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Bosonization of the Low Energy Excitations of Fermi Liquids

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We bosonize the low energy excitations of Fermi liquids in any number of dimensions in the limit of long wavelengths. The bosons are a coherent superposition of electron-hole pairs and are related with the displacements of the Fermi surface in some arbitrary direction. A coherent-state path integral for the bosonized theory is derived and it is shown to represent histories of the shape of the Fermi surface. The Landau theory of Fermi liquids can be obtained from the formalism in the absence of nesting of the Fermi surface and singular interactions. We show that the Landau equation for sound waves is exact in the semiclassical approximation for the bosons.

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Attempts to describe fermionic systems by bosons date to the early days of second quantization. In the early 1950s Tomonaga [1], generalizing earlier work by Bloch [2] on sound waves in dense Fermi systems, gave an explicit construction of the Bloch waves for systems in one dimension. His work was subsequently generalized by many authors [3] who derived an explicit Fermi-Bose transmutation in one-dimensional systems. These works uncovered deep connections in relativistic field theories (both fermionic and bosonic) and with condensed matter systems.

The success of the bosonization approach in one dimension is related to phase space considerations. Even for noninteracting fermions, two excitations with arbitrarily low energies moving in the same direction move at the same speed (the Fermi velocity) and, hence, are almost a bound state. Consequently, even the weakest interactions can induce dramatic changes in the nature of the low-lying states. These effects are detected even in perturbation theory and result in the presence of marginal operators. In dimensions higher than one, phase space considerations change the physics of the low-lying states and there are no relevant or marginally relevant operators left. This observation is at the root of the stability of the Landau theory of the Fermi liquid [4]. It is, thus, hardly surprising that very few attempts have been made to generalize the bosonization approach to dimensions higher than one [5]. The earliest serious attempt at bosonization in higher dimensions was carried out by Luther [6]. Luther constructed a generalized bosonization formula in terms of the fluctuations of the Fermi sea along radial directions in momentum space for the noninteracting problem. Interest in the construction of bosonized versions of Fermi liquids has been revived recently in the context of strongly correlated systems [7]. That particle-hole excitations have bosonic character is well known since the early days of the Landau theory, e.g., the sound waves (zero sound collective modes) of the Fermi surface of neutral liquids or plasmons in charged Fermi liquids [8]. Haldane [9] has recently derived an algebra for density fluctuations of a Fermi liquid in the form of a generalized Kac-Moody algebra (see also Ref. [10]).

The main point of this Letter is to derive a description of the Fermi liquids as the physics of the dynamics of the Fermi surface. The main reason to believe that the Fermi surface is a *real* dynamical entity is based on the following observation: The total energy of an interacting electronic system can be written as an integral in momentum space of the form [11] $E = \sum_{\mathbf{k}} E_{\mathbf{k}}$. For the case of Fermi liquids, $E_{\mathbf{k}}$ has a discontinuity at the Fermi surface (exactly as for the case of the occupation number [12]). We can think of this discontinuity as due to the difference of energy density across the Fermi surface [13]. Hence, we can define an energy per unit area of the Fermi surface, i.e., a surface tension. It readily follows that this surface tension is proportional to the quasiparticle residue and therefore it vanishes for non-Fermi-liquid behavior. From this point of view we see the Fermi sur-

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face as a drumhead where the elementary excitations are the sound waves (and in particular the zero sound) which propagate on it. In this paper we will derive an effective bosonized theory for the dynamics of these excitations.

Our starting point to approach this problem resembles the microscopic approaches for the foundations of Fermi liquid theory [14]. However, instead of working with the dynamics of the response functions, we will work directly with the dynamics of operators as in the standard procedures of bosonization in one dimension. Our main result is the derivation of a bosonized theory of the fluctuations of the shape of the Fermi surface. The main ingredients of our construction are an effective algebra for the local (in momentum space) particle-hole operators valid in a Hilbert space restricted to the vicinity of the Fermi surface and a boson coherent-state path integral constructed from these states. In particular, we get a bosonized version of Landau's theory of the Fermi liquid in all dimensions. The coherent-state path integral can be viewed as a sum over the histories of the shape of the Fermi surface, a bosonic *shape field.* The Landau theory can be found in the absence of nesting of the Fermi surface and singular interactions between the fermions as resulting from a bosonic action which is quadratic in the shape fields. Moreover, the semiclassical approximation yields the Landau equation for the sound modes.

For simplicity we will consider a system of interacting spinless fermions (the spin index can be introduced without problems in the Abelian version of the formalism). In this paper we will concentrate on operators of the form [14]

$$n_{\mathbf{q}}(\mathbf{k},t) = c^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(t)c_{\mathbf{k}+\mathbf{q}/2}(t), \tag{1}$$

where $c^{\dagger}_{\mathbf{k}}$ and $c_{\mathbf{k}}$ are the creation and annihilation operator of an electron at some momentum $\mathbf{k}$ which obey the fermionic algebra, $\left\{c^{\dagger}_{\mathbf{k}}, c_{\mathbf{k}'}\right\} = \delta_{\mathbf{k},\mathbf{k}'}$, where $\{\cdots\}$ is the anticommutator and all other anticommutation relations are zero. In particular, $n_0(\mathbf{k})$ is the number operator in momentum space.

We are interested in the behavior of systems with a Hilbert space restricted to the vicinity of the Fermi surface where $\mathbf{k}$ is at some Fermi momentum $\mathbf{k}_F$ and long wave fluctuations around this point. For the moment we are not going to discuss processes related with large momentum transfer such as backward or umklapp scattering which lead inevitably to nonlinear theories as is well known from the bosonization in one dimensional systems [15]. This is an open problem which must be investigated further [16]. Moreover, we assume that the Fermi surface has always finite curvature (absence of nesting) and that the interactions are regular at the Fermi surface. Our aim is to discuss only the Gaussian fixed point associated with the Landau theory. In the limit of long wave fluctuations around the Fermi surface the commutation relation between the operators defined in (1) can be easily obtained in the limit of $q \to 0$,

$$\begin{aligned}[n_{\mathbf{q}}(\mathbf{k}), n_{-\mathbf{q}'}(\mathbf{k}')] = & -\delta_{\mathbf{k},\mathbf{k}'}\delta_{\mathbf{q},\mathbf{q}'}\mathbf{q}\cdot\nabla n_0(\mathbf{k}) \\ & +2n_0(\mathbf{k})\delta_{\mathbf{q},-\mathbf{q}'}\mathbf{q}\cdot\nabla\delta_{\mathbf{k},\mathbf{k}'}. \end{aligned} \tag{2}$$

The terms ignored in (2) vanish as $\left(\frac{q}{k_F}\right)^2$ as the size of the Fermi surface diverges or, what is equivalent, for operators which create pairs very close to the Fermi surface (i.e., small q) [17].

We define the Hilbert space to be the filled Fermi sea $|\mathrm{FS}\rangle$ and the tower of states obtained by acting finitely on it with local fermion operators. We will replace (2) by a weaker identity valid in the restricted Hilbert space. In order to do that we define a cutoff, Λ, in the direction normal to the Fermi surface. We will make the explicit assumption that we are both in the thermodynamic limit (the momenta form a continuum) and that the Fermi surface is *macroscopically large* ($q_N \ll \Lambda \ll p_F$, where q_N is the normal component of $\mathbf{q}$). Observe that no restrictions are imposed on fluctuations tangent to the Fermi surface. In the restricted Hilbert space it is possible to replace the right hand side (r.h.s.) of (2) by its expectation value in the filled Fermi sea $|\mathrm{FS}\rangle$

$$\begin{aligned} n_0(\mathbf{k}) &\to \langle n_0(\mathbf{k})\rangle = \Theta(\mu-\epsilon_{\mathbf{k}}), \\ \nabla n_0(\mathbf{k}) &\to \nabla\langle n_0(\mathbf{k})\rangle = -\mathbf{v}_{\mathbf{k}}\delta(\mu-\epsilon_{\mathbf{k}}), \end{aligned} \tag{3}$$

where μ is the chemical potential, $\epsilon_{\mathbf{k}}$ is the one-particle fermion spectrum (from which the Hilbert space is constructed), and $\mathbf{v}_{\mathbf{k}} = \nabla\epsilon_{\mathbf{k}}$ the velocity of the excitations. These are exactly the same assumptions that enter in the construction in one dimension. Notice that the state $|\mathrm{FS}\rangle$, which is used to normal order the operators, is not necessarily the ground state of the system of interest (as in the one-dimensional case).

Hence, within this approximation the commutators of the operators $n_{\mathbf{q}}(\mathbf{k})$ become c numbers, namely,

$$[n_{\mathbf{q}}(\mathbf{k}), n_{-\mathbf{q}'}(\mathbf{k}')] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\mathbf{q},\mathbf{q}'}\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}}\delta(\mu-\epsilon_{\mathbf{k}}), \tag{4}$$

where we drop the last term in the r.h.s. of (2) because its matrix elements near the Fermi surface are down by powers of $\frac{\Lambda}{k_F}$.

We now normal order the operator (1) relative to the reference state as

$$a_{\mathbf{q}}(\mathbf{k}) = n_{\mathbf{q}}(\mathbf{k})\Theta(\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}}) + n_{-\mathbf{q}}(\mathbf{k})\Theta(-\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}}), \tag{5}$$

where $\Theta(x)$ is the usual step function. It is important to notice that, by construction, the operator defined in (5) annihilates the filled Fermi sea

$$a_{\mathbf{q}}(\mathbf{k}) \mid \mathrm{FS}\rangle = 0. \tag{6}$$

Moreover, from the commutation relations (4) we easily obtain

$$[a_{\mathbf{q}}(\mathbf{k}), a^{\dagger}_{\mathbf{q}'}(\mathbf{k}')t] =\mid \mathbf{q}\cdot\mathbf{v}_{\mathbf{k}} \mid \delta(\mu-\epsilon_{\mathbf{k}})\delta_{\mathbf{k},\mathbf{k}'}(\delta_{\mathbf{q},\mathbf{q}'}+\delta_{\mathbf{q},-\mathbf{q}'}). \tag{7}$$

Equations (6) and (7) show that the elementary excitations have bosonic character, electron-hole pairs, and are created and annihilated close to the Fermi surface by these operators, moreover, they span the Hilbert space of low energy. It is natural now to define the coherent state [18]

$$| u_{\mathbf{q}}(\mathbf{k})\rangle = U(\mathbf{k}) \mid \mathrm{FS}\rangle, \tag{8}$$

where

$$U(\mathbf{k}) = \exp\left(\sum_{\mathbf{q}} \frac{v_{\mathbf{k}}}{2 \mid \mathbf{q}\cdot\mathbf{v_k} \mid} u_{\mathbf{q}}(\mathbf{k})\, a^{\dagger}_{\mathbf{q}}(\mathbf{k})\right), \tag{9}$$

where in the sum (and all other sums that follow) is implicitly assumed that $q \neq 0$ since we want the fluctuations with respect to the background. Observe that from the definition (5) we have $a^{\dagger}_{-\mathbf{q}}(\mathbf{k}) = a^{\dagger}_{\mathbf{q}}(\mathbf{k})$ and we choose $u_{\mathbf{q}}(\mathbf{k}) = u_{-\mathbf{q}}(\mathbf{k})$ for simplicity. Using this property and the commutation relation (7) we find

$$U^{-1}(\mathbf{k})\, a_{\mathbf{q}}(\mathbf{k})\, U(\mathbf{k}) = a_{\mathbf{q}}(\mathbf{k}) + \delta(\mu - \epsilon_{\mathbf{k}}) v_{\mathbf{k}} u_{\mathbf{q}}(\mathbf{k}) \tag{10}$$

which, together with (6), leads us to the eigenvalue equation

$$a_{\mathbf{q}}(\mathbf{k}) \mid u_{\mathbf{q}}(\mathbf{k})\rangle = \delta(\mu - \epsilon_{\mathbf{k}})\, v_{\mathbf{k}}\, u_{\mathbf{q}}(\mathbf{k}) \mid u_{\mathbf{q}}(\mathbf{k})\rangle. \tag{11}$$

It is easy to see that $u_{\mathbf{q}}(\mathbf{k})$ is the displacement of the Fermi surface at the point $\mathbf{k}$ in the direction of $\mathbf{q}$. Indeed, suppose we change the shape of the Fermi surface at some point $\mathbf{k}$ by an amount $u(\mathbf{k})$. The occupation number changes up to leading order by $\delta\langle n_0(\mathbf{k})\rangle = -\frac{\partial\langle n_0(\mathbf{k})\rangle}{\partial \mathbf{k}} u(\mathbf{k}) = v_{\mathbf{k}}\delta(\mu - \epsilon_{\mathbf{k}})u(\mathbf{k})$ which is precisely the quantity which appears in (11). Hence, the coherent states of Eq. (8) represent *deformed Fermi surfaces* parametrized by the bosonic field $u_{\mathbf{q}}(\mathbf{k})$.

Next we define a many-body state which is given by

$$| \{u\}\rangle = \prod_{\mathbf{k}} \otimes U(\mathbf{k}) \mid \mathrm{FS}\rangle = \Xi[u] \mid \mathrm{FS}\rangle, \tag{12}$$

where, due to the commutation relation at different $\mathbf{k}$'s,

$$\Xi[u] = \exp\left(\sum_{\mathbf{k},\mathbf{q}} \frac{v_{\mathbf{k}}}{2 \mid \mathbf{q}\cdot\mathbf{v_k} \mid} u_{\mathbf{q}}(\mathbf{k})\, a^{\dagger}_{\mathbf{q}}(\mathbf{k})\right). \tag{13}$$

The adjoint is simply

$$\Xi^{\dagger}[u] = \exp\left(\sum_{\mathbf{k},\mathbf{q}} \frac{v_{\mathbf{k}}}{2 \mid \mathbf{q}\cdot\mathbf{v_k} \mid} u^{*}_{\mathbf{q}}(\mathbf{k})\, a_{\mathbf{q}}(\mathbf{k})\right). \tag{14}$$

From the above equations we obtain the overlap of two of these coherent states,

$$\langle \{w\} \mid \{u\}\rangle = \langle \mathrm{FS} \mid \Xi^{\dagger}[w]\Xi[u] \mid \mathrm{FS}\rangle = \exp\left(\sum_{\mathbf{k},\mathbf{q}} \frac{v^2_{\mathbf{k}}\delta(\mu - \epsilon_{\mathbf{k}})}{2 \mid \mathbf{q}\cdot\mathbf{v_k} \mid} w^{*}_{\mathbf{q}}(\mathbf{k})\, u_{\mathbf{q}}(\mathbf{k})\right). \tag{15}$$

It is also possible to find the resolution of the identity for this Hilbert space,

$$1 = \int \cdots \int \prod_{\mathbf{k},\mathbf{q}} \left(\frac{v^2_{\mathbf{k}}\delta(\mu - \epsilon_{\mathbf{k}})}{2\pi \mid \mathbf{q}\cdot\mathbf{v_k} \mid} du_{\mathbf{q}}(\mathbf{k})\, du^{*}_{\mathbf{q}}(\mathbf{k}) \mid u_{\mathbf{q}}(\mathbf{k})\rangle\langle u_{\mathbf{q}}(\mathbf{k}) \mid\right) \exp\left(-\sum_{\mathbf{k},\mathbf{q}} \frac{v^2_{\mathbf{k}}\delta(\mu - \epsilon_{\mathbf{k}})}{2 \mid \mathbf{q}\cdot\mathbf{v_k} \mid} \mid u_{\mathbf{q}}(\mathbf{k}) \mid^2\right) \tag{16}$$

and we conclude, as expected, that they are overcomplete [18].

In order to study the dynamics of these modes we can construct, from (15) and (16), a generating functional as a sum over the histories of the Fermi surface in terms of these coherent states in the form $Z = \int D^2u\, \exp\{i\, S[u]\}$ where S is the action whose Lagrangian density is given by ($\hbar = 1$)

$$L[u] = \sum_{\mathbf{k},\mathbf{q}} \frac{v^2_{\mathbf{k}}\delta(\mu - \epsilon_{\mathbf{k}})}{2 \mid \mathbf{q}\cdot\mathbf{v_k} \mid}\, i\, u^{*}_{\mathbf{q}}(\mathbf{k},t) \frac{\partial u_{\mathbf{q}}(\mathbf{k},t)}{\partial t} - \frac{\langle \{u\} \mid H \mid \{u\}\rangle}{\langle \{u\} \mid \{u\}\rangle}, \tag{17}$$

where H is the Hamiltonian written in the restricted Hilbert space.

Consider now the simplest Hamiltonian possible, for instance, one that is quadratic in the operators $a^{\dagger}$ and a,

$$H = \frac{1}{2V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} G^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'} a^{\dagger}_{\mathbf{q}}(\mathbf{k})\, a_{\mathbf{q}}(\mathbf{k}'), \tag{18}$$

where V is the volume of the system. Using (11) we can rewrite the Lagrangian density in the form

$$L[u] = \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \frac{v^2_{\mathbf{k}}\delta(\mu - \epsilon_{\mathbf{k}})}{2\,\mathbf{q}\cdot\mathbf{v_k}} u^{*}_{\mathbf{q}}(\mathbf{k},t) \left(i\delta_{\mathbf{k},\mathbf{k}'} \frac{\partial}{\partial t} - \frac{1}{V} W^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'} \delta(\mu - \epsilon_{\mathbf{k}'})\right) u_{\mathbf{q}}(\mathbf{k}',t), \tag{19}$$

where

$$W^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'} = \frac{\mathbf{q}\cdot\mathbf{v_k}\; v_{\mathbf{k}'}}{v_{\mathbf{k}}} G^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'}. \tag{20}$$

Notice that this action is quadratic in the fields and, therefore, the semiclassical approximation is exact. The semiclassical equations of motion for this action are given by the saddle point equation derived from L, namely,

$$i\frac{\partial u_{\mathbf{q}}(\mathbf{k},t)}{\partial t} = \frac{1}{V}\sum_{\mathbf{k}'} W^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'}\delta(\mu-\epsilon_{\mathbf{k}'})u_{\mathbf{q}}(\mathbf{k}',t), \tag{21}$$

a partial integral equation. Observe that the vectors $\mathbf{k}$ are at the Fermi surface and, therefore, the above equation only depends on the solid angle, Ω, and the velocity, $\mathbf{v}_{\mathbf{k}}$. We can therefore integrate over the magnitude of $\mathbf{k}'$ and use the fact that the density of states at the Fermi surface is given by $N(0)V = \sum_{\mathbf{k}}\delta(\mu-\epsilon_{\mathbf{k}})$. Therefore, in the thermodynamic limit we replace (21) by

$$i\frac{\partial u_{\mathbf{q}}(\Omega,t)}{\partial t} = \frac{N(0)}{S_d}\int d\Omega' W^{\mathbf{q}}_{\Omega,\Omega'}\, u_{\mathbf{q}}(\Omega',t), \tag{22}$$

where $S_d = \int d\Omega$. In particular, we choose

$$G^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'} = \frac{\delta_{\mathbf{k},\mathbf{k}'}}{N(0)} + f_{\mathbf{k}-\mathbf{q},\mathbf{k}'+\mathbf{q}} \tag{23}$$

and define the angle θ by $\mathbf{q}\cdot\mathbf{v}_{\mathbf{k}} = qv_F\cos\theta$. It is easy to see that Eq. (22) becomes

$$i\frac{\partial u_{\mathbf{q}}(\Omega,t)}{\partial t} = qv_F\cos\theta\, u_{\mathbf{q}}(\Omega,t) + qv_F\cos\theta\int\frac{d\Omega'}{S_d}F_{\Omega,\Omega'}\, u_{\mathbf{q}}(\Omega',t) \tag{24}$$

with $F_{\mathbf{k},\mathbf{k}'} = N(0)f_{\mathbf{k},\mathbf{k}'}$. Equation (24) is the Landau equation of motion for sound waves (the collective modes) of a neutral Fermi liquid where $f_{\mathbf{k},\mathbf{k}'}$ is the scattering amplitude for particle-hole pairs [14].

To understand the reason for this result we rewrite the Hamiltonian H in terms of the original operators (1). Using (5) we find

$$H = \frac{1}{2V}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} G^{\mathbf{q}}_{\mathbf{k},\mathbf{k}'}n_{-\mathbf{q}}(\mathbf{k})\, n_{\mathbf{q}}(\mathbf{k}'). \tag{25}$$

Observe that the Hamiltonian is interacting in terms of the original electrons. We can show [13] that the first term in (23) gives the noninteracting part of the Hamiltonian which is related with the particle-hole continuum and the second term is due to the interaction between the electrons.

We conclude therefore that our bosons represent vibrations which propagate around the Fermi surface. The solution of (24) gives the possible values for the frequencies of oscillation for these modes and they will depend essentially on the Landau parameters of the theory. The Landau equation, (24), yields solutions which represents both stable collective modes ("sounds") as well as the particle-hole continuum. This behavior is a direct consequence of the phase space. Notice that in one dimension only the collective mode is left.

The generating functional introduced in (17) is a powerful tool and we can generate all the results of the Landau theory of Fermi liquids [13]. We can also rewrite the theory in the imaginary time and calculate thermodynamic properties. In particular, the specific heat is shown to be exactly the one expected in the Fermi liquid theory [13]. Besides the Landau theory we can also use our formalism to discuss possible new fixed points not described in this paper. This is, for instance, the case of presence of nesting of the Fermi surface or the presence of singular interactions (such as the ones mediated by gauge fields) [16].

In summary, in this paper we have shown that it is possible to bosonize the particle-hole elementary excitations at small momentum transfer of a Fermi liquid in any number of dimensions consistently with the Landau theory. We have shown that the bosons are a coherent superposition of particle-hole pairs close to the Fermi surface. We also have shown that the coherent states associated with these excitations are related with waves which propagate in the Fermi surface. Our approach gives for simple Hamiltonian models, in the semiclassical limit, the Landau equation which describes the propagation of these sound waves.

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