Alpha-Cluster Structure in ^{212}Po

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We investigate the α- and γ-decay properties of states belonging to the ground state "band" of ^{212}Po, using an α-cluster model with parameter values taken from an earlier study of α decay of heavy nuclei. The known lifetimes of these states, which vary from 5.5×10^{-10} s to 45 s, are reproduced to within a factor of $\sim$2. The (approximately) known α branching ratios, which vary from 3.3×10^{-4} to 0.71, are reproduced to within a factor $\sim$5. These results strongly suggest a large amount of α clustering in this heavy nucleus.

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Recently Varga *et al.* [1] calculated the half-life for α decay from the ground state of ^{212}Po in a microscopic model which incorporates both explicit shell-model and cluster-model configurations. They found that a significant α-cluster component of ~ 0.3 in the ground state of ^{212}Po was necessary to produce a result in accord with experiment. This immediately raises the question of whether other states of ^{212}Po also show evidence of α-cluster structure.

We have shown [2] that the large body of data on lifetimes for favored α decay of heavy nuclei can be reproduced satisfactorily using a simple α-cluster model. We now apply the same model to calculate lifetimes for the α and γ decay of members of the low-lying positive parity "band" of ^{212}Po, to determine whether these states can be adequately described as α-cluster states. Members of this band are described in our model as ^{208}Pb–α-cluster states having a common value of $G = 2n + L$, where n is the number of nodes and L the orbital angular momentum of the cluster-core relative motion wave function. The energies and spins of the experimentally known band members are given in Table I (N.B. some as yet undetected states with $J^\pi = 12^+, 14^+, 16^+$ are also predicted). The presence of an $L = 18$ member indicates that G must have at least this magnitude. A large value of G is consistent with the expectation that the α cluster must be built up from valence nucleons in orbit above the Fermi surface of ^{208}Pb so that the Pauli principle is satisfied. Thus G is taken to be near the value $\sum_i 2n_i + L_i$, where n_i and L_i are the quantum numbers of the α-cluster component nucleons. It is, however, difficult to pin down a unique value for G.

The model uses an α-core nuclear potential of the form

$$V_N(r) = -V_0 \frac{1 + \cosh(R/a)}{\cosh(r/a) + \cosh(R/a)} \tag{1}$$

with depth V_0, diffuseness a, and radius R. The α-core Coulomb potential is taken to be that for a point α particle of charge Z_1 interacting with a uniformly charged spherical core of radius R and charge Z_2,

$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{r} & \text{for } r \geq R, \\ \frac{Z_1 Z_2 e^2}{2R} \left[3 - \left(\frac{r}{R}\right)^2\right] & \text{for } r \leq R. \end{cases} \tag{2}$$

The total α-core potential is thus

$$V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2}(L + 1/2)^2, \tag{3}$$

where we use a Langer modified centrifugal term appropriate to our semiclassical description of α decay. In our model the values of the nuclear potential depth V_0 and diffuseness a are kept constant for all decays [2]. The value of the remaining potential parameter R is tailored to each decay in turn, as discussed below, so as to satisfy the Bohr-Sommerfeld condition for a quasistationary state at the appropriate decay energy Q.

For all decays considered here we choose a fixed value of the global quantum number $G = 2n + L$, where n is the number of nodes in the radial wave function for relative motion, and L is the orbital angular momentum. A quasibound state specified by G, L, and the decay energy Q satisfies the condition

$$\int_{r_1}^{r_2} dr \sqrt{\frac{2\mu}{\hbar^2}[Q - V(r)]} = (2n+1)\frac{\pi}{2} = (G-L+1)\frac{\pi}{2}, \tag{4}$$

where r_1, r_2 (and r_3) are the classical turning points in order of increasing distance from the origin. The values of R and of the turning points can be determined from Eq. (4) using the condition $V(r) = Q$, for fixed values of V_0 and a.

In semiclassical approximation [3], the α-decay width Γ_α is given by

$$\Gamma_\alpha = PF\frac{\hbar^2}{4\mu} \exp\left[-2\int_{r_2}^{r_3} dr k(r)\right], \tag{5}$$

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TABLE I. Excitation energies E^* and internal conversion coefficients α_T of states belonging to the ground state "band" of ^{212}Po [5]. Values of the potential radius R, and α- and γ-decay widths Γ_α and Γ_γ, respectively, have been calculated using the parameter values of Eq. (8).

J^π	E^* (MeV)	α_T	R (fm)	Γ_α (MeV)	Γ_γ (MeV)
0^+	0.000	$\cdots$	7.058	3.80×10^{-15}	0
2^+	0.727	0.014	7.022	8.00×10^{-14}	5.19×10^{-11}
4^+	1.133	0.055	6.985	1.30×10^{-13}	3.93×10^{-12}
6^+	1.356	0.329	6.942	4.15×10^{-14}	2.49×10^{-13}
8^+	1.476	3.310	6.887	4.15×10^{-15}	3.38×10^{-14}
10^+	1.834	0.077	6.808	4.91×10^{-16}	1.56×10^{-12}
(18^+)	2.922	$\gg 1$	6.449	5.65×10^{-24}	$0^{\rm a}$

[a]Not calculated. Multipolarity of primary transition uncertain, with $\Gamma_\gamma \ll \Gamma_\alpha$ [5].

where, in accordance with our earlier treatment [2], we set the preformation probability $P = 1$. The normalization factor F may be approximated by

$$F \int_{r_1}^{r_2} \frac{dr}{2k(r)} = 1, \tag{6}$$

where the wave number $k(r)$ in Eqs. (5) and (6) is

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}\,|Q - V(r)|}. \tag{7}$$

Given the appropriate Q value, and having set the values of the parameters V_0, a, and G as described below, the formalism enables us to calculate the width Γ_α for α emission from an initial state (G, L_i) belonging to the ground state band $L_i = 0^+, 2^+, 4^+, \ldots$ of ^{212}Po, to the ground state of ^{208}Pb. The values of V_0, a, and G are taken from our best fit to α-decay lifetimes for heavy nuclei [2],

$$V_0 = 152.5 \text{ MeV}, \quad a = 0.55 \text{ fm}, \quad G = 20, \tag{8}$$

so that we have no free parameters in the present calculation.

Considering next the $E2$ decays from initial states (G, L_i) to final states (G, L_f) of this ground state band, we calculate the reduced matrix elements

$$B(E2;\ G, L_i \to L_f) = \left|\langle G, L_f \| \beta_2 r^2 Y_2^0(\theta,\phi) \| G, L_i\rangle\right|^2, \tag{9}$$

where the recoil coefficient [4] has the form

$$\beta_2 = \frac{Z_1 A_2^2 + Z_2 A_1^2}{(A_1 + A_2)^2} \tag{10}$$

with (Z_1, A_1) and (Z_2, A_2) the (charge, mass) of the cluster and core, respectively. The radial wave function for each state (G, L) is obtained by solving the Schrödinger equation using a potential $V(r)$ identical to the one used above for α decay from the same state. For the electromagnetic quadrupole transition ($E2$) width we have

$$\Gamma_\gamma(\text{MeV}) = \frac{\hbar E_\gamma^5 (1 + \alpha_T) B(E2;\ L_i \to L_f,\ \ e^2\,\text{fm}^4)}{8.156 \times 10^{-10}}, \tag{11}$$

where $\hbar = 6.582 \times 10^{-22}$ MeV s, E_γ is the γ-ray energy in MeV, and α_T is the appropriate internal conversion coefficient.

Table I lists the experimental information [5] on the known members of the ground state band of ^{212}Po together with the calculated values of R, Γ_α, and Γ_γ. The Q value for α decay from a state (G, L) in ^{212}Po to the ground state of ^{208}Pb is given by

$$Q(G, L) = Q(G, 0) + E^*(G, L), \tag{12}$$

with $Q(G, 0) = 8.986$ MeV, and $E^*(G, L)$ the excitation energy of the state in ^{212}Po. This, together with the internal conversion coefficients α_T and the parameter values of Eq. (8), is sufficient to obtain values for R, Γ_α, and Γ_γ within our model.

We note that these values of R change from state to state, pointing to a deficiency in the shape assumed in Eq. (1) for the α-core nuclear potential. Our technique of fitting R to Q is designed to compensate for such inadequacies by assuring that the condition for a quasistationary state at the required energy is satisfied. Although this technique has proved successful in describing the α decay of heavy nuclei [2], it would be more consistent to find an improved form of the α-core nuclear potential that would result in unchanged values of R for α-cluster states in the same nucleus. Preliminary investigations of this point are promising [6], and confirm the validity of our conclusions using the present method and potential.

In terms of the widths Γ_α and Γ_γ, the half-life $T_{1/2}$ and α branching ratio b_α are given by

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_\alpha + \Gamma_\gamma}, \qquad b_\alpha = \frac{\Gamma_\alpha}{\Gamma_\alpha + \Gamma_\gamma}. \tag{13}$$

Using the values of the widths from Table I, we calculate $T_{1/2}$ and b_α, and in Table II compare our results with the corresponding experimental values. For the half-lives $T_{1/2}$ the agreement is good (to within a factor ~ 2) and this enables us to make confident predictions of $T_{1/2}$ for the 2^+ and 4^+ members of the band. For the branching ratios, although we reproduce the general trend, the agreement is substantially poorer (to within a factor

TABLE II. Excitation energies E^*, experimental half-lives $T_{1/2}$(expt), and α-branching ratios b_α(expt) of states belonging to the ground state "band" of ^{212}Po [5]. Theoretical values $T_{1/2}$(theor) and b_α(theor) have been calculated from Eq. (13) using the results for Γ_α and Γ_γ in Table I.

J^π	E^* (MeV)	$T_{1/2}$(expt)	$T_{1/2}$(calc)	b_α (%) (expt)	b_α (%) (theor)
0^+	0.000	0.30 μs	0.12 μs	100	100
2^+	0.727	⋯	8.8 ps	0.033	0.15
4^+	1.133	⋯	0.11 ns	$\sim$ 27	3.2
6^+	1.356	0.76 ± 0.21 ns	1.6 ns	$\sim$ 71	14
8^+	1.476	17 ns	12 ns	$\sim$ 42 (or 6 ± 1)	11
10^+	1.834	0.55 ± 0.14 ns	0.29 ns	no α particle detected	0.036
(18^+)	2.922	45 s	81 s	99.93	100

$\sim$ 5). We note, however, that the branching ratios are either very small, or imperfectly known [5], and further experimental work may well reduce these discrepancies. We note further that our predictions for the lifetimes are independent of the observed values of these branching ratios.

To summarize these results: Our agreement with the observed lifetimes of the 0^+ (g.s.) and (18^+) (2.922 MeV) states depends solely on the values of Γ_α obtained for these states, whereas that for the 10^+ (1.834 MeV) state depends solely on the corresonding value of Γ_γ. For the 6^+ (1.356 MeV) and 8^+ (1.476 MeV) states both Γ_α and Γ_γ play a role, with the latter predominant in our calculation. Thus our simple α-cluster model, without reliance on adjustable parameters or effective charges, exhibits a remarkable degree of internal consistency and agreement with experiment for the properties of the low-lying positive parity states of ^{212}Po. On the other hand, shell-model calculations [7] (which throw no light on the α widths) obtain γ widths which are a factor $\sim$ 10 too small, even when considerable effective charges $e_p = 1.5$ and $e_n = 0.9$ are used.

We have checked the stability of our calculated numbers by replacing the parameter values of Eq. (8) by other sets [2]. All our results are stable against such changes, with the exception of that for the lifetime for α decay of the (18^+) state. This varies considerably (by a factor of $\sim$ 5) depending on the input, in part because of the rapidly changing value of the pre-exponential factor F in Eq. (6) with node number, for small node number. Notwithstanding this reservation, an assignment of $J^\pi = 16^+$ or 20^+ to the 2.922 MeV isomeric state of ^{212}Po results in lifetimes which differ by a factor $\sim 10^3$ from the observed value, so that we have strong confirmation of the (18^+) assignment to this state.

In conclusion, an α-cluster model, with parameter values which optimize the fit to α-decay lifetimes for a wide range of nuclei [2], results in good agreement with the α- and γ-decay properties of states belonging to the ground state band of ^{212}Po. This, taken together with a recent result for the properties of the ground state [1], is strong evidence for a large amount of α clustering for these states, similar to that already established for light nuclei [8].

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