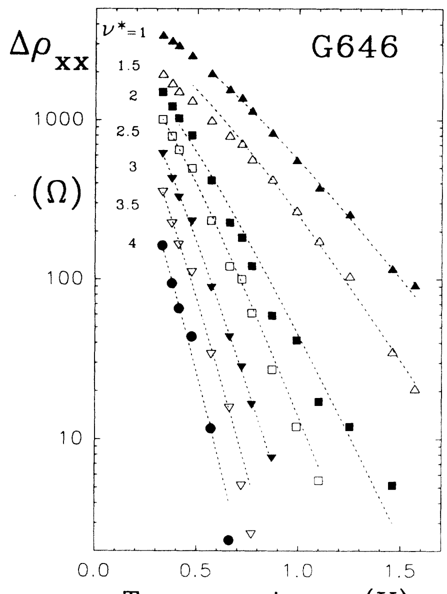

FIG. 2. Temperature dependence of the resistivity oscillations for sample G646 between $\nu = 1/2$ and $1/3$. Data are fitted by Eq. (2) for each minimum (filled symbol) and maximum (open). Values of ν^* are indicated in the figure.

dence of the mass only becomes significant for features at high effective fields, where it can be described by

$$M^* = 0.51 + 0.074\Delta B \qquad (3)$$

in units of m_e with ΔB in tesla. This is also consistent with the measurements of Du *et al.* if values are deduced only from their higher field activation energies, where the Landau level broadening is a relatively small perturbation on the measured energy gaps. Using their activation gaps for 1/3 as an upper limit on M^* gives values of 1.21 and 0.80 at $\Delta B = 9.5$ and 4.62 T while Eq. (3) predicts $M^* = 1.21$ and 0.85. We therefore have strong evidence that the proposition by Du *et al.* of a constant effective mass, determined only by carrier density, is not correct. A filling factor ν^* dependence of the mass was predicted by HLR, however, our absolute values are considerably higher than those estimated by HLR as $\sim 0.27m_e$ for $\nu = 1/2$. This may be due to the finite thickness of the 2D layer, which reduces the strength of electron-electron interaction effects [15].

Using the mass values we can deduce an effective cyclotron gap E^*_{CR} from $e\Delta B/M^*$ (Fig. 4). This is equivalent to what was previously described as the FQHE quasiparticle gap, and studied by activation measurements, but now includes data from a large number of fractions simultaneously. Values deduced from oscillation maxima give similar results to data from minima, confirming that the CF picture provides a consistent description for all filling factors. By contrast only FQHE minima are well described in the hierarchical model.

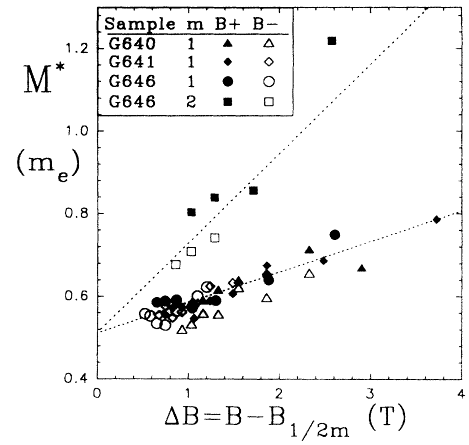

FIG. 3. Composite fermion effective mass M^* deduced by fitting maxima and minima of ρ_{xx} by Eq. (2).

A similar series of oscillations for the $m = 2$ CF generation around $\nu = 1/4$ is shown in Fig. 1(b). In this case there is considerably more uncertainty in our fits due to the strong underlying increase in resistivity in the region $\nu < 1/3$. We can remove the effect of the background by normalizing the oscillation amplitudes as $\Delta\rho_{xx}/\rho_{xx}$, as originally derived in the Ando formula. The underlying temperature dependence of ρ_{xx} suggests, however, that there may be some temperature dependence of $\mathcal{T}_q$, so the values deduced for M^* will be more uncertain. An analysis based only on the temperature dependence of $\Delta\rho_{xx}$ gives M^* values $0.2m_e$ higher. (For $m = 1$ CFs there is no temperature variation of the background resistivity and so the same results are obtained by either method.)

The feature at $\nu = 1/3$ could be considered as $\nu^* = 1$ for the $m = 1$ series or $\nu^* = -1$ for $m = 2$, but the mass values we deduce shows that it belongs to the $m = 1$ generation. Similarly, the feature at $\nu = 1$ ($\nu^* = -1$ for

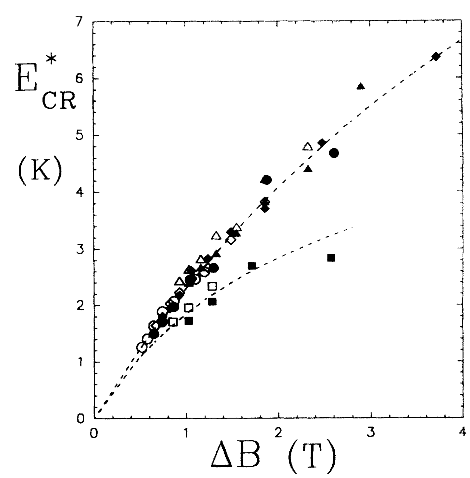

FIG. 4. Effective energy gap E^*_{CR} of the Landau quantized composite fermion levels. Lines use M^* from fits to Fig. 3.

$m = 1$ CFs) is properly described by the single particle ($m = 0$) picture. We conjecture that whenever a state may be formed in more than one way it will reveal the characteristics of the lowest generation of fermions.

From the masses shown in Fig. 3 we see a much larger dependence of M^* on ΔB for the $m = 2$ generation. Indeed for the 1/5 minimum ($\nu^* = 1$) the value of $M^* = 1.2m_e$ is exceptionally large. In this region ρ_{xx} is also increasing rapidly which is often taken as a sign of Wigner crystallization [16]. Although there is more uncertainty in the interpretation of these data, it appears that the mass is diverging with the onset of Wigner crystallization, consistent with the idea of an infinite mass when the particles are locked into a lattice. The mass deduced for the maximum between $\nu = 1/5$ and 2/9 is in line with the other values, so our data do not indicate a re-entrant solid phase in this region at the temperature of our experiments. The data are also plotted in terms of E^*_{CR} in Fig. 4 where the energy gap is seen to saturate more quickly with ΔB than for the $m = 1$ data.

Equation (2) has also been used to deduce values for the quantum lifetime of the composite fermions, and for the electrons from the low field SdH oscillations. Table I shows that in each sample $\mathcal{T}_q$ is the same for both directions of ΔB, which again supports the notion of new particles in an effective field and shows that the same states may be considered for $\pm\Delta B$. $\mathcal{T}_q$ for the $m = 1$ CFs is also identical to τ_q for the ($m = 0$) electrons to within the experimental accuracy. This provides strong evidence to support the conclusions of Du *et al.*, that in high fields where short range scattering is dominant, there is no additional scattering of the composite fermions. However, the transport lifetimes τ_t and $\mathcal{T}_t$ are much smaller for CFs than for electrons at $B = 0$. This suggests the momentum transferred in each CF scattering event is greater.

In conclusion, we have shown that the FQHE can be analyzed accurately in terms of the Shubnikov–de Haas oscillations of a series of renormalized composite fermion states. Their effective masses are weakly dependent on the single parameter of effective field ΔB. The quantum lifetime of the composite particles is found to be the same as the underlying single particle states.

One of Jain's original reasons for considering composite fermions was that the FQHE plateaus structure *looked* just like the IQHE structure. We have shown that not only is this true of ρ_{xy} but also for ρ_{xx}. The conductivity can now be explained in terms of the density of states and scattering properties of composite fermions without having to invoke fractional charge and fractional statistics to describe FQHE quasiparticles.

TABLE I. Transport and quantum lifetimes (in ps) for composite fermions at $\nu = 1/2$ ($m = 1$) and $1/4$ ($m = 2$) and for electrons at $B = 0$. $\mathcal{T}_t$ at $\nu = 1/4$ is for 320 mK and increases with temperature, the rest are constant.

Sample	$B, m = 0$		$m = 1$		$m = 2$	
	τ_t	τ_q	$\mathcal{T}_t$	$\mathcal{T}_q(-,+\Delta B)$	$\mathcal{T}_t$	$\mathcal{T}_q(-,+\Delta B)$
G640	265	2.1	35	2.2, 2.3	...	...
G641	156	3.5	36	3.7, 3.2	15.4	1.5, ...
G646	80	8.3	33	8.2, 8.2	6.2	3.9, 3.6

[1] D.C. Tsui, H.L. Stormer, and A.C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[2] R.L. Willett *et al.*, Phys. Rev. Lett. **59**, 1776 (1987).
[3] J.R. Mallett *et al.*, Phys. Rev. B **38**, 2200 (1988).
[4] T. Chakraborty and P. Pietilainen, *The Fractional Quantum Hall Effect* (Springer-Verlag, New York, 1988).
[5] R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[6] J.K. Jain, Adv. Phys. **41**, 105 (1992).
[7] B.I. Halperin, P.A. Lee, and N. Reed, Phys. Rev. B **47**, 7312 (1993).
[8] R.L. Willett *et al.*, Phys. Rev. Lett. **47**, 7344 (1993).
[9] R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **70**, 2944 (1993).
[10] C.T. Foxon *et al.*, Semicond. Sci. Technol. **4**, 582 (1989).
[11] T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).
[12] The quantum lifetime τ_q is the dephasing time that includes all scattering events, as opposed to the transport lifetime τ_t which also accounts for the angle of scattering.
[13] B.I. Halperin, Helv. Phys. Acta **56**, 75 (1983); X.G. Wu, G. Dev, and J.K. Jain, Phys. Rev. Lett. **71**, 153 (1993).
[14] R.G. Clark *et al.*, Phys. Rev. Lett. **62**, 1536 (1989); J.P. Eisenstein *et al.*, Phys. Rev. Lett. **62**, 1540 (1989).
[15] F.C. Zhang and S. Das Sarma, Phys. Rev. B **33**, 2903 (1986).
[16] H.W. Jiang *et al.*, Phys. Rev. Lett. **65**, 633 (1990); V.J. Goldman *et al.*, Phys. Rev. Lett. **67**, 1630 (1990).