Quasi Fermi Distribution and Resonant Tunneling of Quasiparticles with Fractional Charges

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We study the resonant tunneling of quasiparticles (anyons) through an impurity between the edges of a fractional quantum Hall sample. These quasiparticles are anyons, but their equilibrium one-particle correlation functions are shown to have some quasi Fermi properties. The broken symmetry of many-particle states at the impurity yields a new selection rule for the resonant tunneling: The resonance is suppressed unless an integer number of *electrons* occupies the impurity. This rule allows an explanation of the scaling behavior observed in the mesoscopic fluctuations of the conductivity in the fractional quantum Hall effect.

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The question to what extent the fractionally charged quasiparticles proposed by Laughlin [1] are real, and whether they can be observed individually, was recently treated experimentally [2,3]. In these experiments the fractional quantum Hall effect (FQHE) samples with a constriction were studied, in order to observe the essentially one-particle tunneling processes of the quasiparticles. In particular, the frequencies of mesoscopic fluctuations of the longitudinal resistance in the FQHE with $\nu = 1/3$ were compared to those in the integer QHE (IQHE).

Previously, Jain and Kivelson [4] suggested that the resonant tunneling of electrons from one edge to another through an impurity could cause an enhancement of the dissipative resistance in the IQHE samples with a narrow constriction. Kivelson and Pokrovsky [5] proposed an analogous mechanism for the fractionally charged quasiparticles in the FQHE. Their model implied simple scaling laws for periods of the mesoscopic oscillations in the vicinity of the state with filling factor $\nu \equiv p/q$: $\Delta B \propto q$ at fixed gate voltage V_G, and $\Delta V_G \propto p$ at fixed magnetic field B. Both predictions have been confirmed experimentally [2,3].

Nevertheless, the theoretical understanding of this scaling cannot be considered satisfactory. In particular, Kivelson [6] derived quasiclassical quantization rules for a multianyon bound state at the impurity allowing for their statistical interaction; his scaling relations are different from the observed ones and from those of [5]. Lee [7] suggested that the difference could be explained if Coulomb interaction of the quasiparticles is dominant. Later experiments [3] have shown that it is not true.

Another problem in understanding these experiments is that the mechanism of resonant tunneling usually implicates the existence of the Fermi level for excitations. It clearly exists for the case of the IQHE but is much less obvious for the FQHE. Recently Haldane [8] defined the generalized Pauli principle for anyons. This principle, however, does not imply the existence of the distinct Fermi level required to explain the resonances in tunneling.

The purpose of this work is to elucidate these general questions and give a new explanation for the experimental result. We start from the one-particle semiclassical description of the resonant tunneling and then show that the same physical picture may be obtained from the analysis of the effective many-particle tunneling Hamiltonian of the system.

As we already mentioned, we suppose that the tunneling for anyons is permitted only near some quasi Fermi energy, corresponding to the potential near the edges of the sample. One would then expect that the quasiparticle tunneling from one edge of a narrow constriction to another edge is resonantly enhanced if it proceeds through a proper bound state of an impurity, described by some smooth potential relief. We believe that this potential distribution is mostly determined by the impurities and is the same near all FQHE states. We ignore the Coulomb interaction between quasiparticles accounting the Coulomb forces implicitly in the very fact of the existence of fractionally quantized Hall states and fractionally charged quasiparticles.

In a strong magnetic field the quasiparticles are moving along the lines of equal potential. Kivelson [6] proposed to treat the many-body bounded state at the impurity as consisting of one-particle orbits lying completely one inside another, so that the orbit of an additional quasiparticle encloses the flux,

$$\Phi = m\phi_0^* + N\phi_0, \tag{1}$$

where $\phi_0 = hc/e$ is the flux quantum for an electron, $\phi_0^* = q\phi_0$ is the flux quantum for a quasiparticle (anyon)

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with the charge $e^* = e/q$, N is the number of quasiparticles captured by the impurity, and m is the angular momentum of the tunneling quasiparticle. The first term on the right hand side of Eq. (1) is required by the gauge invariance, while the second one simply shows that each quasiparticle is bound with one flux quantum. The energy of N-anion states characterized by two integer parameters N and m, which semiclassical meaning is defined by (1), has been found exactly for a special case of quadratic impurity potential [9].

At a given gate voltage V_G, the area A enclosed in the trajectory corresponding to the Fermi level is the same for any quantized value of Hall conductivity. Therefore, the interval in the magnetic field between two consequent bound states of quasiparticle is $\Delta B_q = \phi_0^*/A$ if the number N of quasiparticles is fixed. The scaling, consistent with these intervals, was observed experimentally.

However, during the tunneling the number of quasiparticles N coupled with the impurity changes by one. It corresponds to the change of the flux Φ by a single flux quantum ϕ_0 instead of $\phi_0^* = q\phi_0$. Corresponding periods $\Delta B_1 = \phi_0/A$ have *not* been observed experimentally.

The solution of this puzzle lies in the fractional statistics of quasiparticles. Consider the situation when N quasiparticles are initially bound by the impurity, and the tunneling quasiparticle arrives at an orbit enclosing all of them. In the quasiclassical approximation, the wave function of this quasiparticle will gain a phase factor $z = \exp(2\pi i N/q)$ after each complete revolution over the quantized orbit. More accurately, it is multiplied by $z(1-\gamma/2)$, where $\gamma \ll 1$ is the total probability of tunneling from the impurity to either the left or right edge. The total tunneling amplitude contains a series

$$t_{LR} = \sum_k z^k (1-\gamma/2)^k = \frac{1}{1 - z(1-\gamma/2)}. \tag{2}$$

Usually the resonant enhancement of the tunneling happens when all the amplitudes corresponding to different numbers of revolutions in (2) are coherent, i.e., $z = 1$. This is obviously the case for the usual Fermi quasiparticles ($\nu = 1$). For a fractional value of ν, the contributions of q consequent revolutions almost cancel each other. Thus the *resonant* tunneling is suppressed unless N/q is an integer. In other words, the tunneling of an anyon is resonantly enhanced only if an integer number of electrons are already bound by the impurity. This simple selection rule restores the scaling suggested in Ref. [5] and is in agreement with the experiment.

The scaling of the oscillation intervals versus the gate voltage ΔV_G [5] is also easily reproduced. As we change the gate voltage, the geometry of the charge distribution changes, but this change is the same for all values of ν. At a fixed magnetic field B the change ΔV_G corresponding to a new resonance is determined by the change of the area

$$\Delta A = \Delta\Phi/B, \tag{3}$$

where $\Delta\Phi$ is the change of the flux through the trajectory to reach the next resonance. As we have already established, $\Delta\Phi = q\phi_0$ for the resonant tunneling at $\nu = p/q$. On the other hand, the value of the magnetic field B_ν, corresponding to the filling factor ν, is approximately $1/\nu$ times B_1. As a result we obtain $\Delta A_\nu = p\Delta A_1$ and $\Delta V_G^\nu = p\Delta V_G^1$.

These intuitive and semiclassical arguments are supported by direct calculations in the framework of Wen's theory of edge excitations [10]. Simultaneously we find the distribution of edge quasiparticles over momenta to confirm the conjecture on its Fermi-like character [5]. All calculations have been performed for special values of $\nu = 1/q$, where q is an odd integer.

In a restricted geometry without impurities, the only low-lying excitations are edge excitations [11]. In the simplest case $\nu = 1/q$, q odd, there exists only one boson branch of edge excitations, usually called magnetoplasmons. Physically, such excitation corresponds to a charge drifting along the edge in external magnetic field B and edge electric field E with the velocity $v = cE/B$. Because of the incompressibility of the FQHE liquid, any quasiparticle created in the bulk causes the deposit of a corresponding amount of charge at the edge, effectively changing the vacuum of edge phonons.

The Hamiltonian associated with such edge waves [12] is extremely simple: It is just a combination of a rotator, labeling different edge vacuum states, and an infinite set of oscillators, describing the edge magnetoplasmons,

$$\hat{H}_{\text{edge}} = \hbar\omega\nu\frac{\pi_0^2}{2} - e\nu\pi_0\mu + \hbar\omega\sum_{m=1}^{\infty} m a_m^\dagger a_m. \tag{4}$$

Here $\omega = 2\pi v/L$ is the classical frequency of rotation of the quasiparticle around the edge; the operator of "angular momentum" π_0, counting the increment of the number of quasiparticles from some reference state, obeys the usual commutation relationship with associated angular variables $[\phi_0, \pi_0] = i$, operators $a_n^\dagger, a_n$ are usual Bose operators for the edge magnetoplasmon with momentum $k_n = 2\pi n/L$, and L is the total length of the edge. Compared to the usual chiral edge Hamiltonian, we introduced the effective chemical potential μ, accounting for the possible shift of vacuum state with the change of the magnetic field or the gate voltage.

The operator of creation of quasiparticle

$$\psi^\dagger(x,t) \equiv :e^{i\phi(x,t)}: \tag{5}$$

at the point x of the edge is associated with the chiral Bose field

$$\phi(x,t) = \phi_0 + \pi_0\frac{2\pi}{Lq}(x - vt) + \sum_{n>0}\left(\frac{a_n^\dagger e^{ik_n(x-vt)} - a_n e^{-ik_n(x-vt)}}{i\sqrt{nq}}\right), \tag{6}$$

obeying the commutation relationship

$$\left[\phi_{x,t}, \frac{\partial \phi_{x',t}}{\partial x'}\right] = 2\pi i \nu\, \delta(x - x'), \qquad (7)$$

and related with the charge density $\rho = (e/2\pi)\,\partial\phi/\partial x$ at the edge. The permutation relations for anyon operators (5) are

$$\psi^\dagger(x,t)\psi^\dagger(x',t) = e^{i\pi\nu\,\mathrm{sgn}(x-x')}\psi^\dagger(x',t)\psi^\dagger(x,t). \qquad (8)$$

To find the distribution of edge quasiparticles over momenta, it is necessary to calculate the Fourier transformation $\tilde{G}_p$ of the simultaneous correlation function $G(x - x') = \langle \psi^\dagger(x,t)\psi(x',t)\rangle$. Straightforward calculation shows that $\tilde{G}_p$ can be represented as a product

$$\tilde{G}(p') = g_T(p')\frac{1}{\exp(\beta p' v) + 1}, \qquad (9)$$

where the momentum $p' = p - p_F$, p_F is the Fermi momentum, v is the drift velocity along the edge for both the chiral field and anyons, and

$$g_T(p') = \mathrm{const} \times \left(\frac{2\pi T}{v}\right)^{\nu-1} \mid \Gamma(\nu/2 + ip'v/2\pi T) \mid^2 \times \cosh(p'v/2T). \qquad (10)$$

The second factor in (9) is the usual Fermi distribution, while the first one can be treated as a temperature-dependent density of states; it is an even function of p'. At $T = 0$ the density of states $g_T(p')$ has a singularity $\propto |p'|^{\nu-1}$ [11] and diverges at the Fermi level. The singularity is smeared out at a finite temperature.

To investigate the tunneling, we model the impurity as an additional edge with associated Hamiltonian of the form (4). The perimeter L_i of this edge is supposed to be small enough, so that the minimal phonon energy $\hbar\omega_i$ is large compared to the temperature. The outer edge, on the contrary, is supposed to be in the thermodynamical limit; it serves as a thermostat.

The many-body quantum mechanical states at the impurity are well defined at the limit of a small tunneling coupling

$$H_t = \int dx\, dy\, t(x,y)\, \psi^\dagger(x)\psi_i(y) \,+\, \mathrm{H.c.}, \qquad (11)$$

where $\psi^\dagger(y)$ is the creation operator of quasiparticles at the outer edge and $\psi_i(y)$ is the annihilation operator for the edge quasiparticles at the impurity (inner edge). This allows us to reduce the evolution equations of the density matrix at the impurity to a set of kinetic equations, describing evolution for probabilities $W_N = \langle \hat{\mathcal{P}}_N \rangle$ to have exactly N quasiparticles at the impurity, where $\hat{\mathcal{P}}_N$ is the appropriate projection operator. Relationship (9) makes these equations look *exactly* like kinetic equations for fermions, thus explaining the success of a naive one-particle approach.

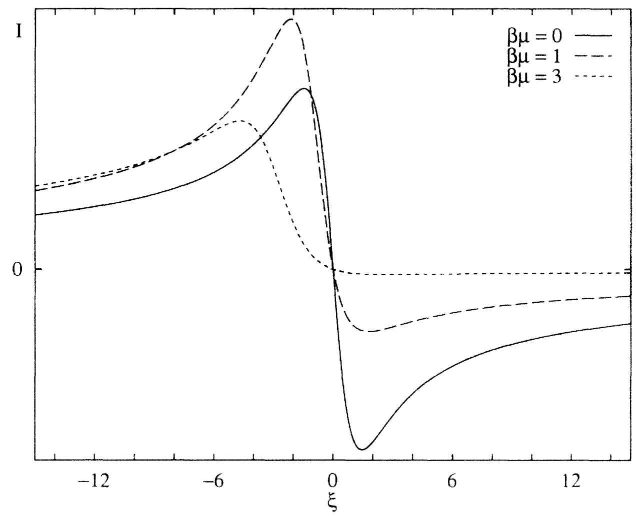

FIG. 1. Nonlinear resonant tunneling current I (arbitrary units) versus the interedge voltage $\xi = \beta V$ expressed in the units of temperature at different values of the bound state energy μ.

One would expect the resonant enhancement of the tunneling when the energy gap $\epsilon_N \equiv E_{N+1} - E_N = \nu[\hbar\omega(N+1/2) - e\mu_i]$ required to transfer the particle from the edge to the impurity is small compared with the interlevel distance. However, in the generic case it turns out that the equilibrium averages similar to $\langle \psi_i^\dagger(y)\psi_i(y')\hat{\mathcal{P}}_N\rangle$ acquire nonzero phase $2\pi N/q$ when one of the variables makes a complete loop around the boundary. This phase breaks the symmetry and is the Berry phase in this model; it is the exact consequence of the fractional statistics and does not depend on the distribution of edge phonons. As usual, the broken symmetry leads to a selection rule for the allowed transitions; namely transitions $N \longrightarrow N + 1$ are suppressed unless N/q is an integer. This statement coincides with our conclusion extracted from the semiclassical model. In contrast to the previous derivation, we made no assumptions about the geometrical properties of the orbit of the tunneling quasiparticle.

The factorization (9) grants the Boltzmann distribution for the probabilities of different states of the impurity in equilibrium. In the presence of the interedge potential difference, however, the impurity population depends on the tunneling probabilities. The current-voltage dependence (see Fig. 1) is highly nonlinear and asymmetric, especially in the vicinity of the resonance. Details of the calculations will be published elsewhere.

In conclusion, we have found new selection rules for the resonant tunneling of quasiparticles in the FQHE, arising from the broken symmetry specific to anyons. The equilibrium momenta distribution of the edge quasiparticles has quasi Fermi properties with the temperature-dependent density of states. This explains the appearance of the resonant tunneling effects in the anyonic systems and, in particular, the scaling properties of the mesoscopic pattern measured in the experiment [2,3].

Upon the completion of this work, we received the preprint [13], where the renormalization group equations for the problem of resonant interedge tunneling are solved numerically in a different geometry. The authors did not consider either scaling properties of the resonant effects or the momentum distribution of quasiparticles. Their main emphasis was the line shape.

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