Dynamic Spatial Solitons

A. W. Snyder, S. J. Hewlett, and D. J. Mitchell
Optical Sciences Centre, Australian National University, Canberra 0200, Australia
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A generalization of self-guided beams (spatial solitons) is discussed. Each beam is characterized by an axially uniform intensity profile whose polarization state changes with propagation. The soliton therefore has internal dynamics. It is composed of two orthogonally polarized modes of the linear waveguide induced by the dynamic soliton which, in general, do not exist as solitons independently. Our insight and exact analytical results for the threshold nonlinearity are found from the elementary physics of linear optical waveguides via a nonlinear self-consistency procedure.

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There is growing interest in self-guided beams (spatial solitons), in part because of the possibility of guiding and manipulating light with light itself [1-7]. This motivates us to search for new spatial solitons by juggling the physical properties of light, thus introducing new degrees of freedom. Our present study considers the physics underlying the expanded class of spatial solitons that exist when the soliton is composed of two orthogonally polarized beams. Here, we accordingly build upon and extend earlier important studies [8-10].

In particular, we discuss a general class of one- and two-dimensional spatial solitons propagating in a homogeneous isotropic medium whose (scalar) refractive index has an arbitrary dependence on intensity. As with classical solitons, their intensity profile remains axially uniform but, in addition, their polarization state now changes continuously with propagation. We refer to this new class of self-guided waves as "dynamic solitons" because of their internal field dynamics. Dynamic solitons can have an arbitrary number of intensity peaks and thus exhibit novel intensity profiles. Their common salient property is that they are composed of two orthogonal beams, neither of which is, in general, a soliton on its own. Each beam is, however, a mode of the (axially uniform) linear optical waveguide induced by the dynamic soliton.

An important aspect of this paper is the conceptual presentation of the underlying physical principles involved. In particular, both our insight and our exact analytical results are derived directly from linear optical waveguide theory via a straightforward self-consistency procedure, without the need to solve a nonlinear differential equation. The logic is as follows. Classical solitons are the fundamental modes of the linear optical waveguide they induce [1,11]. By inverting this self-consistency relation, we infer that the fundamental mode of a linear optical waveguide is the soliton of some nonlinear medium which can be found via an elementary inversion process, as is fully discussed elsewhere [12].

Our purpose here is to generalize the inversion procedure to include dynamic solitons. These are composed of more than one beam, each of which is a mode of the (axially uniform) linear optical waveguide induced by the dynamic soliton. The soliton field $\mathbf{E}$ is therefore expressed as $\mathbf{E}(x,y,z)=a\mathbf{E}_a+b\mathbf{E}_b+c\mathbf{E}_c\ldots$, where $\mathbf{E}_a$, $\mathbf{E}_b,\mathbf{E}_c\ldots$ are the respective complex vector modal fields and $a,b,c\ldots$ are their associated modal amplitudes. If we now impose the condition that the soliton intensity, $I=|\mathbf{E}|^2$, remains axially uniform, the expansion is restricted to only two modes, "a" and "b" say, if we discount accidental degeneracies. Furthermore, these modes must obey the complex orthogonality relation $\mathbf{E}_a\cdot\mathbf{E}_b^*=0$. It is, therefore, apparent that the polarization state changes continuously as the dynamic soliton propagates because the two modes have distinct propagation constants and thus beat.

In summary, the vector field of a dynamic soliton is composed of two modes, a and b, of the linear waveguide characterized by $n^2(I)$ and has the form

$$\mathbf{E}(x,y,z)=a\Psi_a(x,y)\exp(i\beta_a z)\hat{\mathbf{e}}_a + b\Psi_b(x,y)\exp(i\beta_b z)\hat{\mathbf{e}}_b\,, \tag{1}$$

where a,b are (real) amplitude coefficients, Ψ_a,Ψ_b are (real) modal field distributions, β_a,β_b are (real) modal propagation constants, and $\hat{\mathbf{e}}_a,\hat{\mathbf{e}}_b$ are (complex) unit vectors representing an arbitrary polarization state. The polarization unit vectors obey the complex orthogonality relation

$$\hat{\mathbf{e}}_a\cdot\hat{\mathbf{e}}_b^*=0\,, \tag{2}$$

such that the soliton intensity profile remains axially uniform:

$$I(x,y)=|\mathbf{E}(x,y,z)|^2=a^2\Psi_a^2(x,y)+b^2\Psi_b^2(x,y)\,. \tag{3}$$

The simplest nonlinear medium for demonstrating the physics of dynamic solitons is one without intrinsic birefringence [13]. In this case, the refractive index, $n^2(I)$, is a scalar quantity and the nonlinear induced waveguide is isotropic. It is well known [14] that the modes of such a waveguide are solutions of the scalar wave equation (provided the maximum and minimum refractive indices are nearly equal) and that $\hat{\mathbf{e}}_a$ and $\hat{\mathbf{e}}_b$ are arbitrary, apart from satisfying Eq. (2). Thus,

$$\{\nabla_t^2+k^2n^2[I(x,y)]-\beta_a^2\}\Psi_a(x,y)=0\,, \tag{4a}$$

$$\{\nabla_t^2+k^2n^2[I(x,y)]-\beta_b^2\}\Psi_b(x,y)=0\,, \tag{4b}$$

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where ∇_t^2 denotes the transverse Laplacian and $k=2\pi/\lambda$ is the free-space wave number. We note that the physics of dynamic solitons in a medium exhibiting nonlinear induced birefringence is conceptually the same as that described here [15].

It remains for us to give specific examples of dynamic solitons which normally requires solving Eq. (4). For the special case of a Kerr-law medium, Eq. (4) is identical to a set of equations derived in a different physical context [8] for which exact analytical solutions have been found. While this establishes the existence of dynamic solitons in Kerr material, it does not provide an elegant example for conveying the physical principles involved. The ideal example in this regard, and one in the spirit of our intuitive presentation, directly capitalizes on the fact that a field composed of two modes of a linear optical waveguide is the dynamic soliton of some nonlinear medium. Accordingly, we invert [11] the modes of the familiar step-profile optical waveguide shown in Fig. 1(a) which, in turn, leads to solitons of the threshold nonlinearity. This represents an idealization of a sigmoidal saturating system and is illustrated in Fig. 1(b). For a given power, there exist two *classical* solitons—one stable, the other unstable [11,16]—whose shapes change with power. Furthermore, the soliton-induced waveguide can support an arbitrary number of modes depending on the power. None of these properties exist in a Kerr-law medium.

The threshold nonlinearity also supports a rich variety of *dynamic* solitons. These soliton fields are composed of two orthogonally polarized modes and can exist on either a single step-profile waveguide, as is the case for classical solitons, or on any composite step-profile waveguide, each of which has identical (refractive index) profile height but arbitrary width and arbitrary spacing. In our present analysis, we study one-dimensional dynamic solitons which induce a single isolated waveguide [17]. Not only does this provide the most straightforward example but, more importantly, it gives rise to the family of dynamic solitons that are the most highly localized.

The modal fields within the step-profile waveguide of Fig. 1(a) are well known to have the form [14]

$$\Psi_q(x)=\begin{cases}\cos(U_q x/\rho), & q=0,2,4\ldots,\\ \sin(U_q x/\rho), & q=1,3,5\ldots,\end{cases} \tag{5}$$

where $q=0,2,4\ldots$ denote even modes, $q=1,3,5\ldots$ denote odd modes, and U_q is found by solving the eigenvalue equation

$$U_q=V\cos[U_q-(q\pi/2)]\,. \tag{6}$$

Here, the parameter V represents the dimensionless frequency which fully characterizes the modal properties of the waveguide and is defined as

$$V=k\rho\sqrt{n_0^2-n_\infty^2}\,, \tag{7}$$

where ρ denotes the waveguide half-width and $n_0^2-n_\infty^2$ is the waveguide height. The summed power P of the a and

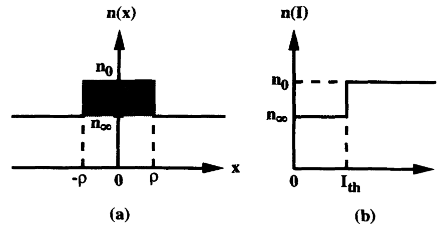

FIG. 1. (a) Step refractive index profile waveguide with a specified maximum index n_0, minimum index n_∞, and half-width ρ. (b) Any two orthogonally polarized modal fields a and b of the step-profile waveguide can form dynamic solitons of the threshold nonlinearity, provided two self-consistency conditions are satisfied. The parameters n_0, n_∞, and I_{th} are constants.

b modes in Eq. (1) is thus given by the expression

$$P=\int_{-\infty}^{\infty}|\mathbf{E}(x,z)|^2dx=a^2N_a+b^2N_b\,, \tag{8}$$

where $N_q=\rho(1+W_q)/W_q$, $W_q=\sqrt{V^2-U_q^2}$, and q refers to either the a or b mode. The fields outside the waveguide decay exponentially as $\exp(-W_q x/\rho)$ while the constant of proportionality is found by demanding that Ψ and $d\Psi/dx$ be continuous. Finally, the modal propagation constants are defined as $\beta_q=[(kn_0)^2-(U_q/\rho)^2]^{1/2}$.

The compound field formed by modes a and b of the step-profile waveguide is a dynamic soliton of the threshold nonlinearity of Fig. 1(b) provided two self-consistency conditions hold. First, the soliton intensity $I(x)=|\mathbf{E}(x,z)|^2$ should be greater than or equal to the threshold intensity, I_{th}, within the waveguide. This, for example, excludes the possibility of having a dynamic soliton composed of two odd modes. Second, self-consistency demands that $I=I_{th}$ at the waveguide boundary, $x=\pm\rho$. Thus, assuming the dynamic soliton is composed of one even and one odd mode, we deduce from Eqs. (3) and (5) that

$$a^2\cos^2(U_a)+b^2\sin^2(U_b)=I_{th}\,, \tag{9}$$

where a^2 and b^2 are the maximum intensities of the a and b modes, respectively.

We now ask what dynamic solitons exist *for a fixed power*, *P*, and consider only those solitons which are most highly localized, i.e., those which induce a single waveguide only [17]. Figure 2 shows examples of dynamic solitons, *all with the same power* (10 times the minimum power required for one classical soliton alone). The four solitons in Fig. 2(a) are composed of the $q=0$ and $q=1$ modes of the soliton-induced waveguide which, in turn, have maximum intensities a^2 and b^2, respectively. In this case, the ratio a/b takes on a continuum of values between two extremes. At one extreme, $b=0$ and only the fundamental soliton exists, i.e., soliton "1" of Fig. 2(a) and the broken curve of Fig. 3(a). At the other extreme, $a^2=I_{th}$ and the contribution of the fundamental mode is the smallest possible for the existence of a dynamic soliton. This leads to soliton "4" of Fig. 2(a) and the solid curve of Fig. 3(a). One important finding is

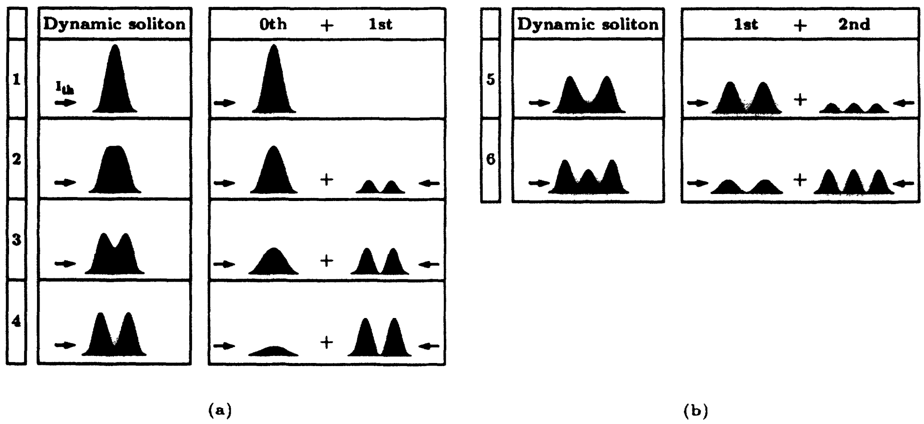

FIG. 2. Intensity profiles of dynamic solitons. All solitons have the same power, $P=10P_{\min}$, but they induce waveguides with different V values, i.e., $V=3.36$, 4.09, 4.90, 5.56, 6.49, and 7.40 for solitons 1–6, respectively. $P_{\min}$ is defined below Eq. (10). These solitons are composed of two (one even and one odd) orthogonally polarized modes, a and b, of the induced waveguide. In (a) the dynamic solitons are composed of the zeroth and first modes while in (b) they are composed of the first and second modes. Arrows indicate the position where $I=I_{\text{th}}$.

that *the greater the available power, the more nearly this soliton appears like the first mode propagating on its own. Thus, this two-peaked soliton appears much like the two (π out of phase) fundamental solitons traveling in parallel with each other, each with (approximately) the same polarization.* Solitons "2" and "3" in Fig. 2(a) illustrate two intermediate situations, one for a flattop soliton and the other for a soliton composed of equal-amplitude modal components.

Another important insight comes from comparing the fundamental soliton (soliton 1) with the dynamic soliton (soliton 4) *when both solitons have the minimum power necessary for them to exist*, i.e., positions "*d*" and "*D*" in Fig. 3(a), respectively. The power of the two-peaked soliton is then approximately twice that of the fundamental soliton. This gives an indication of the minimum power required to most closely pack two solitonlike objects which maintain their individuality and yet travel in parallel. Finally, examples of three-packed dynamic solitons are shown in Fig. 2(b). They are composed of the $q=1$ and $q=2$ modes of the soliton-induced waveguide. Of course, dynamic solitons with an arbitrary number of intensity peaks are possible for sufficiently large power because the induced waveguide can then propagate higher-order modes with multiple intensity peaks. In all cases we discuss, the valley created by one polarization is compensated by a mountain due to the other polarization.

It is possible [18] to derive simple approximate expressions for the fields and power of dynamic solitons which are sufficiently accurate to describe all their physical properties over the parameter range of greatest interest, i.e., the solitons associated with the curves of positive slope in Fig. 3(a). As a specific example, we consider dynamic solitons formed by the zeroth and first mode. The modal amplitudes, a and b, then take on a continuum of values as discussed above. In the extreme case of $b=0$,

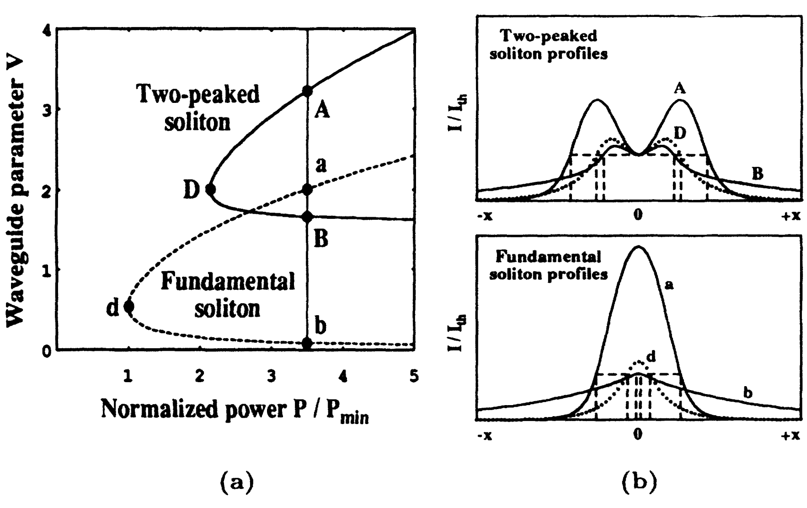

FIG. 3. (a) Dependence of the induced waveguide parameter V on the power P of a dynamic soliton composed of the zeroth and first modes (solid line) and the zeroth mode only (broken line). These curves are derived from Eqs. (8) and (9) and have the functional forms $P=(I_{\text{th}}/\alpha)V[(1+W_a)/W_a+(W_a^2/U_b^2)(1+W_b)/W_b]$ and $P=(I_{\text{th}}/\alpha)(V^3/U_a^2)\times(1+W_a)/W_a$, respectively. Expressions for α and $P_{\min}$ are given below Eq. (10). The broken curve represents the minimum power required for the dynamic soliton to exist. (b) Representative examples of soliton intensity profiles which correspond to the labeled points on the curves in (a). Broken lines denote the positions of the soliton-induced waveguide.

we obtain an expression for the intensity profile of a fundamental soliton in the form

$$I(x)=I_{\rm th}(\gamma\hat{P})^{2/3}\cos^2[x\alpha/(\gamma\hat{P})^{1/3}]\,, \qquad (10)$$

where $\hat{P}=P/P_{\rm min}$ is the relative power, $P_{\rm min}=3.44I_{\rm th}/\alpha$, $\alpha=k\sqrt{n_0^2-n_\infty^2}$, and $\gamma=6.88/\pi$. Thus, the amplitude and width of the soliton *field* both depend on $P^{1/3}$, whereas in Kerr-law material, the soliton amplitude is proportional to P while the width is inversely proportional to P. At the other extreme, when $a^2=I_{\rm th}$, the dynamic soliton has an intensity profile of the form

$$I(x)=I_{\rm th}\{\cos^2[x\alpha/(4\gamma\hat{P})^{1/3}] + (\gamma\hat{P}/2)^{2/3}\sin^2[x\alpha(2/\gamma\hat{P})^{1/3}]\}\,, \qquad (11)$$

to leading order in P.

Dynamic solitons are analogous to temporal vector solitons [8,9,19–21] in that both are composed of orthogonal polarization states. Vector solitons usually refer to stationary waves in a birefringent medium whose fields remain axially uniform. In contrast, dynamic solitons can propagate in a medium without intrinsic birefringence. Although their intensity profile remains axially uniform, their polarization state changes with propagation. The analogous temporal dynamic solitons have the property that each polarization, when viewed independently, is a solitary wave but the two waves beat due to different phase velocities. Vector solitons occur when the two modes comprising the dynamic soliton are degenerate. Although the mathematics of time and one-dimensional space are similar [10], the physical approach developed here applies only to the spatial domain.

In summary, classical fundamental solitons are the modes of the linear optical waveguides they induce. Dynamic solitons are composed of two orthogonally polarized beams, each of which is a mode of the soliton-induced waveguide but need not be a soliton on its own. The beating of these two modes gives rise to the polarization dynamics [22]. While the concept applies to any arbitrary nonlinearity, $n^2(I)$, the one-dimensional threshold nonlinearity provides the simplest example possible. A dynamic soliton of circular cross section could also be derived from the well-known modes of (circular) step-profile optical fibers [11].

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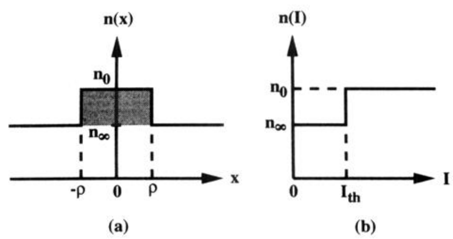

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