

Corrections to Scaling in the Integer Quantum Hall Effect

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Finite-size corrections to scaling laws in the centers of Landau levels are studied systematically by numerical calculations. The corrections can account for the apparent nonuniversality of the localization length exponent ν . In the second lowest Landau level the irrelevant scaling index is $y_{\text{irr}} = -0.38 \pm 0.04$. At the center of the lowest Landau level an additional periodic potential is found to be irrelevant with the same scaling index. These results suggest that the localization length exponent ν is universal with respect to the Landau level index and an additional periodic potential.

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The transitions between different plateaus in the integer quantum Hall effect can be understood as disorder-driven metal-insulator transitions in the centers of Landau levels. These transitions are characterized by finite-size scaling laws [1]. Experimental measurements of the Hall and longitudinal resistivities showed that the corresponding localization length exponent $\nu = 2.3 \pm 0.1$ independent of the Landau level provided the spin splitting of the levels was resolved [2,3]. Numerically it was found that the localization length $\lambda_M(E)$ for cylinders of circumference M behaves near the critical energies E_c as

$$\lambda_M(E) = M\Lambda(M/\xi(E)), \quad (1)$$

with $\xi(E) \propto |E - E_c|^{-\nu}$ and $\nu = 2.35 \pm 0.03$ [4-10]. This universal behavior was observed for the lowest ($n = 0$) Landau level independent of the correlation length of the disorder potential and in the second lowest ($n = 1$) Landau level provided that the correlation length was not smaller than the magnetic length. For shorter correlation length no universal scaling behavior was observed and the numerical data were inconclusive. It remained an open question whether the localization length exponent ν was dependent on the Landau level index [6,9-11] or the available systems were too small to observe the scaling behavior [8].

Chalker and Eastmond observed that deviations from scaling behavior in an extension of the network model with a distribution of node parameters can be analyzed in terms of irrelevant scaling fields [12,13]. They found that the deviations of Λ from its fixed point value scaled like $M^{-0.38 \pm 0.02}$.

In this paper it is shown that their ideas can more generally explain the observed deviations from scaling. It is found that deviations from the finite-size scaling law Eq. (1) scale by themselves and can be described by an irrelevant scaling index y_{irr} . In terms of a field theory describing the transition the corrections are due to irrelevant scaling fields [14,15]. In particular, it is shown that the localization length is a function of at least two scaling fields,

$$\lambda_M(E, \sigma, \dots) = M\Lambda(M/\xi(E), M/\xi_{\text{irr}}, \dots), \quad (2)$$

where ξ_{irr} is a function of the correlation length σ of the disorder potential. The function Λ is an analytic function of the relevant scaling field $\Delta E = (E - E_c)/\Gamma$ and an irrelevant scaling field ζ_{irr} that is related to the correlation length σ . Γ is a measure of the disorder. In the present context the Fermi energy plays the role of the temperature in thermodynamic phase transitions. Scaling implies that the scaling variables are proportional to powers of the system size with the exponents being the scaling indices

$$\lambda_M(E, \sigma, \dots) = M\Lambda(M^y \Delta E, M^{y_{\text{irr}}} \zeta_{\text{irr}}, \dots). \quad (3)$$

$y = 1/\nu$ is the only relevant, i.e., positive, scaling index, y_{irr} is the largest irrelevant scaling index, and “...” represents possible further irrelevant scaling fields with smaller scaling indices. For small arguments the function Λ can be expanded in a Taylor series [16,17]

$$\Lambda = \Lambda_c + a(M^y \Delta E)^2 + bM^{y_{\text{irr}}} \zeta_{\text{irr}} + \dots \quad (4)$$

A linear term in ΔE is missing since Λ is symmetric in ΔE due to the coincidence of the mobility edges at $E = E_c$. Equation (4) is used in the following to extract the irrelevant scaling index y_{irr} from the numerical data. In the absence of any analytic information about the scaling function Λ it cannot be ruled out that b is zero at the critical point. In this case the first nonvanishing term of the series expansion would be quadratic in $M^{y_{\text{irr}}} \zeta_{\text{irr}}$ and the numerically determined y_{irr} would be twice the scaling index of the field theory.

In order to study the corrections to scaling, $\lambda_M(E_c)$ was calculated for $\beta^2 = (\sigma^2 + l_c^2)/l_c^2$, where l_c is the magnetic length \hbar/eB , ranging from 1 to 2 while M varied between 16 and 128 (in multiples of $\sqrt{2\pi}l_c$) [18]. For every value of β^2 the length ξ_{irr} was adjusted in order to make $\lambda_M/M(E_c)$ a function of a single variable $M/\xi_{\text{irr}}(\beta^2)$. The resulting function is shown in Fig. 1. By performing this fit the overall scale of ξ_{irr} cannot be fixed and hence is arbitrary in these calculations. The dependence of the length scale ξ_{irr} on β^2 is shown in Fig. 2. It grows by more than 10^4 when the correlation length σ is decreased from $0.8l_c$ to 0. This large increase in the cutoff length scale for finite-size corrections is the reason why previous

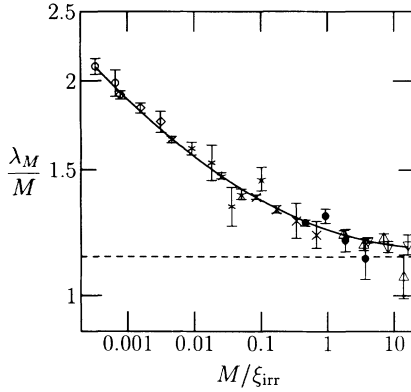


FIG. 1. The renormalized exponential decay length λ_M/M for $\beta^2 = 1.0$ (\circ), 1.1 (\diamond), 1.2 ($*$), 1.3 (\star), 1.4 (\times), 1.5 (\bullet), 1.6 (\triangle), and 1.7 (∇). The dashed line represents the asymptotic value $\Lambda_c = 1/\ln(1 + \sqrt{2})$.

finite-size scaling studies were unable to observe the true asymptotic scaling behavior [8–10]. The observation of the corrections to scaling implies that not only the fixed point value Λ_c of the scaling function but also the localization length exponent ν are universal and independent of the Landau level index and microscopic details of the disorder. However, in order to observe the scaling as a function of ΔE the system width M would have to exceed 10^6 for $\sigma = 0$, considerably larger than the presently accessible $M = 256$ [10]. It is not clear why the length scale ξ_{irr} becomes so large in the $n = 1$ Landau level while it seems to be unnoticeably small in the $n = 0$ Landau level.

Figure 3 shows a doubly logarithmic plot of $\Lambda - \Lambda_c$ as a function of M/ξ_{irr} . $\Lambda_c = 1/\ln(1 + \sqrt{2}) = 1.13459\dots$ was used which is close to the best fit estimate of $\Lambda_c = 1.14 \pm 0.02$ [13]. The slope of the dashed line is given by the irrelevant scaling index $y_{irr} = -0.38 \pm 0.04$.

Another situation where the scaling function $\Lambda(\Delta E =$

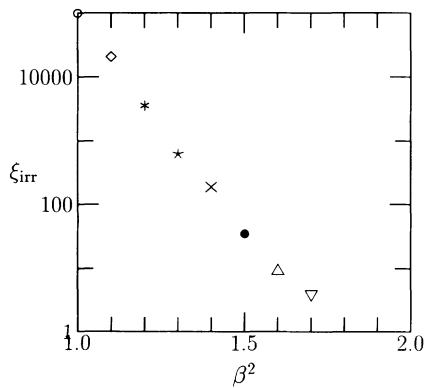


FIG. 2. The length scale ξ_{irr} (in units of $\sqrt{2\pi}l_c$) as a function of β^2 . The value at $\beta^2 = 1$ has been arbitrarily fixed to $\xi_{irr} = 50\,000$.

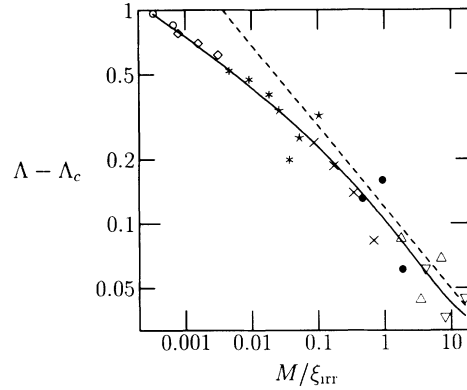


FIG. 3. Deviation of the scaling function Λ from its asymptotic value Λ_c . The data and fitted scaling function of Fig. 1 are shown. The scatter of the data for large M/ξ_{irr} is due to the statistical errors of the data that become comparable to the deviation $\Lambda - \Lambda_c$. The dashed (shifted) line with slope $y_{irr} = -0.38$ serves as a guide to the eye.

0) at the metal-insulator transition does not take on its critical value Λ_c even for the largest numerically accessible systems arises in the presence of a sufficiently strong additional periodic potential [19–21]. Here the Hamiltonian is modified by an additional term

$$V(\mathbf{r}) = 4E_0 \cos(\sqrt{2}\pi x/a) \cos(\sqrt{2}\pi y/a), \quad (5)$$

where the period a is chosen commensurable with the system width M , i.e., $\alpha = 2\pi l_c^2/a^2 = q/p$, with integer p and q . The strength E_0 of the periodic potential is assumed to be small compared to the cyclotron energy $\hbar\omega_c$ so that the single Landau band approximation remains justified, but need not be small compared to the disorder Γ . The calculations were performed for δ -correlated disorder potential in the lowest Landau level and two different values of α . For $\alpha = 1/3$ the Landau band splits into 3 subbands and the only critical energy is situated

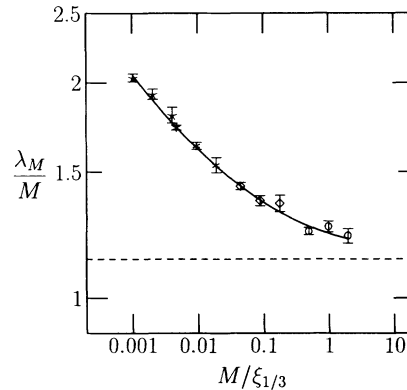


FIG. 4. The renormalized exponential decay length λ_M/M for $\alpha = 1/3$ and $E_0 = 0.75$ (\circ), 1 (\diamond), 1.25 ($*$), and 1.5 (\star). The dashed line represents the asymptotic value Λ_c .

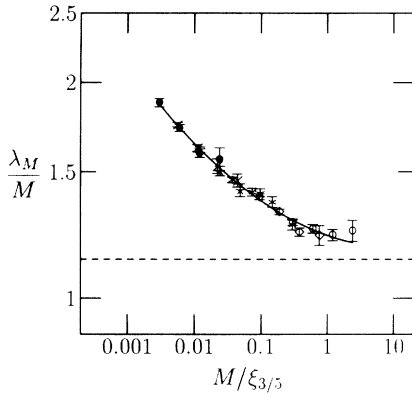


FIG. 5. The renormalized exponential decay length λ_M/M for $\alpha = 3/5$ and $E_0 = 1$ (\circ), 1.25 (\diamond), 1.5 ($*$), 1.75 (\star), 2 (\times), and 2.25 (\bullet). The dashed line represents the asymptotic value Λ_c .

at the center of the band. For $\alpha = 3/5$ the Landau band splits into 5 subbands that each contain at least one critical energy for sufficiently strong periodic potentials [19–21]. In both cases the energy of the critical point at the center of the band is not changed by the periodic potential. The fitted scaling functions $\lambda_M(E_c)/M$ are shown in Figs. 4 and 5 for $\alpha = 1/3$ and $3/5$, respectively. The irrelevant length scales $\xi_{1/3}$ and $\xi_{3/5}$ diverge approximately proportional to E_0^m with $m \approx 8.7$ and $m \approx 5.8$, respectively. The irrelevant scaling indices y_α can be deduced from Figs. 6 and 7 to be $y_{1/3} = -0.38 \pm 0.04$ and $y_{3/5} = -0.42 \pm 0.04$. Based on these data there is no significant difference between the scaling indices for $\alpha = 1/3$ and $3/5$. Furthermore, the scaling indices y_{irr} , y_α , and the one observed by Chalker and Eastmond [12] agree within the numerical uncertainties [22].

In conclusion, the observation of corrections to scaling according to Eq. (3) strongly supports the notion of *universal* metal-insulator transitions at the centers of

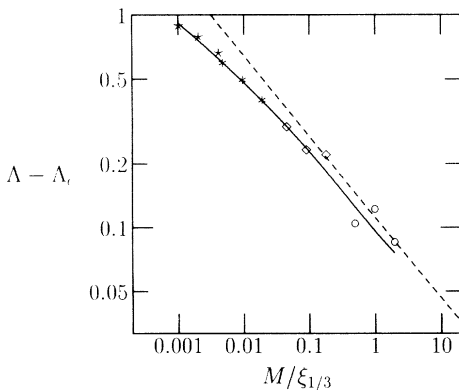


FIG. 6. Deviation of the scaling function Λ from its critical value $\Lambda_c = 1/\ln(1 + \sqrt{2})$ for $\alpha = 1/3$. The dashed line has slope $y_{1/3} = -0.38$ (cf. Fig. 3).

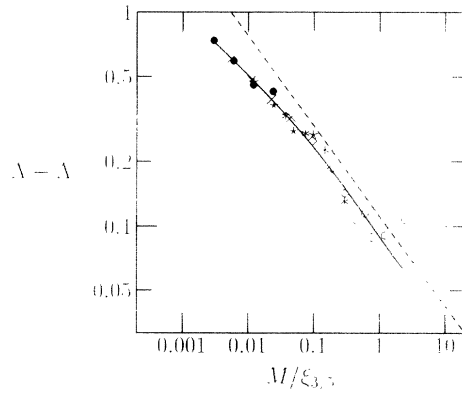


FIG. 7. Deviation of the scaling function Λ from its critical value $\Lambda_c = 1/\ln(1 + \sqrt{2})$ for $\alpha = 3/5$. The dashed line has slope $y_{3/5} = -0.42$ (cf. Fig. 3).

Landau levels in the integer quantum Hall effect. The occurrence of very large irrelevant length scales explains why the universality of the localization length exponent ν could not be observed directly in previous calculations. According to an argument by Lee, Wang, and Kivelson the scaling function Λ is related to the longitudinal conductivity σ_{xx} [13]. The universal value of Λ_c would thus imply that the peak value of σ_{xx} in the center of each Landau level is $\frac{1}{2}e^2/h$, independent of the Landau level index. It is further shown that an additional periodic potential is an irrelevant perturbation at the critical point even though it can create additional critical states in each Landau level [19–21]. The observed values for the largest irrelevant scaling indices, $y_{irr} = -0.38 \pm 0.04$, $y_{1/3} = -0.38 \pm 0.04$, and $y_{3/5} = -0.42 \pm 0.04$, are further important parameters, besides the localization length exponent $\nu = 2.35 \pm 0.03$ [8], that could be used to check an analytic theory of the integer quantum Hall effect.

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