Elastic Theory of Pinned Flux Lattices

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The effect of weak impurity disorder on flux lattices at equilibrium is studied in the absence of free dislocations using both the Gaussian variational method and, to $O(\epsilon = 4 - d)$, the functional renormalization group. We find universal logarithmic growth of displacements for $2 < d < 4$: $\overline{\langle u(x) - u(0)\rangle^2} \sim A_d \ln|x|$ and persistence of algebraic quasi-long-range translational order. When the two methods can be compared they agree within 10% on the value of A_d. We compute the function describing the crossover between the "random manifold" regime and the logarithmic regime. A similar crossover could be observable in present decoration experiments.

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There is considerable interest in describing the effect of microscopic disorder on the Abrikosov vortex lattice in type II superconductors. It is generally agreed, as predicted long ago by Larkin [1], that the long-range translational order of the lattice is destroyed by weak disorder. However, the precise way in which the order is destroyed is beyond the reach of Larkin's original Gaussian model, and despite much recent work [2–5] a correct quantitative theory is still lacking. In this Letter we provide such a *quantitative* description of the static properties of a lattice in the presence of weak disorder, using the two main analytical methods developed to study manifolds in random media, the Gaussian variational method (GVM) [6] and the functional renormalization group (FRG) [7]. It also applies to other pinned elastic systems studied experimentally (magnetic bubbles, Wigner crystal [8,9]).

An important quantity to be computed is the translational correlation function $C(r) = \overline{\langle e^{iK_0\cdot[u(r)-u(0)]}\rangle}$ [$u(r)$ is the displacement from the perfect lattice and K_0 one of the first reciprocal lattice vectors]. $C(r)$ is directly measured in Bitter decoration experiments [10]. These experiments have been analyzed using Larkin's model, in which weak random forces acting independently on each vortex are added to the conventional elastic energy [2]. However, this Gaussian theory is too simple to approximate correctly the full nonlinear problem. As a result, the exponential decay of $C(r)$ it predicts in $d = 3$ becomes inexact beyond the Larkin-Ovchinikov length where the lattice behaves collectively as an elastic manifold in a random potential with many metastable states [3]. This was also pointed out by Bouchaud, Mézard, and Yedidia (BMY), who first used the GVM to study this problem, and found a power-law roughening of the lattice and stretched exponential decay of $C(r)$ [4]. However, the *periodicity* of the lattice was not properly taken into account. Indeed, as was first shown by Nattermann using qualitative Flory arguments, periodicity leads to logarithmic roughening [5].

Here, we retain all the features missing in the previous approaches: nonlinearities, metastable states, *and* periodicity of the lattice. We compute, for an elastic theory with disorder, the correlation function of the relative displacements of two lines $\tilde{B}(r) = \frac{1}{2}\overline{\langle[u(r) - u(0)]^2\rangle}$ as a function of their separation r, as well as $C(r)$. Within our approach these quantities are related through $C(r) = e^{-K_0^2\tilde{B}(r)/2}$. We are primarily interested in the triangular Abrikosov lattice ($d = 2+1$), but also mention the case of $d = 2 + 0$ (thin films) or $d = 1 + 1$ (lines in a plane). We find that at large r, $\tilde{B}(r) \sim A_d \ln|r|$, where A_d is a universal amplitude depending on dimension only, and compute this amplitude using both the GVM and, in a $\epsilon = 4 - d$ expansion, the FRG. These two rather different methods agree at order ϵ within 10%. $C(r)$ has a slow algebraic decay in $d > 2$, $C(r) \sim (1/r)^{A_d}$, and quasi-long-range order persists. Our approach therefore confirms Nattermann's prediction [5], and allows in addition the calculation of the exponent A_d. Our results are valid provided that there are no dislocations in the lattice and that the system has enough time to reach equilibrium. The former is motivated by decoration pictures showing remarkably large regions free of dislocations. Since the core energy of a dislocation loop in $d = 3$ increases with its size, the possibility remains that at higher fields the vortex lattice is free of dislocations. Whether the second hypothesis is realized can only be decided within the context of a given experiment, but our assumptions will always hold below a certain length scale.

We also compute the full crossover function in $d = 3$, choosing for simplicity dispersionless elastic constants (Fig. 1). There are *three* different regimes as a function of the separation r. (i) When $\tilde{B}(r)$ is shorter than the square of the Lindemann length $l_T^2 = \langle u^2\rangle$, the thermal wandering of the lines averages enough over the random potential and the model becomes equivalent to Larkin's model for which $\tilde{B}(r) \sim |r|^{4-d}$. At low temperature, l_T is replaced by the superconducting coherence length ξ_0 (i.e., the correlation length of the random potential [3,4]). (ii) For $l_T^2 \ll \tilde{B}(r) \leq a^2$, $\tilde{B}(r) \sim r^{2\nu}$ where $\nu \sim 1/6$: this

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is the random manifold regime where each line sees effectively an independent random potential. (iii) For $r > \xi$, where ξ corresponds to a relative displacement of the order of the lattice spacing a, $\tilde{B}(r=\xi) \sim a^2$, the periodicity of the lattice becomes important, and one enters the asymptotic logarithmic regime. Nonlocal elasticity and 2d-3d crossover effects relevant for decoration experiments can easily be incorporated in the method by introducing new crossover length, such as the London length. Details can be found in [11]. In $d = 2$, thermal fluctuations are important ($l_T = \infty$) and the random manifold regime is much reduced. We find a modified Larkin regime with T-dependent exponents $\tilde{B}(x) \sim |x|^{2\nu(T)}$ and a long distance logarithmic regime.

We denote by R_i the equilibrium position of the lines labeled by an integer i, in the xy plane, and by $u(R_i, z)$ their in-plane displacements. z denotes the coordinate perpendicular to the planes. For weak disorder $a/\xi \ll 1$ it is legitimate to assume that $u(R_i, z)$ is slowly varying on the scale of the lattice and to use a continuum elastic energy, in terms of the continuous variable $u(x, z)$. Impurity disorder is modeled by a Gaussian random potential $V(x, z)$ with correlations $\overline{V(x,z)V(x',z')} = \Delta(x - x')\delta(z - z')$, where $\Delta(x)$ is a short-range function of range ξ_0 and Fourier transform Δ_q. The total energy is

$$H_{\rm el} = \frac{1}{2}\int d^2x\, dz[(c_{11} - c_{66})(\partial_\alpha u_\alpha)^2 + c_{66}(\partial_\alpha u_\beta)^2 + c_{44}(\partial_z u_\alpha)^2] + \int d^2x\, dz V(x,z)\rho(x,z), \quad (1)$$

where α, β denote in-plane coordinates and the density is $\rho(x,z) = \sum_i \delta(x - R_i - u(R_i, z))$. Although we have also performed the calculations directly on the Hamiltonian (1) [11], it is more enlightening to use the following decomposition of the density that keeps track of the discreteness of the lines. In the absence of dislocations, generalizing [12], one introduces the slowly varying field $\phi(x,z) = x - u(\phi(x,z), z)$. The density can be rewritten as $\rho(x,z) = \rho_0 \det[\partial_\alpha \phi_\beta] \sum_K e^{iK\cdot\phi(x,z)} \simeq \rho_0[1 - \partial_\alpha u_\alpha(\phi(x,z),z) + \sum_{K\neq 0} e^{iK\cdot x}\rho_K(x)]$, where $\rho_K(x) = e^{-iK\cdot u(\phi(x,z),z)}$ is the usual translational order parameter defined in terms of the reciprocal lattice vectors K, and ρ_0 is the average density.

Using the replica trick on (1), the above decomposition for the density leads to our starting model:

$$H_{\rm eff} = \int \frac{d^2q\, dq_z}{2(2\pi)^3} \sum_a G^{-1}_{0,\alpha\beta} u^a_\alpha(q,q_z) u^a_\beta(q,q_z) - \int d^2x\, dz \sum_{a,b} \left(\frac{\rho_0^2 \Delta_0}{2T} \partial_\alpha u^a_\alpha \partial_\beta u^b_\beta + \sum_{K\neq 0} \frac{\rho_0^2 \Delta_K}{2T} \cos\{K \cdot [u^a(x,z) - u^b(x,z)]\} \right), \quad (2)$$

with $G_0^{-1} = (c_{44}q_z^2 + c_{66}q^2)P^T_{\alpha\beta} + (c_{44}q_z^2 + c_{11}q^2)P^L_{\alpha\beta}$ in the case of (1), where $P^T_{\alpha\beta} = \delta_{\alpha\beta} - q_\alpha q_\beta/q^2$ and $P^L_{\alpha\beta} = q_\alpha q_\beta/q^2$. To be rigorous, (2) should be written in terms of $u(\phi(x,z),z)$ but this has no effect on our results. It leads only to corrections of higher order in ∇u that we can neglect in the elastic limit $a/\xi \ll 1$ [11]. For clarity we present the calculation for the isotropic version of (2) in d dimensions with $G_0^{-1} = cq^2\delta_{\alpha\beta}$, where c is an elastic constant and $\rho_0^2\Delta_K = \Delta$ for all K [$x \equiv (x,z)$]. The results for (1) are presented at the end. A related single cosine model was studied by Villain and Fernandez [13] using a real space RG. Our results confirm and extend their analysis. The crossover from the random manifold to the logarithmic regime can only be captured by including *all* harmonics in (2).

One can get an idea of the effect of various terms in (2) by using arguments similar to [14]. Assuming that u varies $\sim a$ over a length ξ, the density of kinetic energy is $\sim c(a/\xi)^2$. The long wavelength part of the disorder is $H^{\rm dis}_{q\sim 0} \sim \int d^dx\, V(x)\partial_\alpha u_\alpha(x) \sim \Delta^{1/2}/\xi^{1+d/2}$ and $H^{\rm dis}_{q\sim K_0} \sim \int d^dx\, V(x)\cos[K_0(x - u(x))] \sim \Delta^{1/2}/\xi^{d/2}$. There are thus two length scales $\xi_{q\sim 0} \sim a\left(c^2a^d/\Delta\right)^{1/(2-d)}$ and $\xi \sim a\left(c^2a^d/\Delta\right)^{1/(4-d)}$. The $q \sim 0$ component of the disorder is therefore relevant only for $d \le 2$ and the second term in (2) can be dropped for $d > 2$. Higher Fourier components $V_{q\sim K}$ disorder the lattice below $d = 4$.

We now look for the best trial Gaussian Hamiltonian

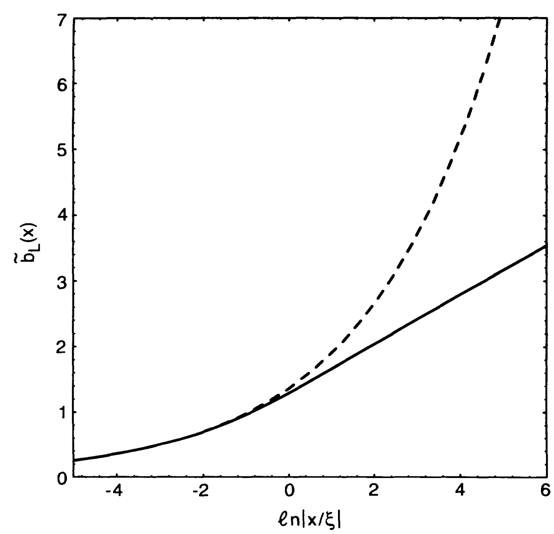

FIG. 1. Plot of $\tilde{b}_L$ versus $\ln|x/\xi|$ (solid line), as defined in the text, for the Abrikosov triangular lattice. The dashed line is the random manifold result.

H_0 in replica space, of the form [6] $H_0 = \frac{1}{2}\int \frac{d^dq}{(2\pi)^d} \times G_{ab}^{-1}(q)u_a(q)u_b(-q)$. Defining the self-energy $G_{ab}^{-1} = cq^2\delta_{ab} - \sigma_{ab}$, and $G_c^{-1}(q) = \sum_b G_{ab}^{-1}(q)$ minimization of the variational free energy $F_{\rm var} = F_0 + \langle H - H_0\rangle_{H_0}$ leads to

$$G_c(q) = \frac{1}{cq^2}, \quad \sigma_{a\neq b} = \frac{\Delta}{nT}\sum_K K^2 e^{-\frac{K^2}{2}B_{ab}(x=0)}, \tag{3}$$

$$B_{ab}(x) = \frac{1}{n}\langle [u_a(x) - u_b(0)]^2\rangle \tag{4}$$

$$= T\int \frac{d^dq}{(2\pi)^d}[G_{aa}(q) + G_{bb}(q) - 2\cos(qx)G_{ab}(q)],$$

n being the number of components of u. The stability of the symmetric solution $B_{a\neq b}(x) = B(x)$, which mimics the distribution of displacements by a single Gaussian, is governed by the replicon eigenvalue λ [6]:

$$\lambda = 1 - \frac{\Delta}{n}\sum_K K^4 e^{-K^2T\int \frac{d^dp}{(2\pi)^d}G_c(p)}\int \frac{d^dq}{(2\pi)^d}G_c^2(q). \tag{5}$$

Introducing a small regularizing mass in G_c, $G_c(q)^{-1} = cq^2 + \mu^2$ we find, when $\mu \to 0$, that for $d < 2$ the replica symmetric solution is always stable and disorder is irrelevant. For $d = 2$ the condition becomes $\mu^{-2(1-TK_0^2/4\pi c)} < 1$ for small μ. Thus there is a transition at $T = T_c = 4\pi c/K_0^2$ between a stable high-T phase where disorder is irrelevant and a low-T glassy phase where the symmetric solution is unstable. For $2 < d < 4$ it is *always unstable* and disorder is always relevant.

We now find a replica symmetry breaking solution for $2 < d < 4$, $d = 2$ to be discussed later. We denote $\tilde{G}(q) = G_{aa}(q)$ and parametrize $G_{ab}(q)$ by $G(q,v)$ where $0 < v < 1$, and similarly for $B_{ab}(x)$. (3) and the algebraic rules for inversion of hierarchical matrices [6] give

$$B(0,v) = B(0,v_c) + \int_v^{v_c} dw \int \frac{d^dq}{(2\pi)^d}\frac{2T\sigma'(w)}{[G_c(q)^{-1} + [\sigma](w)]^2}, \tag{6}$$

where $[\sigma](v) = u\sigma(v) - \int_0^v dw\sigma(w)$ and v_c is the breakpoint such that $\sigma(v)$ is constant for $v > v_c$. $B(0,v_c)$ is a nonuniversal quantity $B(0,v_c) \simeq \xi_0^2 + l_T^2$.

To discuss the large distance behavior $x \gg \xi$ it is enough to keep $K^2 = K_0^2$ in (3) since $B(0,v) \gg a^2$. In that case, using (3), (6), and $[\sigma]'(v) = v\sigma'(v)$, one finds the effective self-energy $[\sigma](v) = (v/v_0)^{2/\theta}$ and $v_0 = 2K_0^2Tc_dc^{-d/2}/(4-d)$. The energy fluctuation exponent is $\theta = d-2$. Energy fluctuations are of order T/v and the large scale behavior is controlled by small v. One can now compute the correlation functions,

$$\overline{\langle [u(x) - u(0)]^2\rangle} = 2nT\int \frac{d^dq}{(2\pi)^d}[1 - \cos(qx)]\tilde{G}(q), \tag{7}$$

$$\tilde{G}(q) = \frac{1}{cq^2}\left(1 + \int_0^1 \frac{dv}{v^2}\frac{[\sigma](v)}{cq^2 + [\sigma](v)}\right) \sim \frac{Z_d}{q^d}, \tag{8}$$

with $Z_d = (4-d)/TK_0^2S_d$ and $1/S_d = 2^{d-1}\pi^{d/2}\Gamma[d/2]$. Thus for $2 < d < 4$ we find *logarithmic* growth,

$$\overline{\langle [u(x) - u(0)]^2\rangle} = \frac{2n}{K_0^2}A_d \ln|x| \tag{9}$$

with $A_d = 4 - d$.

We now give the full crossover function for $d = 3$ for model (2) with $\rho_0^2\Delta_{K_0} = \Delta$. The crossover length is $\xi = a^4c^2/2\pi^3\Delta$. Defining $h(z) = \sum_P(\Delta_K/\Delta_{K_0})P^4e^{-zP^2}$, where $K = 2\pi P/a$, the solution is, in parametric form,

$$v = v_\xi h'(z), \qquad [\sigma]^{1/2} = c^{1/2}\xi^{-1}h(z), \tag{10}$$

with $B(0,v) = a^2z/2\pi^2$ and $v_\xi = 2\pi^4T\Delta/a^6c^3 \sim l_T/\xi$. The mean square displacement $\tilde{B}$ is $\tilde{B}(x) = \frac{a^2}{2\pi^2}\tilde{b}\left(\frac{x}{\xi}\right)$, with

$$\tilde{b}(x) = \int_0^\infty dt\frac{h''(t)h(t)}{h'(t)^2}f(xh(t)), \tag{11}$$

$$f(x) = 1 - \frac{1}{x}(1 - e^{-x}). \tag{12}$$

For $x \ll \xi$, $B(0,v)$ is very small and $h(z) \sim 1/z^{n/2+2}$ and therefore $\tilde{B}(x) \sim x^{2\nu}$ with $\nu = 1/6$ for $n = 2$. This corresponds to the random manifold regime [4]. The crossover function [(10) and (11)] was derived assuming $v_\xi \ll v_c$, equivalent to $l_T \ll \xi$. At scales such that $\tilde{B}(x)$ is smaller than l_T^2 or ξ_0^2, one is in the regime $v > v_c$ and one recovers the replica symmetric propagator $\tilde{G}(q) \sim 1/q^4$ for $q^2 \gg [\sigma](v_c)$, and Larkin's model behavior.

In the vortex lattice (1), assuming $c_{66} \ll c_{11}$, the crossover length becomes $\xi = c_{66}^{3/2}c_{44}^{1/2}a^4/2\pi^3\Delta$. $\tilde{B}_{\alpha\beta}(x) = \tilde{B}_TP_{\alpha\beta}^T(x) + \tilde{B}_LP_{\alpha\beta}^L(x)$ depends on the direction of x and we find

$$\tilde{B}_{L,T}(x) = \frac{a^2}{2\pi^2}\left\{\tilde{b}_{T,L}\left(\frac{x}{\xi}\right) + \frac{c_{66}}{c_{11}}\tilde{b}_{L,T}\left(\frac{x}{\xi}\sqrt{\frac{c_{66}}{c_{11}}}\right)\right\},$$

where $\tilde{b}_{L,T}$ are similar to (11) with f replaced by

$$f_L(x) = \frac{1}{2} - \frac{1}{x^2} + \left(\frac{1}{x} + \frac{1}{x^2}\right)e^{-x}, \quad f_T(x) = f(x) - f_L(x).$$

The crossover is shown in Fig. 1. We find that if $\xi/a \gg 1$ all curves should scale when plotted in units of x/ξ. The ratio $R = \tilde{B}_T(x)/\tilde{B}_L(x)$ crosses over at $x = \xi$ from $R = 4/3$ to $R = 1$ as isotropy is restored at large scale.

As $d \to 2^+$ the function $[\sigma](v)$ vanishes for $v < v_0 = TK_0^2/4\pi c$. Thus in $d = 2$ for $T < T_c$ there is a one-step replica symmetry broken solution with $[\sigma](v) = 0$ for $v < v_c$ and $[\sigma](v) = [\sigma](v_c)$ for $v_c < v < 1$ [11]. In the glassy phase $T < T_c$ the variational method predicts the following two regimes. For T moderate, the short distance Larkin regime $\tilde{B}(x) \sim x^2$ crosses over directly towards the asymptotic logarithmic growth (9) at a length $\xi = a(c^2a^2/\Delta)^{1/(2-TK_0^2/2\pi c)}$. At low T, $T\ln(\xi/a)/ca^2 \ll 1$, there is, in addition, an intermediate random manifold regime. In $d = 1 + 1$ the starting model (2) becomes exact due to the absence of dislocations and a RG calculation for $T \approx T_c$ [13,15,16] finds $\tilde{B}(x) \sim \ln^2(x)$ at large x. If one believes in the validity of this RG in a glassy phase, the simple Gaussian ansatz does not give the exact long-range behavior, due to the importance of fluctuations in $d = 2$. However, it gives T_c exactly and captures correctly the crossover towards a slower logarithmic regime. Using RG we have shown [11] that in $d = 2$ the Larkin regime is in fact *anomalous* with a continuously variable exponent: $\tilde{B}(x) \sim a^2(x/\xi)^{2-\frac{TK_0^2}{2\pi c}}$ for $x < \xi$ (at low T ξ is replaced by another length). Model (2) will apply in $d = 2 + 0$ at scales shorter than the distance between dislocations. It could also describe a polymerized membrane on a disordered substrate, with very high dislocation core energy.

A previous application of the variational method by BMY [4] led to the conclusion, which we believe is erroneous, that the fluctuations are enhanced at large scales. They applied the same method to a model in which each line sees a different disorder. This amounts to introducing an extra and unphysical disorder in the original model (1) with correlations decaying as $1/|R_i - R_j|^\lambda$. The long wavelength part of such a disorder dominates since large global translations of the lattice can improve the bulk energy, whereas in the physical model the energy gain from the long wavelength components can only come from surface terms and is thus irrelevant for $d > 2$, as shown in the discussion following (2). Indeed for $\lambda \to 0$ the amplitude they obtain vanishes. In fact one can simplify the saddle point equations of [4] by noting that the x dependence of $B(x, u)$ in these equations is unimportant, up to higher orders in ∇u, and find results identical to those of the local model (2).

To complement the variational method we perform a functional renormalization group calculation on the isotropic model. To simplify we take u to be a scalar field ($n = 1$) and $c = 1$. The replicated Hamiltonian is $H_{\rm imp} = -\frac{1}{2T}\sum_{ab}\int d^dx\,\Delta(u_a(x) - u_b(x))$. The function $\Delta(z)$ is periodic of period 1 ($2\pi K_0 = 1$). The RG equations to order $\epsilon = 4 - d$ have been derived by Fisher [7] for the random manifold problem:

$$\frac{d\Delta}{dl} = (\epsilon - 4\zeta)\Delta + \zeta z\Delta' + \frac{1}{2}(\Delta'')^2 - \Delta''\Delta''(0). \quad (13)$$

A factor S_d was absorbed into Δ. The periodicity of the function implies that $\zeta = 0$. We obtain the fixed point function in $[0, 1]$,

$$\Delta^*(z) = \frac{\epsilon}{72}\left(\frac{1}{36} - z^2(1-z)^2\right). \quad (14)$$

Values for other z are obtained by periodicity. The fixed point is stable except for a constant shift. The linearized spectrum is discrete and the eigenvectors are Jacobi polynomials. This function has a nonanalyticity [7] $\Delta^{*(4)}(0) = \infty$. To compute the correlation function $\tilde{\Gamma} = T\tilde{G}$ one uses the RG flow equation, $\tilde{\Gamma}(q, T, \Delta) = e^{dl}\tilde{\Gamma}(qe^l, Te^{(2-d)l}, \Delta(l))$. Choosing $e^{l^*}q = 1/a$ we obtain perturbatively $\tilde{\Gamma}(q) = -\Delta''^*(0)/q^4 = a^{(4-d)}\epsilon/36S_dq^d$. Thus we find at order ϵ a logarithmic growth (9) of line displacement with $A_{d,\rm RG} = \epsilon(2\pi)^2/36 = 1.10\epsilon$. This compares within 10% with $A_{d,\rm VAR} = \epsilon$ which slightly underestimates fluctuations [17]. For $2 \leq d < 4$ the real space RG of [13] also predicts a log but does not give the universal prefactor A_d or the crossover function. The agreement between these methods, none being rigorous, lends credibility to the additional results in $d = 3$ obtained using the GVM.

To conclude, we have shown that due to the periodicity of the lattice, the pinning by impurities becomes less effective at large length scales. Although more work is needed to extend this approach to the problem of a driven lattice, this is likely to have consequences for the transport properties. For instance, if there is a regime of flux creep where plastic deformations can be neglected, energy barrier arguments [3], and the exponents found here, lead to a voltage-current relation $V \sim \exp(-1/Tj^\mu)$, where μ crosses over from $\mu \approx 0.7 - 0.8$ to $\mu = 1/2$ as j decreases.

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Note added.— After submission, we received a preprint by S. Korshunov who considered the single cosine model and also obtained (8) using the variational ansatz.

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