Raman Forward Scattering of Short-Pulse High-Intensity Lasers

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Raman forward scattering of short-pulse relativistic-intensity laser pulses is investigated. Differential equations which model the instability for arbitrarily large pump strengths are derived. Exact solutions are obtained for a set of physically relevant initial conditions. The growth rate is found to asymptotically approach zero for ultrarelativistic laser intensities. The relevance of the results to present experiments and the limitations of the quasistatic equations are discussed.

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A fundamental phenomenon in laser-plasma interactions is Raman forward scattering (RFS). In this interaction an electromagnetic wave (ω_0,k_0) incident upon a plasma scatters into two copropagating electromagnetic sidebands $(\omega_p \pm \omega_0,k_p \pm k_0)$ and a plasma wave (ω_p,k_p). RFS has the potential to generate relativistic electrons because the resulting plasma wave (ω_p,k_p) has a phase velocity nearly equal to the speed of light c. As a result, RFS has been actively investigated for nearly twenty years because of its relevance to pellet preheat in laser fusion [1,2], to cosmic ray generation in pulsars [3], and to plasma accelerators and light sources [4,5].

Past theoretical research has primarily been concerned with long pulse lasers interacting with plasmas of lengths smaller than the pulse length and with laser intensities for which $eA_0/mc^2 \ll 1$, where A_0 is the vector potential of the laser [1]. Consequently, only the temporal growth or the steady state spatial growth was considered. There has also been little experimental evidence [2,5] of RFS because of its relatively small temporal growth rate. The development of the chirped pulse amplification technique [6] has led to high-intensity lasers with pulse lengths which are typically less than a Rayleigh length. In experiments and applications with these lasers [7], the effects of spatial-temporal growth and relativistic pump strengths will become important.

In this Letter we analyze the spatial-temporal growth of RFS. We start from a set of equations valid for weakly relativistic pumps. From these equations a single differential equation for the plasma wave (ω_p,k_p) is derived. This equation is solved exactly for a relevant set of initial conditions. The exact solution permits a precise determination of the time at which the asymptotic solutions obtained from the pole-pinch condition [8] are valid. We next use the quasistatic equations to obtain the growth rate for arbitrarily large laser intensities. Last we comment on the implications of this work for present and future experiments.

We emphasize that this work differs from two recent papers by Antonsen and Mora [9] and Sprangle *et al.* [10]. Each numerically solved a set of quasistatic equations combined with the paraxial-ray approximation. Each observed that the laser pulse breaks up longitudinally into beamlets of length $2\pi c/\omega_p$ after propagating roughly a Rayleigh length. Each also attributed the breakup to 2D effects. Upon examining the asymptotic impulse response for the weakly nonlinear pumps, Antonsen and Mora argued that the number of e-foldings per Rayleigh length depends only on the ratio of incident power to threshold power for self-focusing and the pulse length in units of c/ω_p. However, the use of the paraxial-ray approximation precludes direct forward scatter and incorrectly models near forward scatter, while the asymptotic response is invalid for near term experiments. Sprangle *et al.* imply that breakup only occurs when the relativistic self-focusing power threshold (P_c) is exceeded. We show that for 4-wave RFS (forward and near forward) the number of e-foldings per Rayleigh length additionally depends on ω_p/ω_0. These results indicate that beam breakup can occur in 1D from RFS and that it can occur for power less than P_c. Furthermore, we give an exact solution, rather than an asymptotic response, for the evolution of RFS, calculate the growth rate to all orders in $eA_0/mc^2 \equiv a_0$, and examine the validity of the quasistatic approximation.

We begin by making the 1D quasistatic approximation [11]. In this approximation a mathematical transformation is made from (x,t) coordinates to the $(\psi \equiv t - x/c$, $\tau \equiv t)$ coordinates, and $\partial/\partial\tau$ derivatives are neglected in the fluid equations but not Maxwell's equations. This is valid if fluid quantities at a fixed position relative to the front of the pulse, i.e., fixed ψ, do not change much during the duration of the pulse. The resulting equations are [11]

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$$-2\frac{\partial}{\partial\psi}\frac{\partial}{\partial\tau}a=\frac{a}{\chi}, \tag{1}$$

$$\frac{\partial^2}{\partial\psi^2}\chi+\frac{1}{2}\left[1-\frac{1+a^2}{\chi^2}\right]=0, \tag{2}$$

where $\chi\equiv 1+\phi$ is the scalar potential of the plasma response, a is the vector potential of the electromagnetic wave, and time is normalized to ω_p^{-1}, space to c/ω_p, and current to en_0c.

In RFS the density perturbations grow from noise so we linearize Eqs. (1) and (2) to obtain

$$-2\frac{\partial}{\partial\psi}\frac{\partial}{\partial\tau}a=a(1-\phi), \tag{3}$$

$$\left(\frac{\partial^2}{\partial\psi^2}+1\right)\phi=\frac{a^2}{2}. \tag{4}$$

The quasistatic approximate equations differ from the exact linearized equations in two places. On the right-hand side of Eq. (3), n/γ is replaced by $1-\phi$, and on the left-hand side of Eq. (4) $\partial^2/\partial t^2$ is replaced by $\partial^2/\partial\psi^2$, i.e., $\partial/\partial\tau$ derivatives are neglected. In what follows, we make the quasistatic approximation in Eq. (3) but not in Eq. (4), giving

$$-2\frac{\partial}{\partial\psi}\frac{\partial}{\partial\tau}a=a(1-\phi), \tag{5}$$

$$\left(\frac{\partial^2}{\partial\psi^2}+2\frac{\partial}{\partial\psi}\frac{\partial}{\partial\tau}+1\right)\phi=\frac{a^2}{2}. \tag{6}$$

We note that Eqs. (5) and (6) can be derived [12] without making the quasistatic approximation. Later we comment on how the inclusion of the $\partial/\partial\tau$ derivative in Eq. (6) alters the result. This provides a condition for the validity of the quasistatic approximation.

We next carry out a stability analysis of Eqs. (5) and (6) by letting

$$a=\frac{a_0}{2}e^{i\theta_0}+\frac{a_+}{2}e^{i\theta_+}+\frac{a_-}{2}e^{i\theta_-}+\text{c.c.}$$

and

$$\phi=\tfrac{1}{2}\phi_s e^{i\theta}+\text{c.c.},$$

where $\theta_\pm\equiv\theta\pm\theta_0$. Rather than letting the θ's contain real and imaginary parts, we assume that each θ is purely real and derive a differential equation for the envelopes of a_+, a_-, and ϕ_s. This makes the relevant initial conditions more recognizable and provides the dispersion relation in the Galilean frame. We assume that the θ's can be written as

$$\theta_0=-[k_0\psi+(\omega_0-k_0)\tau],$$

$$\theta_\pm=-[k_\pm\psi+(\omega_\pm-k_\pm)\tau],\quad \theta=-[k\psi+(\omega-k)\tau],$$

where (ω_0,k_0), (ω_-,k_-), and (ω,k) satisfy their own respective dispersion relations. This leads to the following set of differential equations:

$$i2k_+\frac{\partial a_+}{\partial\tau}=-\frac{a_0\phi_s}{2}+\Delta a_+, \tag{7}$$

$$i2k_-\frac{\partial a_-}{\partial\tau}=-\frac{a_0^*\phi_s}{2}, \tag{8}$$

$$-i2k\frac{\partial\phi_s}{\partial\psi}-i2k\frac{\partial\phi_s}{\partial\tau}=\frac{a+a_0^*}{2}+\frac{a-a_0}{2}, \tag{9}$$

where $\partial a_\pm/\partial\psi$ was neglected compared to $k_\pm a_+$ and $k_\pm^2\partial a_+/\partial\tau$, and $\partial\phi_s/\partial\psi$ was neglected compared to $k\phi_s$. The extra term on the right-hand side of Eq. (7) is a mismatch term which results because the Stokes and anti-Stokes waves cannot both be resonant simultaneously. In the purely 1D limit $\Delta=2k_0^2$, while $\Delta=2k_\perp^2$ if $k_\perp^2>2/k_0$. (Recall that the k's are normalized to ω_p/c.) We note that the $\partial\phi_s/\partial t$ term in Eq. (9) is absent when the $\partial/\partial\tau$ derivatives are neglected in Eq. (4). For the moment we neglect the mismatch term in Eq. (7) and combine Eqs. (7)–(9) into a single differential equation for ϕ_s,

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{\partial}{\partial\tau}\frac{\partial}{\partial\psi}-\gamma_0^2\right)\phi_s=0, \tag{10}$$

where $\gamma_0\equiv|a_0|/\sqrt{8}k_0$ is the well known temporal growth rate [1]. Temporal growth occurs when there is translational invariance in the coordinate ψ. Purely temporal growth is prevented when the quasistatic approximation is made because the $\partial^2/\partial\tau^2$ term is absent. We also note that in the paraxial-ray approximation $k_-=-k_+=-k_0$ and $\Delta\to 2k_\perp^2$ resulting in $\gamma_0^2\to 0$ as $k_\perp\to 0$. Therefore, when the paraxial-ray approximation is used, the mismatch term and growth rate vanish for direct forward RFS, and near forward RFS is qualitatively different.

We solve Eq. (10) by performing a double Laplace transform in $\tau(e^{i\alpha\tau})$ and $\psi(e^{i\beta\psi})$ because $\phi_s=0$ for $\tau<0$ and $\psi<0$:

$$\phi_s(\alpha,\beta)=\frac{\phi_s(\tau=0,\psi=0)-i\alpha\phi_s(\alpha,\psi=0)-i\beta\phi_s(\tau=0,\beta)-\partial\phi_s/\partial\tau(\tau=0,\beta)-i\alpha\phi_s(\tau=0,\beta)}{\alpha^2+\alpha\beta+\gamma_0^2}. \tag{11}$$

Three initial conditions are required, namely, $\phi_s(\tau,\psi=0)$, $\phi_s(\tau=0,\psi)$, and $\partial\phi_s/\partial\tau(\tau=0,\psi)$. We choose the simple yet physically relevant set $\phi_s(\tau,\psi=0)=\phi_s(\tau=0,\psi)=\phi_0$ and $\partial\phi_s/\partial\tau(\tau=0,\psi)=0$. This assumes that the noise source at the head of the pulse is constant.

The solution for ϕ_s is determined by inverting Eq. (11) and is given [12,13] by

$$\phi_0 H(\psi)\left[[H(\tau)-H(\tau-\psi)]\cosh\gamma_0\tau+H(\tau-\psi)\sum_n\left(\frac{\psi}{\tau-\psi}\right)^n I_{2n}\left(2\gamma_0\sqrt{(\tau-\psi)\psi}\right)\right], \tag{12}$$

where H is the Heaviside step function, I_n is the modified Bessel function of the first kind, and n is an integer from 0 to ∞. We note that if the $\partial^2/\partial\tau^2$ term is neglected in Eq. (10), the solution is

$$\phi_s = \phi_0 H(\psi) H(\tau) I_0(2\gamma_0\sqrt{\tau\psi}) . \tag{13}$$

It follows from Eq. (12) that purely temporal growth occurs at a given position, ψ, until information originating at the head of the pulse arrives at ψ. This information moves at the speed of light c in this Galilean frame, because the plasma waves have zero group velocity in the laboratory frame. After this time ($\tau=\psi$), quasistaticlike growth occurs, i.e., growth in the variable $\sqrt{(\tau-\psi)\psi} \simeq \sqrt{\tau\psi}$. Therefore, the quasistatic are reasonable when $\cosh\gamma_0\psi_m \lesssim 1$ and when $\tau \gg \psi_m$, where ψ_m is the position at the back of the pulse. We also note that the $\cosh\gamma_0\tau$ growth is a result of ϕ being uniform in space at $\tau=0$ while the $I_0(2\gamma_0\sqrt{(\tau-\psi)\psi})$ term is the response to the boundary condition at $\psi=0$.

When the pole-pinching (stationary phase) criterion is used, ϕ_s is found to grow asymptotically as $\exp(2\gamma_0\sqrt{\tau\psi})$ initially and as $\exp[(3^{1/2}2^{1/3}3/4)\gamma_0^{2/3}\tau^{2/3}\psi^{1/3}k_0^{-2/3}]$ when the mismatch term in Eq. (7) eventually becomes important. Similar pole-pinch expressions have been obtained by others [10]. We call the first type of growth 4-wave RFS and the second type nonresonant [14] 4-wave RFS. However, by examining Eq. (12), we find that the number of e-foldings, N, which occurs before the mismatch term becomes important, is $N=\psi/2a_0^2$. Therefore, for pulses many c/ω_p long, the so-called 4-wave RFS growth continues until saturation. We note that RFS eventually passes through two other regimes of growth. When $(\partial/\partial\psi)\phi > k\phi$ the higher order terms need to be kept in Eq. (9). The resulting asymptotic growth is generally referred to as strongly coupled RFS. Eventually $k_+(\partial/\partial\tau)a_+ < (k^2/k_+^2)a_+$. At this time the anti-Stokes can be completely neglected and the asymptotic growth scales as 3-wave RFS. Antonsen and Mora [9] have identified analogous regimes for the purely temporal analysis.

To make a connection with the work of Antonsen and Mora [9] and Sprangle *et al.* [10], we rewrite the 1D growth in terms of the relativistic self-focusing power threshold [15], $P_c \equiv a_0^2\sigma^2/32$, and the Rayleigh time, $\tau_R \equiv k_0\sigma^2/2$, where σ is the light wave's spot size. This gives the asymptotic growth as $\exp\{[2(P/P_c)(\tau/\tau_R)\times\psi/\omega_0]^{1/2}\}$. Therefore, the exponentiation can be expressed in terms of 2D quantities (P_c and τ_R), although the analysis and the resulting beam breakup are strictly 1D. This differs from the results of Antonsen and Mora by the extra $\omega_0^{-1/2}$ factor. We identify the scaling of Antonsen and Mora as 3-wave RFS. Furthermore, for the paraxial-ray approximation, $\Delta = 2k_\perp^2/k^2$, resulting in asymptotic growth as $\exp[(2^{1/2}3^{1/2}3/4)(P/P_c)^{1/3}(\tau/\tau_R)^{2/3}\psi^{1/3}(k_\perp\sigma)^{2/3}]$. Assuming a characteristic value for $k_\perp$ of $1/\sigma$ gives growth which scales similarly to those obtained by others [16] for whole beam instabilities of finite width pulses.

We next consider modifications to γ_0 for arbitrarily large a_0. We return to Eqs. (1) and (2) and assume that $\chi=\chi_0+\delta\chi$ where $\delta\chi \ll \chi_0$. This leads to the following set of equations:

$$-2\frac{\partial^2}{\partial\psi\partial\tau}A_\perp = \frac{A_\perp}{\chi_0} - \frac{A_\perp}{\chi_0^2}\delta\chi , \tag{14}$$

$$\frac{\partial^2}{\partial\psi^2}\delta\chi + \frac{\delta\chi}{\chi_0} = \frac{1}{2}\frac{1}{\chi_0^2}(1+A_\perp^2-\chi_0^2) . \tag{15}$$

Guided by the weakly relativistic case, we could add $\partial/\partial\tau$ derivatives into Eq. (15) but, without further justification, we will not because the effect of this term is already known to be replacing τ with $\tau-\psi$ and the $\cosh\gamma_0\tau$ term. As before we let

$$A_\perp = \frac{A_0}{2}e^{i\theta_0} + \frac{A_+}{2}e^{i\theta_+} + \frac{A_-}{2}e^{i\theta_-} + \text{c.c.} \tag{16}$$

and

$$\delta\chi = (\delta\chi_s/2)e^{i\theta} + \text{c.c.} , \tag{17}$$

where the dispersion relation for each θ is appropriately modified and $\chi_0^2 = 1 + A_0^2/2$. The fact that the harmonics of A_0 and χ_0 can be ignored has been previously justified [17]. The ratio of the third harmonic to the fundamental scales as $\frac{3}{256}(\omega_p^2/\omega_0^2)A_0^2/(1+A_0^2/2)^{3/2}$, which is always $\ll 1$. We neglect the higher order $\theta \pm n\theta_0$ couplings because they are nonresonant. Substituting Eqs. (16) and (17) into Eqs. (14) and (15) and keeping the leading order terms yields

$$i2k_+\frac{\partial A_+}{\partial\tau} = -\frac{\delta\chi_s}{2}\frac{A_0}{\chi_0^2} , \tag{18}$$

$$i2k_-\frac{\partial A_-}{\partial\tau} = -\frac{\delta\chi_s}{2}\frac{A_0^+}{\chi_0^2} , \tag{19}$$

$$-i2k\frac{\partial}{\partial\psi}\delta\chi_s = \frac{1}{2}\frac{1}{\chi_0^2}(A_0^* A_+ + A_0 A_-) . \tag{20}$$

We combine Eqs. (18), (19), and (20) into a single equation for $\delta\chi_s$,

$$\left(\frac{\partial^2}{\partial\psi\partial\tau} - \gamma_{\rm nl}^2\right)\delta\chi_s = 0 ,$$

where

$$\gamma_{\rm nl}^2 \equiv \left[\frac{1}{8k_0^2}\frac{A_0^2}{(1+A_0^2/2)^2}\right]^{1/2} \tag{21}$$

is the nonlinear growth rate. The nonlinear evolution of RFS can be obtained by simply relacing γ_0 with $\gamma_{\rm nl}$ in Eq. (13).

An interesting feature of $\gamma_{\rm nl}$ is that it asymptotically approaches 0 as $A_0 \to \infty$. As a result, there can be strongly coupled temporal growth of RFS because the ratio $\gamma_{\rm nl}/\omega_{p\rm nl} \to 1/2k_0 \ll 1$ as $A_0 \to \infty$. The physical reason can be seen as follows: The ponderomotive force is pro-

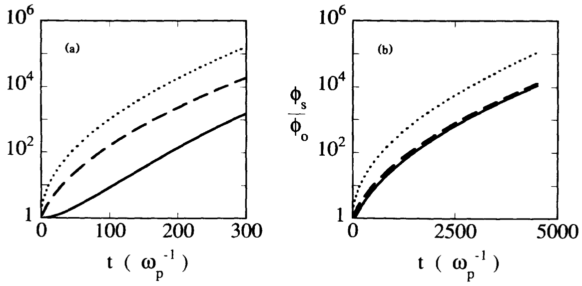

FIG. 1. The growth of 4-wave RFS for (a) $a_0=0.8$, $\omega_0/\omega_p=10$, $\psi_m=150$, and (b) $a_0=0.1$, $\omega_0/\omega_p=5.7$, and $\psi_m=150$. The dotted line, dashed line, and solid line represent the asymptotic, quasistatic, and exact solutions, respectively.

portional to $F_p=\nabla\gamma\propto(1/\gamma)\nabla A_0^2$ where ∇A_0^2 is the lowest order ponderomotive force. As A_0 increases the ponderomotive force decreases from the lowest order value because γ increases and $\nabla\propto k$ decreases. The decrease in k occurs because ω/k must still equal c and $\omega=\omega_{pnl}=\omega_p/\gamma$. The ponderomotive force therefore saturates. However, the Raman coupling is proportional to ∇F_p which asymptotically decreases as k decreases. The scaling of the nonlinear growth rate Eq. (21) has been verified in particle-in-cell simulations [12].

To illustrate the consequences of the results in this Letter, we consider two examples. The first is for the current experimental conditions at the (5 J)/(0.8 ps)(1 μm) laser at Livermore [7]. We assume the laser pulse is propagated through a plasma with density n_e-10^{19} cm^{-3} with a spot size of $\sigma=20$ μm. These numbers correspond to $a_0=0.8$, $\omega_0/\omega_p=10$, $\psi_m=150$, and $\tau_R=1.5$ mm/c. In Fig. 1(a), we plot the exact solution, the quasistatic solution, and the asymptotic solution as functions of τ. This plot demonstrates that using the asymptotic expression can significantly overestimate the number of e-foldings. However, after propagating for a time $\tau=300\omega_p^{-1}$, which corresponds to 0.3 Rayleigh length, the instability has amplified by 10^3 and should be observable.

The second example is for parameters relevant to proposed recombination x-ray lasers. A condition for gain is that the plasma produced by the ionizing laser must have a temperature less than 50 eV. Therefore, RFS needs to be prevented because it produces hot plasmas. We use $I=2.2\times10^{17}$ W/cm^2, $\lambda_0=0.25$ μm, a pulse length of 114 fs, and $n_e=5\times10^{20}$ cm^{-3}. These parameters give $a_0=0.1$, $\omega_0/\omega_p=5.7$, and $\psi_m=150$. For sufficient gain, the laser must propagate [18] 0.1 cm which corresponds to a $\tau=4500$. In Fig. 1(b) we show that the three types of solutions are nearly equal and that we can expect an amplification of any noise by a factor of 10^4. Thus, it would appear that RFS could pose a problem for recombination x-ray lasers.

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[1] D. W. Forslund, J. M. Kindel, and E. L. Lindman, Phys. Fluids **18**, 1002 (1975); K. Estabrook and W. L. Kruer, Phys. Fluids **26**, 1892 (1983).
[2] R. E. Turner *et al.*, Phys. Rev. Lett. **57**, 1725 (1986); D. M. Villenueve and H. A. Baldis, Phys. Fluids **31**, 1790 (1988); S. H. Batha *et al.*, Phys. Rev. Lett. **66**, 2324 (1988).
[3] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[4] D. W. Forslund *et al.*, Phys. Rev. Lett. **54**, 558 (1985); S. C. Wilks, *et al.*, Phys. Rev. Lett. **62**, 2600 (1989); C. J. McKinstrie and R. Bingham, Phys. Fluids B **4**, 2626 (1992).
[5] C. Joshi *et al.*, Phys. Rev. Lett. **47**, 1285 (1981).
[6] P. Maine *et al.*, IEEE J. Quantum Electron. **24**, 398 (1988).
[7] C. B. Darrow *et al.*, Phys. Rev. Lett. **69**, 442 (1992).
[8] A. Bers, in *Handbook of Plasma Physics*, edited by M. N. Rosenbluth and R. Z. Sagdeev, Basic Plasma Physics Vol. 1, edited by A. A. Galeev and R. N. Sudan (North-Holland, New York, 1983), p. 451.
[9] T. M. Antonsen and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992); Phys. Fluids B **5**, 1440 (1993).
[10] P. Sprangle *et al.*, Phys. Rev. Lett. **69**, 2200 (1992); J. Krall *et al.*, Phys. Rev. E **48**, 2157 (1993).
[11] P. Sprangle *et al.*, Phys. Rev. Lett. **64**, 2011 (1990).
[12] C. D. Decker *et al.* (to be published).
[13] E. A. Williams and R. R. McGowan, in *Inertial Confinement Fusion*, edited by K. A. Breckner (AIP, New York, 1992), p. 325; D. L. Bobroff and H. A. Haus, J. Appl. Phys. **38**, 390 (1967).
[14] D. Pesme, International Laser Matiere report, 1986, pp. 21–32; see National Technical Information Service Document No. PB92100304.
[15] P. Sprangle *et al.*, IEEE Trans. Plasma Sci. **15**, 145 (1987); C. E. Max *et al.*, Phys. Rev. Lett. **33**, 209 (1974).
[16] G. Shvets and J. Wurtele, Bull. Am. Phys. Soc. **38**, 1988 (1993); E. Esarey *et al.*, Bull. Am. Phys. Soc. **38**, 2081 (1993).
[17] W. B. Mori *et al.*, IEEE Trans. Plasma Sci. **21**, 110 (1993); J. M. Rax and N. J. Fisch, Phys. Rev. Lett. **69**, 772 (1992); E. Esarey *et al.*, IEEE Trans. Plasma Sci. **21**, 95 (1993).
[18] P. Amendt, Phys. Rev. Lett. **66**, 2589 (1991); D. C. Eder *et al.*, Phys. Rev. A **45**, 6761 (1992).