# Multiple Excitation of Giant Dipole Resonances in Relativistic Heavy Ion Collisions

V. Yu. Ponomarev,[1,2] E. Vigezzi,[1] P. F. Bortignon,[1,3] R. A. Broglia,[1,3,4] G. Colò,[1,3] G. Lazzari,[1,3] V. V. Voronov,[2] and G. Baur[5]

[1]*Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy*
[2]*Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Head Post Office, P. O. Box 79, Moscow, Russia*
[3]*Dipartimento di Fisica, Università di Milano, Via Celoria 16, 20133 Milano, Italy*
[4]*The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark*
[5]*Institut fur Kernphysik, Forschungzentrum Jülich GmbH, Germany*
(Received 23 September 1993)

The relativistic excitation of the double giant dipole resonance in $^{136}$Xe has been calculated, describing the resonance as a harmonic vibration of the mean field, damped through its coupling to doorway states. While the predictions associated with the one-phonon states provide an overall account of the experimental findings, the calculated cross section for the two-phonon states is much smaller than that extracted from the involved analysis of the data. The experimental elucidation of this discrepancy seems essential for assessing the validity of the standard picture of giant resonances.

PACS numbers: 25.75.+r, 24.30.Cz

One of the central questions in the study of the nuclear structure is that of the interweaving of single-particle and collective motion leading to renormalization of the properties of nuclear motion and responsible also for their damping. Among these phenomena, the study of the relaxation of giant resonances provides an excellent testing ground of the role played by the different mechanisms coupling collective modes among themselves and to the single-particle motion. This is because the damping width of giant resonances receives a contribution already at the level of mean field. In fact, giant resonances can exchange energy and angular momentum with the motion of the individual nucleons. For this to take place, the particle excitation has to have the same frequency as that of the giant resonance. This phenomenon, which in the case of infinite systems is known as Landau damping, in the case of finite systems like the atomic nucleus leads to a fragmentation of the mode.

Beyond mean field, giant resonances can couple to more complicated states, and eventually to the compound nucleus. The states which act as doorways in this hierarchy of couplings have two particle–two hole character, and are composed of an uncorrelated particle-hole excitation and a collective surface vibration (cf. e.g., Ref. [1] and references therein).

While this picture seems to be able to provide an overall account of the experimental findings, no direct evidence of the relative role of the two mechanisms is yet available. This is because at the excitation energies typical of the resonances (tens of MeV), the density of levels of the system is so high ($\approx 10^{12}$ MeV$^{-1}$ at 15 MeV) that any evidence of the original couplings is completely blurred. A particularly clear example of this situation is provided by the beautiful work reported in Ref. [2], concerning the high energy-resolution electron-scattering excitation of the giant quadrupole resonance in $^{208}$Pb. Although the observed peaks seemed to have a correspondence with the predicted [3] doorway states, the expected energy-weighted sum rule limit was only achieved assuming a Porter-Thomas distribution of the intensities. In other words, while scars of the wave functions can be found in this zone of complete mixing, it is not possible to obtain detailed information about these wave functions. On the other hand one can try to be more modest and look for more overall consequences of the presence of specific states and couplings in the completely mixed regime. In particular, interference phenomena, one of the expressions of the underlying structure of the wave functions and of their relative phases, may lead to interference patterns with clear experimental consequences.

In the present paper we shall calculate the relativistic Coulomb excitation of the double giant dipole resonance (GDR) of atomic nuclei (cf. also Ref. [4]). In particular, we will study the excitation of $^{136}$Xe on $^{208}$Pb at $E_{\rm lab} = 690$ MeV/$n$, which has recently been measured [5], making use of a microscopic model which takes into account both the Landau breaking of the strength and the coupling to doorway states. It will be concluded that the complicated interference between the Landau damping and the doorway (2p-2h) relaxation mechanism leads to a transition strength from the single- to the double-giant dipole resonance which essentially coincides with the simple harmonic picture. This is also due to the fact that the direct excitation of the two-phonon state is very small. Finally, the associated excitation function reflects closely the structure of the calculated one-phonon cross section, due to the fact that the reaction mechanism displays a rather weak dependence on excitation energy, in the range where the strength function is appreciable.

The microscopic calculation of the giant resonances was carried out making use of a model Hamiltonian $H$ [6], which includes an average Woods-Saxon field $U(r)$, monopole pairing interaction and isoscalar and isovector residual interaction of separable type, with a form factor proportional to $dU/dr$. The one-phonon states were calculated in the random phase approximation (RPA). The creation operator of the $i$th one-phonon state with multipolarity $\lambda\mu$ is given by

$$Q^{\dagger}_{\lambda\mu}(i)=\frac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'}[\alpha^{\dagger}_{j'},\alpha^{\dagger}_{j}]_{\lambda\mu}-(-1)^{\lambda-\mu}\phi^{\lambda i}_{jj'}[\alpha_{j}\alpha_{j'}]_{\lambda-\mu}\}\,. \quad (1)$$

The quantities $\alpha^{\dagger}_{j}$ and $\alpha_j$ are creation and destruction quasiparticle operators. The phonon excitation energies $\omega_{\lambda i}$ and the forward and backward amplitudes $\psi^{\lambda i}_{jj'}$ and $\phi^{\lambda i}_{jj'}$ were determined in the RPA and contain the BCS occupation factors. The sum in (1) runs over both protons and neutrons.

The one-phonon states couple to two-phonon states via the matrix element

$$U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)=\langle Q_{\lambda}(i)\|H\|[Q^{\dagger}_{\lambda_1}(i_1),Q^{\dagger}_{\lambda_2}(i_2)]_{\lambda}\rangle\,. \quad (2)$$

Within the present scheme the doorway states which couple to lowest order to the giant resonance are states containing two phonons. Among them, the most efficient to break the giant resonance strength are, as mentioned above, those containing one collective and one noncollective phonon, being the 2p-2h configurations most likely to lead to on-the-energy-shell transitions.

The Hamiltonian was diagonalized in the space of one- and two-phonon states. Technically, the diagonalization implies finding the roots of the nonlinear determinant

$$F(E_{\nu})=\det\left|(\omega_{\lambda i}-E_{\nu})\delta_{ii'}-\frac{1}{2}\sum_{\lambda_1 i_1,\lambda_2 i_2}\frac{U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-E_{\nu}}\right| =0\,, \quad (3)$$

and leads to the mixed states $|\nu\rangle$, whose wave function is written

$$\Psi_{\nu}(\lambda\mu)=\left\{\sum_{i}R_i(\nu)Q^{\dagger}_{\lambda\mu}(i)+\sum_{\lambda_1 i_1,\lambda_2 i_2}P^{\lambda_1\lambda_2}_{i_1 i_2}[Q^{\dagger}_{\lambda_1\mu_1}(i_1)Q^{\dagger}_{\lambda_2\mu_2}(i_2)]_{\lambda\mu}\right\}|0\rangle_{\rm pho}\,. \quad (4)$$

The amplitudes of one- and two-phonon states into the mixed state are denoted by $R$ and $P$, respectively.

In the dipole case, $\lambda^{\pi}=1^{-}$, the one-phonon states exhaust 107% of the classical oscillator strength and are displayed in the inset of Fig. 1. Of these, 20 states have an oscillator strength which is at least 1% of the strongest strength and together exhaust 104% of the classical energy weighted sum rule (EWSR). We have used these states in the coupling to two-phonon states. We have included all the natural parity phonons $\lambda^{\pi}=1^{-}-8^{+}$ with energy lower or equal to 21 MeV, obtaining 2632 two-phonon configurations. One obtains 1614 states of type (4), in the energy interval from 7 to 19.5 MeV. Their strength is shown in Fig. 1. The $B(E1)$ value associated with each mixed state is calculated through its admixture with one-phonon states, as

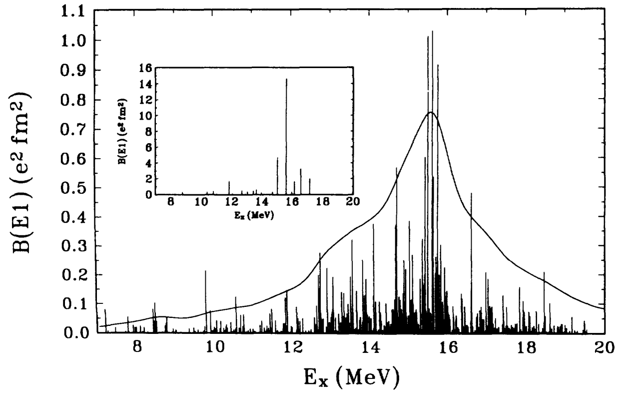


FIG. 1. Fine structure of the giant dipole resonance. The transition strength to each of the microscopically calculated states $\Psi_{\nu}(1\mu)$ is shown in histogram form. Also shown by the continuous curve is the transition strength obtained introducing an averaging parameter $\eta=500$ keV. In the inset, the transition strength obtained in RPA, neglecting the coupling with the two-phonon states, is displayed.

$$|\langle\nu\|M(E1)\|0\rangle|^2=\left|\sum_{i}R_i(\nu)\langle 0\|Q_1(i)M(E1)\|0\rangle\right|^2.$$

Also shown in Fig. 1 is the result obtained adding an averaging parameter of 0.5 MeV to Eq. (3). This parameter represents in some average way the coupling to increasingly more complicated states and eventually to the compound nuclear states. From the resulting smooth response it is easy to directly extract the centroid and the full width at half maximum of the GDR. The corresponding values are $E_{\rm GDR}=15.1$ MeV and $\Gamma_{\rm GDR}=4$ MeV. They can be compared with the values extracted from experiment, $E_{\rm GDR}=15.2$ MeV and $\Gamma_{\rm GDR}=4.8$ MeV.

Making use of the same model, the isoscalar and the isovector giant quadrupole resonance (GQR) have also been calculated. The results are displayed in Fig. 2. The centroid, width, and percentage of the EWSR associated

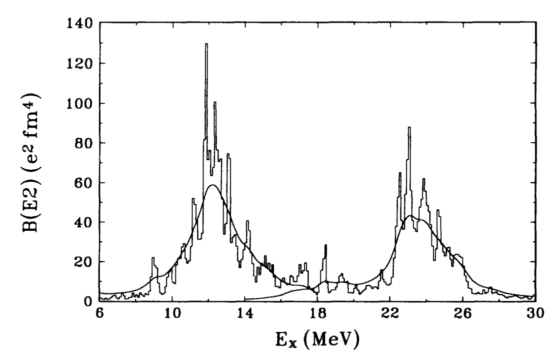


FIG. 2. The transition strength of the giant quadrupole resonance is shown in histogram form. Also shown by the curves are the averaged transition strengths to the left and to the right peaks, corresponding to $\tau=0$ and $\tau=1$ excitations, respectively.

with the isoscalar mode are 12.5 MeV, 3.2 MeV, and 75%, respectively. The corresponding quantities associated with the isovector GQR are 23.1 MeV, 3.6 MeV, and 80%.

Making use of the results discussed above and of the theory of relativistic Coulomb excitation [7], the cross sections associated with the one-phonon giant dipole resonance as well as giant quadrupole resonance ($\tau=0$ and 1) have been calculated. These cross sections can be written in terms of the first- and second-order amplitudes $a^{(1)}$ and $a^{(2)}$, respectively, as

$$\sigma^{(k)}=2\pi\int_{R_{\min}}^{\infty} b\,db\int_0^{\infty} d\omega\left|\sum_{M_f} a^{(k)}_{I_f,M_f;I_0,M_0}(\omega,b)\right|^2, \quad k=1,2. \tag{5}$$

For $k=1$, the final states have angular momentum $I_f=1$, while for $k=2$, both $I_f=0$ and 2 are possible. The amplitudes depend on the impact parameter $b$ and on the excitation energy $\hbar\omega$. The first-order amplitude is equal to

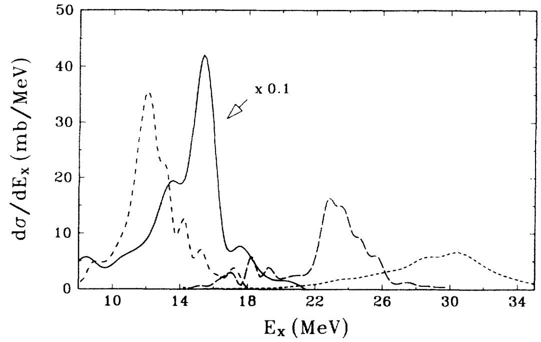


FIG. 3. The cross section for Coulomb excitation of the one-phonon GDR (continuous curve), of the isoscalar GQR (dash-dotted), of the isovector GQR (long dashed), as well as for the double-phonon GDR (short dashed), are shown. They have been calculated at $E_{\rm lab}=681$ MeV/$n$, taking into account the energy reduction of the beam in the target [5]. The one-phonon GDR cross section has been reduced in the figure by a factor 10.

$$a^{(1)}_{I_f,M_f;I_0,M_0}(\omega)=\frac{4\pi Z_t e}{i\hbar(2\lambda+1)}(-1)^{I_0-M_0}\begin{pmatrix} I_0 & \lambda & I_f \\ -M_0 & M_0-M_f & M_f\end{pmatrix}\langle I_0\|M(E\lambda)\|I_f\rangle S_{\lambda,M_0-M_f}(\omega), \tag{6}$$

where $Z_t$ is the charge of the target. The radial integral $S$, carried out on a straight-line trajectory in keeping with the relativistic character of the reaction, is given by

$$S_{\lambda,\mu}=(2\lambda+1)^{3/2}G_{E\lambda,\mu}(c/v)K_\mu(\xi(b))(\omega/c)^\lambda/(4\pi v\gamma).$$

Here $\xi(b)=\omega b/v\gamma$ is the adiabaticity parameter. In the calculations the correction to $b$ in the expression for $\xi(b)$, namely $\delta b=\pi Z_p Z_t e^2/2mv^2\gamma$, has been considered. The functions $K_\mu$ are modified Bessel functions, while the polynomials $G_{E\lambda,\mu}$ are related to the Legendre polynomials. The lower limit of integration over impact parameter is to be taken in such a way as to exclude nuclear reactions.

The second order amplitude needed in the calculation of the double phonon excitation can be written as

$$a^{(2)}_{I_f,M_f;I_0,M_0}(\omega_{f0})=\frac{1}{2}\sum_{I_i,M_i} a^{(1)}_{I_f,M_f;I_i,M_i}(\omega_{fi})a^{(1)}_{I_i,M_i;I_0,M_0}(\omega_{i0})+\frac{i}{2\pi}\sum_{I_i,M_i}\mathcal{P}\int_{-\infty}^{\infty}\frac{dq}{q}a^{(1)}_{I_f,M_f;I_i,M_i}(\omega_{fi}-q)a^{(1)}_{I_i,M_i;I_0,M_0}(\omega_{i0}+q), \tag{7}$$

where $I_iM_i$ denotes the angular momentum and projection of the intermediate state. A central aspect of the above expression is the interference between the different $E1$ amplitudes, and thus between the different components of the dipole response [cf. Fig. 1 and Eq. (4)]. The principal integral in Eq. (7) can be shown to vanish within the present model.

The evaluation of $a^{(2)}$ requires the knowledge of the matrix element $\langle I_f\|M(E1)\|I_i\rangle=\langle v_1v_2\|M(E1)\|v_1\rangle$. Because of the phonon character of the operators $Q_{\lambda\mu}$, it can be shown that

$$\langle v_1v_2\|M(E1)\|v_1\rangle=\sqrt{1+\delta_{v_1v_2}}\sum_i R_i(v_2)\langle 0\|Q_1(i)M(E1)\|0\rangle=\sqrt{1+\delta_{v_1v_2}}\langle v_2\|M(E1)\|0\rangle. \tag{8}$$

This result emerges from the interference of $10^3$ states, and shows that the strength function for the excitation of the double giant dipole resonance can be derived from the one-phonon strength function, considering the multiple excitation of all the $|v\rangle$ states, with the appropriate boson factor and phase which account for the double excitation of the same state.

The differential Coulomb-excitation cross sections as a function of the energy associated with the one-phonon GDR and GQR resonances and the two-phonon GDR are displayed in Fig. 3. It is seen that the centroid of the two-phonon dipole excitation falls at 30.6 MeV, about twice that of the one-phonon states, while the width is $\Gamma\approx 6$ MeV, the ratio to that of the one-phonon excitation being 1.5.

The associated integrated values are displayed in Table I, in comparison with the experimental findings. As can be seen from Eq. (5), the cross sections depend on the choice of the value of $R_{\min}=r_0(A_p^{1/3}+A_t^{1/3})$. In keeping with the standard "safe distance," that is, the distance beyond which nuclear excitation can be safely neglected, we have used $r_0=1.5$ fm. Because their values essentially do not depend on the width of the GDR [8], we view the

TABLE I. Calculated and experimental cross section (in mb) for the excitation of giant resonances in $^{136}$Xe are shown. In the first and second row, the cross sections calculated using microscopic strength functions are shown, for two values of $r_0$, $r_0=1.2$ fm and $r_0=1.5$ fm, respectively. In the last row, the experimental cross sections for Coulomb excitation of one-phonon and two-phonon states from Ref. [5] are shown. The value of the integrated cross section reported in Ref. [5] is $1.85\pm0.1$ b. The nuclear contribution has been estimated in Ref. [5] to be about 100 mb, while about 3% (50 mb) of the cross section is found at higher energy. Subtracting these two contributions and the two-phonon cross section leads to the value $1485\pm100$ mb shown in the table.

| 1-ph ($E1$) | 1-ph $E2$ ($\tau=0$) | 1-ph $E2$ ($\tau=1$) | 1-ph $E1$+1-ph-$E2$ | 2-ph ($E1$) |
|---|---|---|---|---|
| 2180 | 170 | 120 | 2470 | 130 |
| 1480 | 110 | 60 | 1650 | 50 |
| $1025\pm100$ | ... | ... | $1485\pm100$ | $215\pm50$ |

calculated cross section of 1650 mb as a rather accurate value and if anything an upper limit for the one-phonon Coulomb excitation cross section. It is satisfactory that the measured cross section is rather close to this value. Also shown in Table I are the predictions associated with the sequential excitation of the double giant dipole resonance. This result is essentially not modified evaluating the direct Coulomb excitation of the double GDR. In fact, the cross section associated with this process is a factor $10^{-3}$ smaller than that associated with the two-step process. This is because the direct excitation process is possible either through ground state correlations, and thus is controlled by the small backward-going amplitudes $\phi^{\lambda i}_{jj'}$ of the RPA, or through anharmonic processes which are expected to be rather small due to strong cancellation between the different amplitudes.

The calculated value of 50 mb is a factor of 0.25 smaller than experimentally observed [9]. In order to make clearer the seriousness of this discrepancy, we have recalculated all the cross sections using $r_0=1.2$ fm, namely with a much smaller radius than that prescribed in order to respect the safe Coulomb excitation distance of closest approach. The calculated value of 130 mb is still a factor of 0.6 smaller than the reported experimental cross section. At the same time the cross section of the one-phonon states has become a factor 1.7 larger than the empirical value. This factor becomes 1.5 when the coupling to higher multiphonon states is included according to the standard Poisson distribution for the excitation probabilities [10].

One may try to invoke the interaction among the two giant dipole resonances (anharmonicities) as a possible cause for the observed discrepancy between theory and experiment. However, to the extent that perturbation theory is applicable, these anharmonicities are mostly related to the violations of the Pauli principle, which are likely to reduce the harmonic two-phonon cross section.

We conclude that recently reported cross sections for the Coulomb excitation of the double giant dipole resonance are inconsistent with the phonon picture of giant dipole resonances. If these results were to be confirmed they will imply that the theory of linear response of nuclear collective modes has to be seriously revised.

V.Yu.P. thanks the Dipartimento di Fisica of the Università di Milano for the hospitality and INFN for financial support during his stay in Italy where this work was done.

[1] G. F. Bertsch and R. A. Broglia, "Oscillations in Finite Quantum Systems" (Cambridge Univ. Press, Cambridge, to be published).
[2] G. Kilgus *et al.*, Z. Phys. A **326**, 41 (1987).
[3] P. F. Bortignon and R. A. Broglia, Nucl. Phys. **A371**, 405 (1981).
[4] C. A. Bertulani and V. G. Zelevinsky, Phys. Rev. Lett. **71**, 967 (1993).
[5] R. Schmidt *et al.*, Phys. Rev. Lett. **70**, 1767 (1993).
[6] V. G. Soloviev *et al.*, Nucl. Phys. **A288**, 376 (1977).
[7] K. Alder and A. Winther, Nucl. Phys. **A319**, 518 (1979).
[8] We have also calculated the integrated cross sections in the purely harmonic picture, that is, for the excitation of a dipole state of zero width and carrying the same fraction of the EWSR as the full response calculated microscopically. The resulting cross sections differ from those shown in Table I by less than 5%. The small influence of the width can be understood, noting that the cross section depends on excitation energy solely through the product $\omega^\lambda K_\mu$. For the reaction considered here, the excitation is mostly transverse and the component $\omega^\lambda K_\lambda$ dominates. It changes smoothly over the range covered by the microscopic dipole and quadrupole strength functions (cf. Fig. 2 in Ref. [7]); it varies most rapidly in the dipole case, where it leads to a modest distortion of the shape of the cross section as compared to the strength function, enhancing its low-energy side. However, for a symmetric strength distribution, the effects due to the finite width tend to average out in the total cross section.
[9] Two other processes are possible within the sequential excitation of the giant modes which can lead to an excitation energy similar to that of the two-phonon GDR. They are the excitation of the isoscalar GQR mode followed by a GDR mode and vice versa. The resulting cross section is estimated to be an order of magnitude smaller, cf. Table I, and does not change qualitatively this result.
[10] K. Alder and A. Winther, *Electromagnetic Excitation* (North-Holland, Amsterdam, 1975), Chap. VII.