Time and Frequency Domain Analysis of Superradiant Coherent Synchrotron Radiation in a Waveguide Free-Electron Laser

A. Gover
Faculty of Engineering, Tel Aviv University, Ramat-Aviv 69978, Israel

F. V. Hartemann, G. P. Le Sage, and N. C. Luhmann, Jr.
Department of Electrical Engineering, University of California, Los Angeles, California 90024

R. S. Zhang and C. Pellegrini
Department of Physics, University of California, Los Angeles, California 90024
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The coherent synchrotron radiation process in a waveguide is theoretically investigated. A single, short bunch propagating through a wiggler is considered. In a waveguide, two very distinct regimes are possible. At grazing, where the beam velocity matches the wave group velocity, the bunch emits a single, ultrashort chirped pulse whose duration is determined by the interaction bandwidth and the waveguide dispersion. Away from grazing, where slippage dominates, two distinct pulses are radiated at the Doppler upshifted and downshifted frequencies. Both the time and frequency domain expressions for the radiation characteristics are derived.

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One of the central problems of classical electrodynamics is the radiation of electromagnetic waves by an accelerated charge [1,2]. In this Letter, we consider the theoretical description of radiation from an axially extended charge distribution propagating through a helical wiggler. This type of radiation process is currently under investigation by several groups worldwide [3–7]. Preliminary experimental results have been reported [8,9] and are extremely encouraging for future applications, which range from surface and solid state physics, millimeter-wave and far infrared photochemistry and photophysics, to ultrawideband radars and communications.

Superradiant emission is the coherent radiation of a large number (N) of charges, all oscillating in phase with each other, within a spatial region smaller than a wavelength. The fields emitted by individual charges then add up in phase, and the radiated intensity is proportional to N^2 instead of N as in regular undulator radiation of randomly phased oscillating electrons [10]. In a waveguide, the coherent synchrotron radiation process is of particular interest for the following reasons. First, in a waveguide, the electron axial velocity v_z can be adjusted to match the group velocity v_g of the radiated waveguide mode. In this case, the Doppler upshifted and downshifted interaction frequencies are equal. In the beam frame, where the axial velocity is equal to zero, the bunch oscillates about the wiggler axis and radiates along the z axis in a $\sin^2$ pattern similar to a dipole. In this frame, the grazing condition corresponds to cutoff, and the radiated wave packet is confined around the electron bunch. Therefore, an extremely narrow and intense pulse is expected. Since, by definition, slippage is equal to zero at grazing, the question arises of what physical mechanism determines the final output pulse length in this regime. Next, because of transverse mode quantization it is possible to choose waveguide parameters such that the electron bunch will emit undulator radiation only on a single transverse mode and around one (or two) center frequencies. The radiation wave packet will then be entirely coherent—both spatially (diffraction limited) and temporally (Fourier transform limited).

Undulator synchrotron radiation in a waveguide of randomly phased electrons (spontaneous emission) was analyzed in the frequency domain [10]. Periodically bunched superradiant emission was also analyzed in both a waveguide [11] and in free space [12]. However, quantitative analysis in the case of coherent radiation from a single bunch in a waveguide, as measured in [8], was still not available. In particular, there is no quantitative theoretical analysis of the temporal behavior of the emitted coherent wave packet. Such an analysis is especially desirable in the grazing case. This analysis is described in the present Letter, in which explicit expressions for the time dependent fields, instantaneous power, and spectral energy of the radiation are derived. The theoretical results are then used to predict the output radiation characteristics of an ultrashort pulse free-electron laser (FEL) experiment currently underway.

Our theoretical model is based on the assumption that a single transverse electromagnetic mode propagating in the forward direction is excited in the waveguide. The fields are then given by

$$\mathbf{E}(r,t)=a(z,t)\mathcal{E}(x,y),\quad \mathbf{H}(r,t)=a(z,t)\mathcal{H}(x,y)\,, \qquad (1)$$

where $\{\mathcal{E},\mathcal{H}\}$ are the electric and magnetic fields of the waveguide eigenmode under consideration, normalized according to $\frac{1}{2}\int\int\mathcal{E}\times\mathcal{H}\cdot\hat{\mathbf{z}}\,dx\,dy=P_0$, and $a(z,t)$ is a dimensionless field amplitude. After Fourier transforming the fields into the frequency domain $[\breve{F}(\omega)\equiv\mathcal{F}\{f(t)\}\equiv\int_{-\infty}^{+\infty}f(t)e^{i\omega t}dt]$, we use the following excitation equa-

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tion [13] to solve for the field amplitude $\check{a}(z,\omega)$

$$\frac{d}{dz}\check{a} - ik_z\check{a} = -\frac{1}{4P_0}\int\int \check{J}_\perp(r,\omega)\cdot\mathcal{E}_\perp(x,y)\,dx\,dy\,. \tag{2}$$

Here, $k_z(\omega)=(\omega^2-\omega_{\rm co}^2)^{1/2}/c$, $\omega_{\rm co}$ is the cutoff frequency of the mode, and $\check{J}_\perp=\check{\rho}(r,\omega)\mathbf{v}_\perp$ is the driving current density in Eq. (2).

In a helical magnetic wiggler field $\mathbf{B}=\frac{1}{2}\mathbf{B}_w e^{ik_wz}+$c.c., one obtains $\mathbf{v}_\perp=\frac{1}{2}\mathbf{v}_w\exp(ik_wz)+$c.c., where $\mathbf{v}w=e(\hat{\mathbf{z}}\times\mathbf{B}_w)/\gamma mk_w$. The spectral charge density $\check{\rho}$ is the Fourier transform of the charge density of the bunch

$$\rho(r,t)=\frac{q}{A_el_e}f\left(r,t-\frac{z}{v_z}\right), \tag{3}$$

where A_e, l_e, $t_e=l_e/v_z$, and q are the cross-section area, length, duration, and total charge of the electron bunch, respectively. In addition, $\int f d^3r=A_el_e$. Thus, we have

$$\int\int\check{\rho}(r,\omega)\,dx\,dy=\frac{q}{A_el_e}\check{F}(\omega)\exp\left(i\frac{\omega}{v_z}z\right), \tag{4}$$

where $\check{F}(\omega)\equiv\mathcal{F}\{\int\int f(r,t)\,dx\,dy\}$ is the Fourier transform of the bunch axial charge distribution.

In the present Letter, we assume that the bunch is small compared to the radiation wavelength and that the radiated electromagnetic energy remains small compared to the bunch kinetic energy. As a result, we neglect both nonlinear saturation effects and radiation damping (v_z =const). We further assume that the beam width is small relative to the transverse mode gradient. The solution of Eq. (2) for $\omega>0$ is

$$\check{a}(z,\omega)=\mathcal{K}z\check{F}(\omega)\exp\left[\frac{i}{2}\left(\frac{\omega}{v_z}+k_z-k_w\right)z\right]\times\mathrm{sinc}[\theta(\omega)/2]\,, \tag{5}$$

where $\theta=[\omega/v_z-k_z(\omega)-k_w]z$ and $\mathcal{K}=(\mathbf{v}_w^*\cdot\mathcal{E}_\perp)q/8v_zP_0$. For negative frequencies, $\check{a}(z,\omega)=\check{a}^*(z,-\omega)$. The spectral energy (defined for positive frequencies only) is given by

$$\frac{dW}{d\omega}=\frac{2}{\pi}|\check{a}(z,\omega)|^2=\frac{2}{\pi}|\mathcal{K}|^2z^2|\check{F}(\omega)|^2\mathrm{sinc}^2[\theta(\omega)/2]\,. \tag{6}$$

It is easily seen that $\check{a}(z,\omega)$ has a significant amplitude only for $\theta(\omega)\approx0$. Inspection of the equation $\theta(\omega)=0$ reveals that it can have either two distinct solutions, as shown in Fig. 1, or a single degenerate root corresponding to grazing. The explicit solutions for $\theta(\omega_{1,2})=0$ are

$$\omega_{1,2}=\gamma_z^2k_wc\beta_z\left\{1\mp\beta_z\left[1-\left(\frac{\omega_{\rm co}}{\gamma_zk_wc\beta_z}\right)^2\right]^{1/2}\right\}, \tag{7}$$

where we recovered the usual FEL Doppler downshifted and upshifted interaction frequencies, and where we have defined $\beta_z=v_z/c$ and $\gamma_z=1/\sqrt{1-\beta_z^2}$. When there are two solutions $\omega_{1,2}$ the lower frequency ω_1 corresponds to the backward emitted wave when viewed in the rest frame of the oscillating bunch. When $\omega_1=\omega_2$, the beam and waveguide mode dispersion lines are tangent, and the picture in the moving frame is that of a charge bunch oscillating in the waveguide exactly at its cutoff frequency.

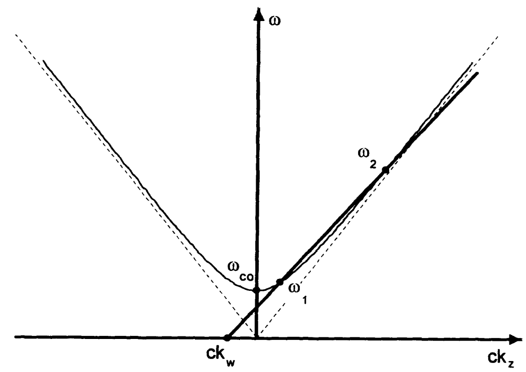

FIG. 1. Schematic of waveguide mode and beam mode dispersion.

To derive the fields in the time domain, it is necessary to take the inverse Fourier transform of Eq. (5). In the case of two well separated solutions, where the respective frequency bandwidths of the sinc function around ω_1 and ω_2 are much smaller than the frequency differences $\omega_2-\omega_1$ and $\omega_1-\omega_{\rm co}$, the following linear expansion can be used:

$$\theta(\omega)=\theta(\omega_{1,2})+\frac{d\theta}{d\omega}(\omega-\omega_{1,2})=t_{{\rm sl}1,2}(\omega-\omega_{1,2})\,. \tag{8}$$

Here, the resonance condition $\theta(\omega_{1,2})=0$ is used and $t_{{\rm sl}1,2}$ is the slippage time between the wave packet and the electron bunch, evaluated at ω_1 and ω_2, respectively. The explicit expression for the slippage time is

$$t_{{\rm sl}1,2}(z)=d\theta/d\omega=(v_z^{-1}-v_g^{-1})z\,. \tag{9}$$

Note that the Doppler upshifted radiation pulse propagates faster than the electron bunch ($t_{\rm sl2}>0$), while the low frequency pulse lags behind it ($t_{\rm sl1}<0$). The bandwidths are simply given by $\Delta\omega_{1,2}=2\pi/t_{{\rm sl}1,2}$. Substituting Eq. (8) into Eq. (5), the inverse Fourier transform leads to

$$a(z,t)=2\mathcal{K}z\sum_{i=1}^{2}\frac{1}{t_{{\rm sl}i}}\check{F}(\omega_i)\cos[k_z(\omega_i)z-\omega_it]\times\mathrm{rect}\left(\frac{t-z/v_z+t_{{\rm sl}i}/2}{|t_{{\rm sl}i}|}\right), \tag{10}$$

where rect(x) is the ate function with both unit amplitude and width. Note that for a bunch length shorter than the radiation wavelength, $\check{F}(\omega_i)\simeq1$, and the emission is coherent. The instantaneous power is proportional to the square of the field amplitude in Eq. (10). In the limit of two well separated solutions, $P(z,t)$ has the following expression:

$$P(z,t) \equiv \frac{dW}{dt} = \sum_{i=1}^{2} 2P_0 \frac{W_i}{|t_{sli}|} \cos^2[k_z(\omega_i)z - \omega_i t] \times \mathrm{rect}\left(\frac{t - z/v_z + t_{sli}/2}{|t_{sli}|}\right), \quad (11)$$

where z is the observation position along the wiggler. The total energy radiated by the electrons at the frequency ω_i is

$$W_i = \int_{-\infty}^{+\infty} P(z,t)\,dt = \frac{\alpha_i}{8}\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2}\left(\frac{a_w q z}{\gamma\beta_z}\right)^2 \frac{|F(\omega_i)|^2}{A_{em} t_{sli}}, \quad (12)$$

which displays the usual quadratic dependence on the wiggler strength a_w, bunch charge q, and interaction length z (coherent radiation process). Here, $A_{em} \equiv 2P_0/\sqrt{\epsilon_0/\mu_0}|\mathcal{E}_\perp|^2$, $a_w = e|B_w|/k_w mc$ is the normalized wiggler parameter and α_i is defined as follows:

$$\alpha_i = \frac{|\mathbf{v}_w \cdot \mathcal{E}_\perp|^2}{|\mathbf{v}_w|^2 |\mathcal{E}_\perp|^2} \times \begin{cases} [1-(\omega_{co}/\omega_i)^2]^{1/2} & \text{for TM modes}, \\ [1-(\omega_{co}/\omega_i)^2]^{-1/2} & \text{for TE modes}. \end{cases}$$

For the degenerate case ($v_g = v_z$), i.e., grazing, the power series expansion in Eq. (4) has to be performed to second order because the first order term of $\theta(\omega)$ is equal to zero. Therefore, we have

$$k_z(\omega) = k_z(\omega_0) + \frac{1}{v_g}(\omega - \omega_0) + \frac{1}{2}D(\omega - \omega_0)^2,$$
$$\theta(\omega) = -\tfrac{1}{2}D(\omega - \omega_0)^2 z, \quad (13)$$

where ω_0 is the grazing frequency and $D = \beta_g(1-\beta_g^2)/\omega_0 c$ corresponds to the group velocity dispersion and $\beta_g = \sqrt{1-\lambda_0^2/\lambda_c^2}$. The frequency bandwidth of the emission line becomes very wide: $\Delta\omega \approx (8\pi/Dz)^{1/2}$. Correspondingly, the radiation pulse becomes very narrow in the time domain: Its temporal width is determined by the interaction bandwidth and by the waveguide dispersion, in contrast with the nongrazing case where slippage determines the pulse duration. This can be found by inverse Fourier transforming Eq. (5),

$$a(z,t) = \mathcal{H} z \check{F}(\omega_i) e^{-i(\omega_0 t - k_z z)} \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} e^{i2\tau_0^2\omega^2} e^{-i\omega(t - z/v_z)} \mathrm{sinc}(\omega^2\tau_0^2)\,d\omega + \mathrm{c.c.}, \quad (14)$$

TABLE I. UCLA experimental parameters.

Parameters	
Beam energy	4 MeV
Bunch charge	1 nC
Bunch width (FWHM)	1.7 mm
Wiggler period	84 mm
Number of periods	20
TE_{11} cutoff frequency	11 GHz

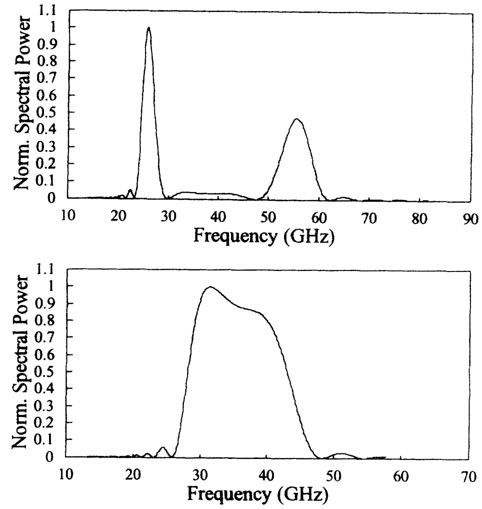

FIG. 2. Normalized spectral power density output. Top: away from grazing ($B_w = 3.0$ kG). Bottom: grazing case ($B_w^* = 3.25$ kG).

where $\tau_0 = \sqrt{Dz/2}$. Although a simple analytical solution to the above integral is not available, the pulse duration at grazing, $\Delta\tau$, can be estimated as follows:

$$\Delta\tau \simeq \frac{L}{c}\frac{1}{\beta_z^3\gamma_z^2}\frac{\Delta\omega}{\omega_0}, \quad (15)$$

where L is the total length of the interaction region.

To illustrate the derivation presented above, we have performed numerical calculations in the case of a Gaussian charge distribution of width Δz and total charge q assumed to propagate along helical trajectories through a helical magnetic wiggler field in a cylindrical waveguide. The excited mode is the fundamental TE_{11} cylindrical waveguide eigenmode. For the UCLA experimental parameters listed in Table I, we obtain the following results. The first figure (Fig. 2, top) shows the normalized spec-

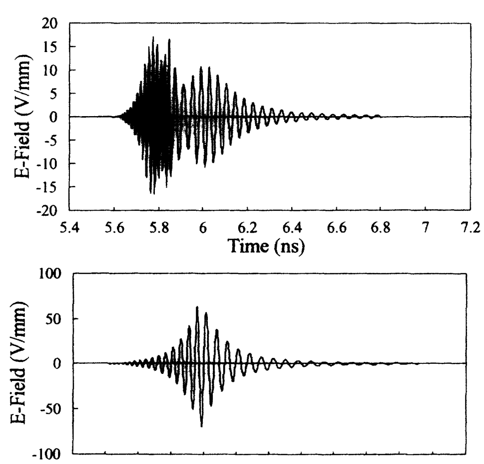

FIG. 3. Time domain electric field. Top: $B_w = 3.0$ kG. Bottom: $B_w^* = 3.25$ kG.

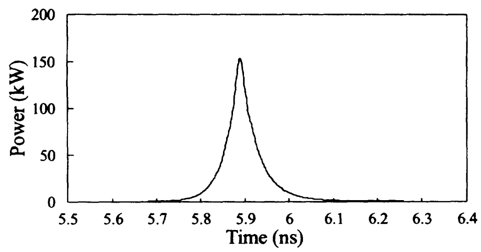

FIG. 4. Instantaneous power flowing at the waveguide output in the grazing case.

tral power density at the FEL output for $B_w = 3$ kG, away from grazing. Two distinct spectral components appear clearly. They correspond to the Doppler upshifted and downshifted lines. The low frequency component has a larger amplitude than its upshifted counterpart because its relative coherence is higher (longer radiation wavelength). The corresponding E field in the time domain is shown in Fig. 3 (top). The higher frequency pulse propagates faster than the bunch and is immediately followed by the lower frequency pulse, which lags behind the bunch. In addition, because the energy emitted is inversely proportional to the slippage time, the higher frequency radiation has a greater output power. The respective time scales of the two output pulses are determined by slippage. For $B_w^* = 3.25$ kG (grazing field), the spectral power density is shown in Fig. 2 (bottom). As expected, the interaction bandwidth is considerably wider than in the previous case, and shows a single spectral feature at $\omega = \omega_0$. The corresponding Fourier transformed E field in the time domain is shown in Fig. 3 (bottom). The pulse is clearly chirped because of waveguide dispersion, and is only a few oscillations long. In contrast with the nongrazing case, the peak electromagnetic energy is centered on the bunch because the group velocity of the main spectral component is exactly equal to the bunch axial velocity. The corresponding ultrashort output instantaneous power pulse is shown in Fig. 4.

In conclusion, the coherent synchrotron radiation process in a waveguide has been investigated theoretically. A single axially extended charge distribution propagating through a helical wiggler was considered. In the case of grazing, the bunch is found to radiate a single ultrashort chirped pulse. Indeed, for the experimental parameters listed in Table I, the ratio of the output power pulse width (FWHM) to the radiation period is extremely small: 1.85. Away from grazing, where slippage dominates, the system radiates two distinct pulses at the Doppler upshifted and downshifted frequencies.

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