Synchronization of Chaotic Diode Resonators by Occasional Proportional Feedback

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Synchronization of a pair of chaotic diode resonators has been experimentally achieved by applying a minute feedback signal to the amplitude of the drive voltage of a slave resonator. The signal is proportional to the voltage difference between the two resonators and is applied for a fraction of the driving cycle. This method is the synchronization analog of the occasional proportional feedback scheme developed by Hunt for controlling unstable periodic orbits in chaotic attractors.

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Synchronizing chaotic signals is presently a topic of active investigation. Practical applications are being realized in the area of secure communications [1] and include the possibility of the development of cardiac regulatory devices and the construction of sets of chaotic dynamical systems whose common output signals are synchronized. Combined with the concept of stabilizing unstable periodic orbits in a chaotic attractor, the latter would be of great interest in the area of arrays of coupled lasers [2].

Currently two experimental methods have emerged in the literature as the predominant techniques for the synchronization of chaotic systems. The first involves weakly coupling two identical systems. The effect of the coupling is to add a new parameter to the governing equations so that the Lyapunov exponents of the system may be altered. By varying the coupling strength, the conditional Lyapunov exponents of the linearized signal difference between the two systems can be made negative causing distances between the orbits of the coupled system to shrink to zero. Perturbations due to the coupling are continuously applied and proportional to the instantaneous difference between two chosen dynamical parameters. Rul'kov *et al.* [3] have demonstrated mutual synchronization by linking two chaotic electrical circuits with a resistor. Roy and Thornburg [4] have reported synchronization between two Nd:YAG lasers by overlapping the intracavity electric fields. Newell *et al.* [5] showed that by adding an additional term only to a slave system, this technique was valid for one-way coupling in a driven diode resonator circuit.

The second method of synchronization, developed by Pecora and Carroll [6], entails the division of the system into two subsystems. The subsystem containing the largest positive Lyapunov exponent can be used to drive the other subsystem (containing only negative Lyapunov exponents) into synchronization with a duplicate subsystem. In this manner, synchronization has been achieved in various electrical circuits [1,7].

In this Letter, we describe the synchronization of chaotic time signals generated from two independently driven matched diode resonators. Our synchronization scheme for a continuous system is drawn from the control theory of Ott, Grebogi, and Yorke (OGY) [8]. The extension of the OGY algorithm to handle the stabilization of a chaotic trajectory of one system about a chaotic trajectory of another has been derived for maps by Lai and Grebogi [9]. This was achieved by applying a minute perturbation to a critical parameter of a slave dynamical system when a point on the orbit of the slave was near an equivalent point on the master trajectory. The applied perturbation was equal to the master-slave signal difference multiplied at each iteration by a factor which depended upon the structure of the Poincaré map at that point. Although their OGY based method intrinsically precludes *a priori* knowledge of the dynamical equations for the system, they still required structural information over the entire first return map in order to calculate the necessary parameter perturbations. The resulting synchronization is one in which neither is a new term added to the governing equations nor is the slave system required to be a subsystem of the master [10].

For the continuous system that we consider in this Letter, we also measure the signal difference between the chaotic master and slave system at some set point in the cycle of the orbit. This difference is multiplied by a gain α. However, the magnitude of our gain, unlike the factor calculated upon each iteration by Lai and Grebogi, is taken to be a constant. The resulting term is used to perturb the amplitude V_0 of the driving voltage, $V_0 \sin(2\pi f t)$, of the slave system for some fraction of the period. Synchronization is obtained when the above process is repeated for each cycle [11]. In this manner our technique is the synchronization analog of the occasional proportional feedback (OPF) scheme developed by Hunt for controlling unstable period orbits [12].

The experimental arrangement utilizes two independent diode resonators each composed of a diode, inductor, and resistor in series which are independently driven by a sinusoidal voltage. This well-studied nonlinear circuit [13,14] is a low-dimensional system which exhibits a period-doubling route to chaos. A block diagram of the experimental arrangement is shown in Fig. 1. Each of our diode resonators consisted of a 1N4004 silicon rectifier diode, a 33 mH inductor with a dc resistance of

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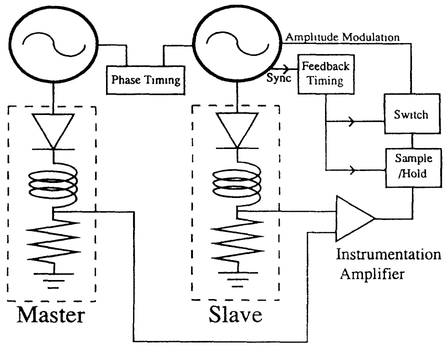

FIG. 1. A block diagram of the experimental apparatus. The setup consists of two diode resonators each composed of a 1N4004 diode, 33 mH inductor, and a 90.5 Ω resistor. The amplified difference between the chaotic voltages dropped by the resistor of each circuit is measured by an instrumentation amplifier. On the hold strobe, this difference is held by a sample and hold device and gated into the amplitude modulation input of the slave sine wave generator.

243 Ω, and a 90.5 Ω resistor driven at 70 kHz. The diodes used were similar in properties, based on the bifurcation diagrams obtained with them. The resonator displays a well-defined bifurcation from period 2 to period 4 and a transition from chaos to a period 3 window as the drive voltage is increased. We use these two points to match the diodes since the transitions will occur at different voltage levels for diodes with different properties. It is important to keep the temperature constant in order to characterize diodes in this manner.

The voltage drop across the resistor of one circuit, designated as the master signal $V^M(t)$, presents a convenient chaotic signal to represent the attractor of the circuit. From a second diode resonator circuit operating under similar conditions we select the corresponding chaotic signal, designated as the slave $V^S(t)$, which is to be synchronized to $V^M(t)$. The feedback necessary to synchronize the two is applied only to the driving amplitude of the slave resonator and is determined in the following manner. The difference in the two voltage signals, $V^S(t)-V^M(t)$, is obtained and amplified by a factor α by an instrumentation amplifier. At an arbitrary holding point in the drive cycle, this difference is frozen by means of a sample-and-hold amplifier and gated into the amplitude modulation input of the wave-form generator which drives the slave resonator. Both the selection of the hold point and the duration of the feedback signal are experimentally adjustable parameters which are set by the timing circuit.

Separate wave-form generators ensure independence of the two systems and allow for adjustment of the amplitudes and dc offsets so that each resonator operates in similar regions of the bifurcation diagram. A phase-timing circuit links the sync output of the master generator with the trigger input of the slave generator, ensuring

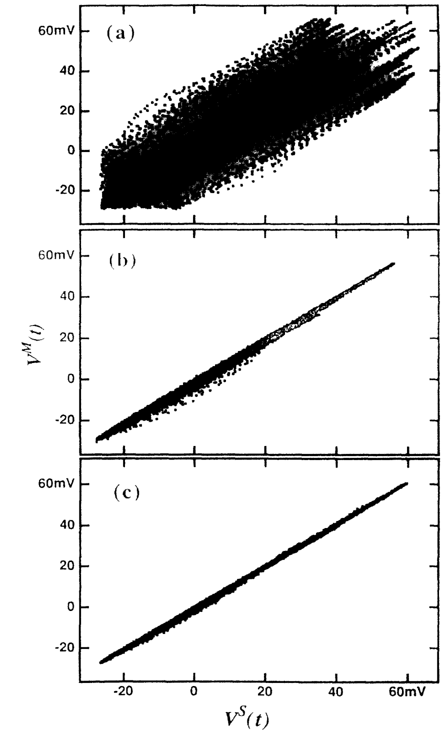

FIG. 2. The voltage across the slave resistor $V^S(t)$ (abscissa) versus that of the master $V^M(t)$ (ordinate) while signals are (a) unsynchronized and (b) synchronized. The imperfect synchronization is primarily due to differences between the properties of diodes. The bulge arises from a slight phase difference existing between the chaotic signals as they traverse from a low trough to a high peak. (c) Synchronization for more closely matched diodes.

that both resonators are driven at the same frequency while allowing for a constant, adjustable relative phase difference. Data are recorded using a Tektronix RTD710 10 bit digitizer with a sampling rate of 5 MHz. We have included no circuitry to restrict the feedback perturbation, $\alpha[V^M(t)-V^S(t)]$. When the feedback signal is applied, its initial magnitude is at most 25% of the peak-to-peak voltage of the driving sine wave. However, during synchronization the feedback signals were only a few percent of the driving voltage.

The amplitudes and dc offset of each wave-form generator are set so that each resonator is operating chaotically just below the period 3 window. Figure 2(a) is a plot of the voltage across the slave resistor versus that of the master while unsynchronized. As the system is nonautonomous and in phase, the area is more hexagonal than square in shape . Feedback pulses are then applied commencing on the peaks of the driving wave and having a duration of 8 μs (56% of the 14.3 μs period). The gain is slowly increased until synchronization is achieved [Fig. 2(b)]. This rather impure synchronization is primarily due to the diodes not being ideally matched. It is also

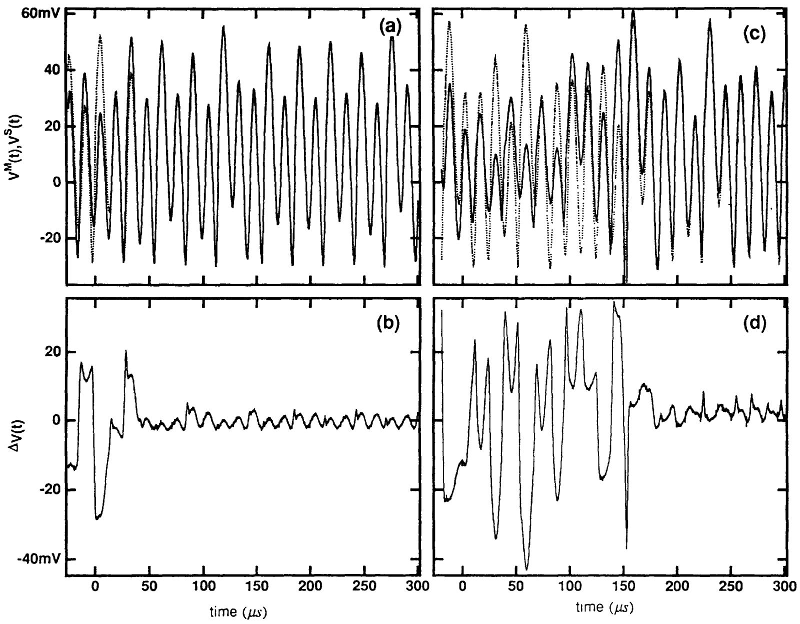

FIG. 3. (a) $V^M(t)$ (dotted line) and $V^S(t)$ (solid line) for closely matched diodes. Feedback commences at $t=0$. (b) $\Delta V(t) = V^S(t) - V^M(t)$. (c) $V^M(t)$ (dotted line) and $V^S(t)$ (solid line) for unmatched diodes. (d) $\Delta V(t) = V^S(t) - V^M(t)$ for unmatched diodes. On average, synchronization takes longer to be achieved than in (b).

due to each resonator being driven in different regions of the bifurcation diagram. Other factors reside in differences between the two driving wave-form generators. These include a slight phase difference between the two generators and small but noticeable differences in the power spectra of each generator. Note the slight bulge on the lower side of the figure. This occurs on those occasions when the resistor voltages are traversing from one of the lowest troughs to the highest peaks. There is a slight phase difference between the two signals and, combined with the relatively large slope, this produces the well-above-average difference. In spite of these variations, the feedback pulses are only 5% of the peak-to-peak voltage of the driving wave. Figure 2(c) is a plot where the components, while not exactly identical, are more closely matched than in the previous figure, and each of the adjustable parameters is optimized. The synchronization process has reduced the root-mean-square difference in the master and slave signals by a factor of 9. Additionally, the average heights of feedback pulses necessary to maintain synchronization have been reduced to only 1% of the peak-to-peak voltage of the driving wave.

Figure 3(a) is a time-domain plot of the slave resistor voltage along with the master. At time $t=0$, feedback is applied and within a few cycles, synchronization is achieved, as seen from the difference of the two signals in Fig. 3(b). Compare this figure to Figs. 3(c) and 3(d), in which the diodes are not as well matched. The time to achieve synchronization is noticeably longer. Note that the time it takes to converge to synchronization depends on the location of the two systems in their respective attractors when the feedback signal is initially applied. On average, closely matched diodes converge to synchronization more rapidly than poorly matched ones. Figure 4 shows the driving slave sine wave along with feedback signal for the optimized case necessary to maintain synchronization. Note that the troughs of the driving wave are only slightly perturbed.

In order to test the robustness of synchronization, we vary each of the experimental parameters until synchronization is lost. By adjusting the phase difference between the driving waves, good synchronization can be main-

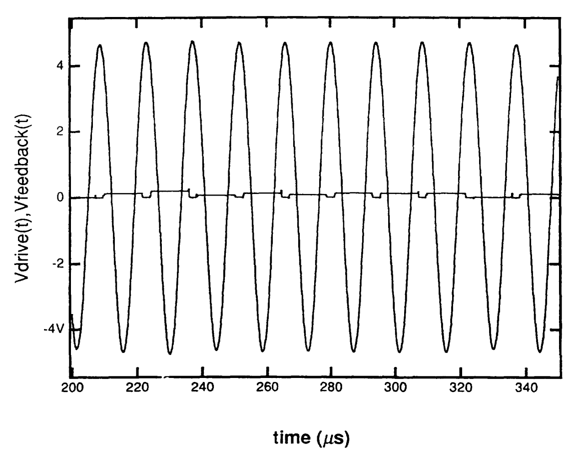

FIG. 4. The feedback signal and the perturbed slave driving wave corresponding to Fig. 3(a). Note that the troughs of the driving wave are only slightly perturbed.

tained with up to a 12° phase difference and a crude synchronization maintained for up to 20°. Since the feedback is a constant evaluated at one point of the cycle, it is not necessary to have the entire time series of the master system, merely the value of the master evaluated at the holding point. This implies that synchronization can be achieved for an arbitrary constant phase difference between the two driving wave-form generators. Hence, it is possible for a computer with a digital-to-analog converter to supply a master signal in order to synchronize the slave resonator.

With the feedback pulses commencing at the peaks, we can apply the feedback for up to 12 μs (84% of the period) after which synchronization can no longer be maintained. Thus, when feedback occurs as the driving wave is approaching its peak, synchronization is lost. This observation is a characteristic of the diode resonators and is due to the polarization of the diodes. Along the positive half of the drive wave, they are switching forward biased. As the wave approaches its peak, any change in the drive amplitude alters the current through the diode accordingly and tends to overcompensate the perturbation needed to maintain synchronization. Continuing this line of observation, we find that the feedback only needs to alter the troughs of the driving wave. While the original trigger point of feedback is on each peak of the sine wave, it can be adjusted until the phase difference between it and the peak is as great as 145°, i.e., just above the trough of the wave. When triggered at this point in the cycle, the duration of the feedback pulse can be reduced to less than 3 μs (20% of the period). This is again due to the asymmetry of the diodes. In this reversed-bias region, the current does not appreciably alter with changes in the amplitude. Hence, by tweaking the minimum voltage level seen by the diode, the necessary minor shifts in the attractor can be obtained.

In conclusion, we have demonstrated that a chaotic slave diode resonator can be synchronized to a master resonator by applying a small perturbation to the amplitude of the voltage driving the slave. For each cycle the minute feedback signal is constant and applied for only a fraction of the period. This method of synchronization could be maintained despite moderate variations in the component properties of the resonators, small phase differences between the driving generators, and a range of amplifier gains.

The occasional-proportional-feedback scheme presented here, along with the iterative dependent theory of Lai and Grebogi, are natural generalizations of the central ideas embodied in the OGY control algorithm extended to the problem of synchronization. A detailed examination of the similarities and differences between our synchronization method and that of Lai and Grebogi will be presented in a future publication.

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[1] K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993).

[2] H. G. Winful and L. Rahman, Phys. Rev. Lett. **65**, 1575 (1990).

[3] N. F. Rul'kov, A. R. Volkovski, A. Rodriguez-Lozano, E. Del Rio, and M. G. Velarde, Int. J. Bifur. Chaos **2**, 669 (1992).

[4] R. Roy and K. S. Thornburg, Jr., "Experimental Synchronization of Chaotic Lasers" (to be published).

[5] T. C. Newell, P. M. Alsing, A. Gavrielides, and V. Kovanis, Phys. Rev. E **49**, 313 (1994).

[6] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990); IEEE Trans. Circuits Syst. **38**, 453 (1991); Int. J. Bifur. Chaos **2**, 659 (1992).

[7] L. O. Chua, L. Kocarev, K. Eckert, and M. Itoh, Int. J. Bifur. Chaos **2**, 705 (1992); T. Endo and L. O. Chua, IEEE Trans. Circuits Syst. **38**, 1580 (1991).

[8] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).

[9] Y. C. Lai and C. Grebogi, Phys. Rev. E **47**, 2357 (1993).

[10] A theory structurally similar to [9] has been proposed in which synchronization is achieved by the addition of a one-way coupling term to the governing system equations. In this theory the proportionality factor of [9] is replaced by a vector, which again is determined dynamically. See P. So, E. Ott, and W. P. Dayawansa, Phys. Lett. A **176**, 421 (1993).

[11] In contrast to this work, the paper by Newell *et al.* [5] described the synchronization of a pair of chaotic circuits by the continuous addition of a feedback signal to the total driving field, $V_0 \sin(2\pi f t)$. That method of synchronization is different from the one described in this paper in which the feedback signal is applied only to the chaotic control parameter V_0, and is constant during the interval between applications.

[12] E. R. Hunt, Phys. Rev. Lett. **67**, 1953 (1991).

[13] P. S. Linsay, Phys. Rev. Lett. **47**, 1349 (1981).

[14] J. Testa, J. Perez, and C. Jefferies, Phys. Rev. Lett. **48**, 714 (1982).