Novel Symmetry of a Random Matrix Ensemble: Partially Broken Spin Rotation Invariance

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(Received 13 October 1993)

A novel type of a random matrix ensemble originating from a partial breaking of the spin rotation invariance is considered. Universal conductance fluctuations, density of states correlation function, and the persistent current in mesoscopic rings are studied for this type of symmetry of a disordered system.

PACS numbers: 73.20.Dx, 71.70.Ej, 72.10.–d

In 1962 Dyson [1] proposed a classification of ensembles of random matrices according to a symmetry of a physical system. Originally introduced to describe a spectrum of a complex nucleus it can be applied to disordered electronic systems as well. This field attracted considerable interest from physicists due to the development of the scaling theory of localization [2–4] and, later on, in connection with the investigation of mesoscopic systems [5–9].

The key point of this classification is the invariance (or noninvariance) of the Hamiltonian with respect to the time reversal (T) and spin rotation [SU(2)] symmetry. Correspondingly, three types of ensembles are known.

(1) Neither T nor SU(2) is broken. This corresponds to an electron subjected to a random potential in the absence of external magnetic field, magnetic impurities, and spin-orbit interaction.

(2) SU(2) symmetry is broken, T is preserved. This situation takes place in a system with the spin-orbit coupling in the absence of magnetic field.

(3) Time reversal symmetry is broken that may result from the external magnetic field or magnetic impurities.

These three ensembles are called orthogonal, symplectic, and unitary, respectively.

In the present Letter we introduce one more possible symmetry of a random matrix ensemble, which implies the specific properties of the corresponding physical systems. Our finding is that while the time reversal symmetry is not broken, SU(2) symmetry may be broken not completely, but only partially, down to U(1). This situation takes place when the random Hamiltonian contains only one (for instance, σ_z) of the Pauli matrices. Physical realization of this symmetry is related to a type of the spin-orbit coupling which arises most naturally in low dimensional systems.

If the electron momentum is quantized in the z direction, perpendicular to a plane of the 2D electron gas, then the spin-orbit skew scattering is described by the matrix element [10]

$$V_{\mathbf{p},\mathbf{p}'} = \alpha\sigma_z\left[\mathbf{p}\times\mathbf{p}'\right]_z\,, \tag{1}$$

with α being the spin-orbit constant, $\mathbf{p}$ and $\mathbf{p}'$ the momenta before and after the scattering, respectively. If the material possesses the inversion center and the electrons are confined to two dimensions by a symmetric potential, then (1) is the only allowed spin-dependent interaction in a system. As a consequence the z projection of the electron spin is conserved, despite of the presence of spin-orbit scattering. It is the first example of partially broken spin rotation invariance. Another one is the quasi 1D system made of a material without the inversion center (e.g., GaAs). In this case the size quantization in two directions takes place [11,12] and the electron Hamiltonian contains a spin-orbit coupling term of the form

$$\mathcal{H}_{\mathrm{so}} = \beta\sigma_z p_z, \tag{2}$$

where β is a constant of the spin-orbit splitting of the conduction band. This term has no consequences in the case of a wire, when it can be removed by a gauge transformation, but is important for a ring.

Let us compare the symmetry properties of a Hamiltonian containing a term of the type (1), (2) with those of the known matrix ensembles. Because of the symmetry restrictions the [2 × 2] blocks (in the spin space) of the full Hamiltonian matrix $\mathcal{H}$ in the case (1), (2) take the form

$$\mathcal{H}_{r,r'} = \begin{pmatrix} z & 0 \\ 0 & z^* \end{pmatrix}, \tag{3}$$

where r and r' are the spatial coordinates, z is a complex number, and z^* denotes the complex conjugate. Table I presents the form of the spin blocks for all types of symmetry of a random matrix ensemble. The first row in the table corresponds to the Dyson orthogonal ensemble, the third to the symplectic, and the fourth and the sixth to the unitary one. The second row describes the newly introduced ensemble with a partially broken spin rotation invariance. The same spin symmetry but in the absence of the time reversal invariance gives rise to the ensemble presented in the fifth row. This situation may occur when the magnetic field is present in addition to the spin-orbit interaction of the type (1), (2).

Now we are going to study several properties of the new ensemble. We start from the consideration of the universal conductance fluctuations variance var(g), which we

0031-9007/94/72(12)/1894(4)$06.00

TABLE I. The form of spin blocks of a random Hamiltonian and the relative value of the conductance variance for different types of symmetry of ensemble. In the second and third columns the sign "+" means that the corresponding symmetry is preserved, the sign "−" denotes the complete symmetry breaking, and U(1) stands for the partially broken spin rotation invariance.

Ensemble	T	SU(2)	Spin blocks	Relative var(g)
1	+	+	$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$	1
2	+	U(1)	$\begin{pmatrix} z & 0 \\ 0 & z^* \end{pmatrix}$	$\frac{1}{2}$
3	+	−	$\begin{pmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{pmatrix}$	$\frac{1}{4}$
4	−	+	$\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}$	$\frac{1}{2}$
5	−	U(1)	$\begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix}$	$\frac{1}{4}$
6	−	−	$\begin{pmatrix} z_1 & z_2 \\ z_3 & z_4 \end{pmatrix}$	$\frac{1}{8}$

calculate for the two-dimensional case (1). The leading contribution to the conductance fluctuations at $l/L \ll 1$, $k_F l \gg 1$ (l being the mean free path, L the system size, k_F the Fermi momentum) is given by the sum of diagrams which are generated systematically by starting with two concentric loops connected by impurity ladders and inserting the two current vertices in all possible ways [9]. Every diagram contains either particle-hole ladders (diffusons), or particle-particle ladders (cooperons). The contribution of each diagram to the variance is given in general by the sum originating from four independent channels (zero total spin channel, and three channels characterized by the total spin equal to unity with different values of the spin projection). The total spin $\mathbf{S}^{(D,C)}$ is a sum of the electron spins $\frac{1}{2}\boldsymbol{\sigma}_1$ and $\frac{1}{2}\boldsymbol{\sigma}_2$ for the Cooperon and is a difference of the spins for the diffuson: $\mathbf{S}^{(D,C)} = (\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2)/2$. The total number of channels for the diffuson and Cooperon diagrams is therefore equal to 8.

In case (1) the equation for the Cooperon $P^{(C)}$ (diffuson $P^{(D)}$) propagator in the momentum representation has the form

$$\left[Dq^2 + \frac{S_z^{(C,D)2}}{\tau_{so}} \right] P^{(C,D)}(\mathbf{q}) = 1. \tag{4}$$

Here D is the diffusion constant, $\tau_{so}^{-1} = \pi\nu\alpha^2 \langle [\mathbf{p} \times \mathbf{p}']_z^2 \rangle$ is the spin-orbit dephasing time (the angular brackets denote the averaging over the Fermi surface, ν is the single electron density of states), the momentum $\mathbf{q} = \mathbf{p}_1 \pm \mathbf{p}_2$, $\mathbf{p}_{1,2}$ being the particles momenta, the upper (lower) sign refers to the Cooperon (diffuson) propagator. The eigenvalues of Eq. (4) can be written as

$$P_{0,0}(\mathbf{q}) = P_{1,0}(\mathbf{q}) = \frac{1}{Dq^2}, \tag{5}$$

$$P_{1,1}(\mathbf{q}) = P_{1,-1}(\mathbf{q}) = \frac{1}{Dq^2 + \tau_{so}^{-1}}, \tag{6}$$

where the first index of P corresponds to the value of the total spin, the second to the value of its projection on the axis z. We see that if $1/\tau_{so} \gg 1/\tau_L$, $\tau_L = L^2/D$, the two (of four) channels of propagation are suppressed for both the Cooperon and the diffuson and the variance is reduced by a factor of 2 in comparison with the orthogonal symmetry case. Therefore, var(g) is 2 times larger than for the symplectic ensemble corresponding to the usual spin-orbit scattering case.

In the presence of magnetic field $H \gg H_c = \hbar c/4eD\tau_L$ perpendicular to the 2D plane all the Cooperon channels are suppressed and the conductance fluctuations are reduced by an additional factor of 2. This situation corresponds to the random matrix ensemble presented in the fifth row of Table I. Let us notice that the same symmetry occurs in the case of a very strong magnetic field $H \gg H_s = \hbar/\tau_L g\mu$ (g being the electron g factor, μ the Bohr magneton), applied to a system without spin-orbit interaction [7]. For such fields not only orbital magnetic field effect but also Zeeman splitting should be taken into account that breaks the spin rotation symmetry down to U(1). However, in our case this kind of spin symmetry breaking holds already in the absence of magnetic field due to the spin-orbit coupling. Consequently, the "spin" magnetic field $H \gg H_s$ perpendicular to the 2D plane does not change the symmetry of the system as compared to the orbital one $H \gg H_c$. Meanwhile, in-plane spin magnetic field lowers the symmetry of our ensemble down to a symplectic one (third row of Table I) in view of the absence of the orbital effect. The relative value for var(g) for all the symmetry types is given in the last column of Table I. Let us note that this value is inversely proportional to a number of independent real variables parametrizing the 2×2 spin blocks of the Hamiltonian.

Another interesting property of the new ensemble is the absence of the one loop weak localization correction to the conductivity [10] since the value

$$C = -\frac{1}{2}P_{0,0} + \frac{1}{2}P_{1,0} + \frac{1}{2}P_{1,1} + \frac{1}{2}P_{1,-1} \tag{7}$$

determining the quantum correction is not divergent in the limit $q \to 0$.

Consider now the implications of the new symmetry in the case of a 1D ring with the spin-orbit coupling term (2) in the Hamiltonian. The physical quantity which is of interest in such a system is the persistent current [13–15] induced by the Aharonov-Bohm magnetic flux Φ. The averaged current is given by [16]

$$I(\Phi) = -\frac{\Delta}{2}\frac{\partial}{\partial\Phi}\langle(\delta N)^2\rangle_{\mu=\langle\mu\rangle}, \tag{8}$$

where Δ is the mean level spacing, μ is the chemical potential, and angular brackets stand for the disorder average. $\langle(\delta N)^2\rangle$ is the number of particle variance in the grand canonical ensemble related to the level correlation function $K(\epsilon_1,\epsilon_2) = \langle\rho(\epsilon_1)\rho(\epsilon_2)\rangle - \langle\rho(\epsilon_1)\rangle\langle\rho(\epsilon_2)\rangle$ [$\rho(\epsilon)$ being the level density] by

$$\langle(\delta N)^2\rangle = \int_0^\mu \int_0^\mu d\epsilon_1 d\epsilon_2 K(\epsilon_1,\epsilon_2). \quad (9)$$

To the leading order of the perturbation theory

$$K(\epsilon_1,\epsilon_2) = \frac{1}{2\pi^2}\mathrm{Re}\sum_{S,S_z,\mathbf{q}} [P^{(D)2}_{S,S_z}(\mathbf{q},\epsilon_1-\epsilon_2) + P^{(C)2}_{S,S_z}(\mathbf{q},\epsilon_1-\epsilon_2)]. \quad (10)$$

Equation (10) is the result of the summation of the usual diffuson and Cooperon diagrams [7]. The Cooperon (diffuson) propagator in case (2) is influenced by the effective spin vector potential $A_z^{(C,D)} = 2\beta m S_z^{(C,D)}$ [11,12]:

$$[D(\mathbf{q}+\mathbf{A}^{(C,D)})^2 + i\omega]P^{(C,D)}(\mathbf{q},\omega) = 1. \quad (11)$$

Applying the periodic boundary condition for a ring of a radius r, we have $q_z = n/r$ with $n = 0, \pm1, \pm2, \ldots$ and

$$P_{0,0}(n,\omega) = P_{1,0}(n,\omega) = \frac{1}{Dn^2r^{-2} + i\omega}, \quad (12)$$

$$P_{1,\pm1}(n,\omega) = \frac{1}{Dr^{-2}(n \pm 2\beta mr)^2 + i\omega}. \quad (13)$$

Equation (11) is valid at $\Phi = 0$. For nonzero Aharonov-Bohm flux we should substitute $q_z + e\Phi/\pi rc$ for q_z in the equation for the Cooperon, while the diffuson remains unaffected. Therefore n in the Cooperon channel should be replaced by $n + 2\Phi/\Phi_0$, $\Phi_0 = hc/e$ being the flux quantum. Only the Cooperon channel contributes to the persistent current given by (7)–(9). Using (10) we obtain the following result:

$$I(\Phi) = \frac{\Delta}{\pi\Phi_0}\left[2\cot\frac{2\pi\Phi}{\Phi_0} + \sum_{s=\pm1}\cot 2\pi\left(\frac{\Phi}{\Phi_0} + \beta mrs\right)\right]. \quad (14)$$

The first term in brackets originates from $S_z = 0$, the second from $S_z = \pm1$.

In the vicinity of the points $\Phi = 0$, $\Phi = \pm\beta mr\Phi_0/2$ where (14) diverges, it loses its validity. This is related with the fact that one of the Cooperon channels becomes almost massless in these regions. The appropriate approximation in such a situation is to take into account the zero spatial mode (with $q = 0$) exactly rather than to consider all the modes perturbatively. It can be done by using the supermatrix nonlinear σ model. In the case of the absence of the spin-orbit interaction this calculation was performed in [17]. Such a consideration modifies the expression (14) in the vicinity of the singularities. As a result the current in three regions is of order of

$$I(\Phi) \sim \frac{\sqrt{\Delta E_c}}{\Phi_0} \quad (15)$$

with E_c being the Thouless energy.

Let us note that the current $I(\Phi)$ has the nonzero limit value as $\Phi \to 0$. In view of the antisymmetry relation $I(\Phi) = -I(-\Phi)$ that means an occurrence of a jump of $I(\Phi)$ at this point. The origin of this quantum jump is the level crossing at $\Phi = 0$ due to removal of the Kramers degeneracy by the Aharonov-Bohm magnetic field. This phenomenon was considered in detail in [18] (see also [19]) for the generic spin-orbit interaction. In [18] the crossover from the symplectic ensemble to the unitary one occurs with switching on the magnetic flux. Our case is characterized instead by a crossover from the second to the fifth ensemble of Table I. Consequently, the exact form of the level correlator and the persistent current $I(\Phi)$ in the crossover region should be different from those of [18], although the parametric dependence remains the same.

The density of states correlation function at $\Phi = 0$, $\Phi = \pm\beta mr\Phi_0$ and away from the transition regions can be expressed through the corresponding two-level correlation function for the Gaussian ensembles. Namely, let us consider the normalized two-level correlator $R_2(r)$,

$$R_2(r) = \Delta^2 K(\epsilon, \epsilon + r\Delta). \quad (16)$$

Then, at $\Phi = 0$ (i.e., for the ensemble 2 of Table I)

$$R_2(r) = R_2^{\mathrm{GUE}}(r/2), \quad (17)$$

where R_2^{GUE} is defined for the Gaussian unitary ensemble [1] without spin degrees of freedom. Therefore, the level statistics for the ensemble 2 is the same as for the unitary one, although the symmetry is essentially different. At a value of Φ far enough from the points $\Phi = 0, \pm\beta mr\Phi_0$ the symmetry of the system is described by the direct product of two unitary ensembles corresponding to two projections of spin (ensemble 5) and we get

$$R_2(r) = \frac{1}{2}R_2^{\mathrm{GUE}}(r/2). \quad (18)$$

Finally, at $\Phi = \pm\beta mr\Phi_0$ we obtain

$$R_2(r) = \frac{1}{4}\left[R_2^{\mathrm{GUE}}(r/2) + R_2^{\mathrm{GOE}}(r/2)\right], \quad (19)$$

where GOE means the Gaussian orthogonal ensemble. In these points the subsystems with different signs of the spin projection belong to the different universality classes: one to the unitary class and another to the orthogonal one.

The asymptotic behavior of the level correlator $K(\epsilon_1,\epsilon_2)$ at $\epsilon_1 - \epsilon_2 \gg \Delta$ is determined by the massless diffuson and Cooperon modes. According to Eq. (9) we get

$$K(\epsilon_1,\epsilon_2) = -\frac{\gamma}{2\pi^2}\frac{1}{(\epsilon_1-\epsilon_2)^2}, \quad (20)$$

where $\gamma = 4$ for $\Phi = 0$, $\gamma = 3$ for $\Phi = \pm\beta m r \Phi_0$ and $\gamma = 2$ away from these points. It is easy to see that these expressions agree with the exact formulas (16)–(19).

In conclusion, we considered a new type of the symmetry of a disordered system which originates from the partial breakdown of the spin rotation invariance. This type of symmetry may most naturally arise in a low dimensional system with the spin-orbit interaction. The variance of the universal conductance fluctuations and the asymptotics of the level-level correlation function are 2 times larger than for the generic spin-orbit interaction. In the case of a quasi 1D ring the persistent current and the density of states correlation function are shown to have an unusual behavior with the variation of the Aharonov-Bohm flux.

We are grateful to A. G. Aronov for fruitful discussions. Yu.L.-G. acknowledges the kind hospitality extended to him at the University of Karlsruhe. A.D.M. acknowledges the financial support by Alexander von Humboldt Foundation.

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