Alpha-Particle Binding Energies for Realistic Nucleon-Nucleon Interactions

W. Glöckle and H. Kamada
Institut für Theoretische Physik II Ruhr-Universität Bochum, D44780 Bochum, Germany
(Received 17 May 1993)

The four-nucleon Yakubovsky equations are solved precisely for various realistic NN interactions. The resulting binding energies underbind the α particle by about 1 to 5 MeV. The one-boson-exchange NN potentials with smallest deuteron d-state probabilities provide the strongest binding. Within the group of NN-force models used the $3N$ and $4N$ binding energies are strongly correlated. The probabilities to find two nucleons at a distance r are compared to each other in the deuteron, the triton, and the α particle.

PACS numbers: 21.45.+v, 21.10.Dr, 21.30.+y, 27.10.+h

What is a useful Hamiltonian for low energy nuclear physics (below the pion threshold) to describe bound and scattering states? Is it a good approximation to use the same nucleon-nucleon (NN) interaction, which acts between two free nucleons? Are three-nucleon forces (3NF) (or even forces between more than three nucleons) required? Is the nonrelativistic framework sufficient? These are old questions. The existence of modern supercomputers combined with powerful algorithms allows us to test existing theoretical concepts quantitatively. At least this helps to guide theory to some extent, which is still unable to control the problem of strong interactions on the meson and/or quark level.

In recent years rigorous solutions [1] of the three-nucleon ($3N$) Faddeev equations based on realistic NN interactions showed that $3N$ scattering data (cross sections and various spin observables in elastic nucleon-deuteron scattering and in the breakup process) can be described extremely well in the most simple dynamical scenario: unperturbed NN forces only in a nonrelativistic Schrödinger equation. This is independent to a large extent of the specific choice of the NN interaction, which demonstrates the "stability" of that result [2]. There are a few exceptions still disturbing that simple picture: the nucleon-deuteron analyzing powers at very low energies, which are very sensitive to the ^3P_J NN forces [3] and possibly some breakup cross sections [2]. While the first defect might be cured by better fine tuning of the NN forces (new dedicated measurements especially in the np system, improvement of the NN phase-shift analysis and corresponding adjustments of the NN force parameters), the latter one, if confirmed experimentally, points possibly to the action of a 3NF. From the very first theoretical insight [4] into the action of a 3NF in the $3N$ continuum, we see that it is quite possible that a signature of the 3NF shows up in some breakup configurations and not in other ones. Thus it will require very careful studies both experimentally and theoretically to manifestly pin down the action of a 3NF. In any case it will be a small effect in $3N$ scattering.

The triton binding energy still poses a problem [5]. Thereby the strength and shape of the NN tensor force in the dominant $^3S_1$-$^3D_1$ state plays a decisive role. Its action shows up in the d-state probability P_d of the deuteron, which unfortunately is not a measurable quantity, but whose value is strongly correlated to the theoretical triton binding energy. The smaller the P_d the larger the triton binding energy. This is exhibited in Table I for the various current realistic NN interactions (34 channel calculations). Thus all the currently available NN potentials underbind the triton by about 0.3–1.0 MeV (this estimate leaves out the Reid potential which should be updated and the Bonn A potential which might be criticized because of its wrong prediction of the ε_1 phase-shift parameter at high energies), which has to be not only compared to the experimental binding energy of 8.48 MeV but also to the typical total potential energy in the triton, which is about 50 MeV. Thus the missing energy is only of the order of 2% of the total potential energy. It is a fact that the nuclear binding energies are differences of big numbers (kinetic versus potential energies), which implies that even small deficiencies in the theory are magnified. Further experimental efforts to narrow down the still existing uncertainty in the $^3S_1$-$^3D_1$ mixing parameter ϵ_1 are under way [6,7]. This will reduce the spread in theoretical binding energy predictions, once the potentials are fitted to the same "final " ϵ_1 value. However, there will still remain a difference between local NN potentials and potentials which have in addition

TABLE I. $3N$ and $4N$ binding energies for various realistic NN potentials in comparison with experimental values. The binding energies increase with decreasing deuteron d-state probabilities P_d.

Potential	P_d	^{3}H	^{4}He
Reid	6.47	7.34	23.45
AV14	6.08	7.68	24.62
Paris	5.77	7.46	24.26
Njimegen	5.39	7.63	25.03
Bonn B	4.99	8.14	27.04
Bonn A	4.38	8.32	28.11
Expt.		8.48	28.30
Without Coulomb			29

0031-9007/93/71(7)/971(4)$06.00

nonlocal parts like the Bonn one-boson-exchange (OBE) potentials and which provide additional binding energy.

Now the α particle has a much higher central density and might therefore possibly pose a harder test for current NN-force models. This has been studied with great care by the Argonne-Urbana group [8,9] and in [10], using variational Monte Carlo and Green's-function Monte Carlo (GFMC) techniques. They investigated the Argonne AV14, the Urbana V14, the Nijmegen, and the r-space Bonn (OBEPR) $2N$ potentials, including on top various $3N$ force models. However, up to now they did not study meson theoretical NN forces, which are nonlocal and not so easily accessible by their configuration space techniques. The variational upper bounds are quoted to be uncertain by about (3–4)% [9], which can lead to up to 1 MeV underbinding in the α particle. The Green's-function Monte Carlo method is in principle an exact technique for bosons and the accuracy is limited only by statistics. For fermions, however, there are still problems and also for certain momentum dependencies, which occur in realistic NN forces [10]. For the model forces Reid V8 and AV8 the GFMC provided the most rigorous ^{4}He calculations [10] and comparison to variational results indicates that the latter can lack up to 1.5 MeV of binding energy. It is therefore very desirable to use directly the generalization of the exact Faddeev formalism for three particles and the Yakubovsky scheme for four particles [11]. We developed techniques to solve the $4N$ Yakubovsky equations for any given NN interaction in [12] and shall present here the final converged results.

Before doing that we would like to briefly sketch Yakubovsky's idea,which is equivalent to the work by Alt, Grassberger, and Sandhas [13]. Those schemes have been worked out even for any general number of particles. One regards all possible fragmentations of the system of four particles into subclusters. Obviously there are two two-body fragmentations of the type 3+1 and 2+2. The three-body subclusters can further be fragmented into two-body subclusters and a third particle. Then one can form chains of consecutive fragmentations until one reaches two-body subclusters: $4 \rightarrow 3+1 \rightarrow 2+1+1$. In the last stage one singles out one specific pair of particles in the two-body subcluster. There are twelve chains of that type. In the case of the 2+2 partitions, there is no further fragmentation. If one singles out one specific two-body subcluster, then there are six possibilities. The art is now to decompose the total four-body wave function into eighteen parts related to the eighteen ways of fragmenting a group of four particles. Those parts are called Yakubovsky (Y) components and obey a set of eighteen coupled equations, the Yakubovsky equations. We refer to the original work [11,13] for the way this is done and to [12,14] for the application to the actual $4N$ problem, our notation, and more formal background. For identical particles the twelve Y components related to the 3+1 partitions are all identical in their functional form and one can choose one Y component and generate the remaining ones through suitable particle permutations; the same is true for the six Y components of the 2+2 partition. Only one is needed and the remaining ones are generated by particle permutations. The two independent Y components, ψ_1 and ψ_2, related to the 3+1 and 2+2 partitions, obey the coupled set [12]

$$\psi_1 = -G_0\ T\ P\ P_{34}\ \psi_1 + G_0\ T\ P\ \psi_2, \tag{1}$$

$$\psi_2 = G_0 \tilde{T} \tilde{P}\ (1 - P_{34})\ \psi_1. \tag{2}$$

In that set occur permutation operators P_{34}, $P \equiv P_{12}\ P_{23} + P_{13}\ P_{23}$, and $\tilde{P} \equiv P_{13}\ P_{24}$, the free four-body propagator G_0 and the operators T and $\tilde{T}$ describing the full dynamics within a three-body subcluster and for two noninteracting two-body clusters, respectively. They obey

$$T = t + T\ P\ G_0\ t, \tag{3}$$

$$\tilde{T} = t + \tilde{T}\ \tilde{P}\ G_0\ t, \tag{4}$$

where t is the off-shell t matrix for the particles 1 and 2. The four-body wave function composed of eighteen Y components is

$$\Psi = (1 + P - P_{34}\ P + \tilde{P})(\psi_1 - P_{34}\ \psi_1 + \psi_2). \tag{5}$$

We solve the set (1),(2) in momentum space and in a partial wave projected basis. The set of all relative orbital angular momenta, spins, and isospins is usually called a channel. Apparently many channels will be active in a complete description of the $4N$ dynamics. We refer to [12] for all those technical details and also for the algorithm to handle the very large number of unknowns (of the order 10^6).

Our techniques have been checked [12] for s-wave model forces against a configuration space calculation [15], which is based on a totally different mathematical structure, partial integral-differential equations. The agreement is very good. A harder test [12] for the local MT-V potential, requiring already many angular momentum combinations (channels), against an exact GFMC [16] result and very precise variational calculations [17,18] was also very successful.

We use current NN potentials, which describe quite well the large amount of NN data and deuteron properties. These are the Nijmegen [19], the Argonne AV14 [20], the Paris [21], and the Bonn B and Bonn A [22] potentials. We also include, more for historical reasons, the Reid potential and its supplemented form [23]. Our results (based on 190 channels) are displayed in Table I. They are based on keeping the NN forces different from zero in all NN partial wave states with total angular momenta $j \leq 3$. This is sufficient in the triton within 20 keV. Since ^{4}He and ^{3}H binding energies are correlated, as will be shown below, and the slope is roughly 5 we estimate the additional contribution to the binding energy of neglected higher partial wave components to be about

100 keV. More details and a more precise estimate on the neglected NN force components for $j \geq 4$ will be given elsewhere. The many channels allow that, for instance, a $3N$ subcluster does not only exist in the state of total $3N$ angular momentum and parity $1/2^+$, which are the ^{3}He or ^{3}H ground state quantum numbers, but also in $1/2^-$, $3/2^-$, and $3/2^+$. Also the accompanying fourth nucleon can have corresponding relative orbital angular momenta 0 and 1. In the case of a 2+2 partition, the two two-body subclusters cannot only be in the state of total angular momentum and parity 1^+, which is realized in the deuteron, but in all states $0^\pm$ up to $3^\pm$. Finally the relative orbital angular momenta of the two two-body subclusters can be 0, 1, and 2. The computer time for one energy is about 3 h on a NEC-SX3 (plus once 5 h for the geometrical coefficients).

In Table I we see a rather large spread in the predictions of the theoretical α-particle binding energies of about 4 MeV. Again the smaller the P_d the larger the binding energy. Since we excluded the pp Coulomb force, which leads to an estimated Coulomb energy of $\approx$ 700 keV [9,10], we should compare our theoretical results to $\approx$ 29 MeV. The meson-theoretical OBE Bonn A and Bonn B results come rather close to the experimental value, within 1 or 2 MeV, whereas the other potentials are off by 4–5 MeV. There is a well established charge-independence breaking of the NN force in the state $^1S_0$, saying that the np force is slightly stronger than the strong pp force. If one takes that into account, the Bonn results have to be reduced by about 0.6 MeV [12]. The missing binding energies, especially for the Bonn potentials, are again only about 2% of the total potential energy ($\langle V \rangle_{\text{Bonn B}} = -104.8$ MeV). That defect is therefore very similar to the one in the triton.

Let us now compare our results to existing variational upper bounds. For AV14 we get 24.62 MeV against 24.24 MeV [9], 24.45 MeV [24], and 23.86 MeV [10], for Nijmegen 25.03 MeV against 23.92 [10], and for Paris 24.26 against 25.5 MeV [25]. With the exception of the last case we are below the variational upper bounds.

Now we demonstrate that $3N$ and $4N$ binding energies are correlated. For simple forces this was shown to be true for the first time in [26]. We display in Fig. 1 $3N$ and $4N$ binding energies for the various realistic NN interactions. We see indeed a narrow band, which would shrink even more in width if the charge-independence breaking in the state $^1S_0$ were taken into account. Thus, one might say unfortunately, the $4N$ binding energies are strongly correlated to the $3N$ binding energies, at least for those NN force models. The $4N$ system does not probe those forces in a different manner as the $3N$ system. Strong deviations from that linear correlation would have been a more interesting insight in order to distinguish the various NN forces. The band in Fig. 1 hits the experimental point. Additional dynamics (other NN forces with new features and/or 3NF, relativistic effects) have to move the theory along the band to the experimental point.

Finally we would like to compare the probability $C(r)$ to find two nucleons at a certain distance, averaged over spin directions, for the deuteron, the triton, and the α particle. $C(r)$ is normalized as $\int_0^\infty C(r)dr = 1$. In Fig. 2 we show our results for the Bonn B NN potential. The different tails result obviously from the different separation energies. More interesting is the nearly parallel behavior of the $C(r)$'s for $r \leq 1$ fm. If one multiplies the deuteron and triton correlation functions by appropriate factors such that their maxima coincide with the maximum of the α-particle correlation function, then for $r \leq 1.7$ fm the three curves are very close to each other. They can be brought to a nearly perfect coincidence if one shifts in addition the deuteron correlation function by 0.1 fm and the triton correlation function by 0.05 fm to the left. This tells us that the three correlation functions are very closely related to each other. Proton-proton densi-

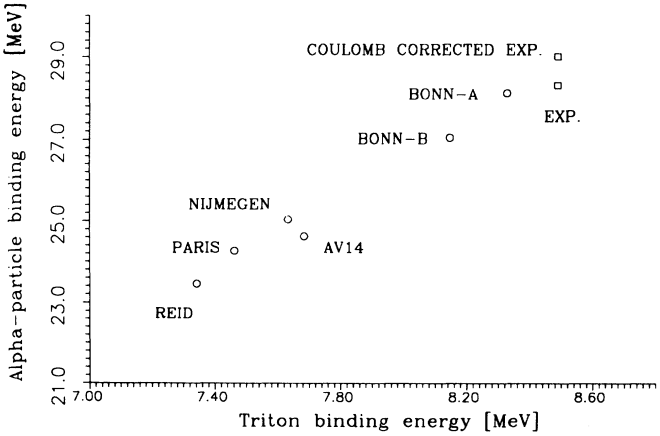

FIG. 1. The α particle against the triton binding energies for various realistic NN forces are correlated in a band, which includes the experimental point.

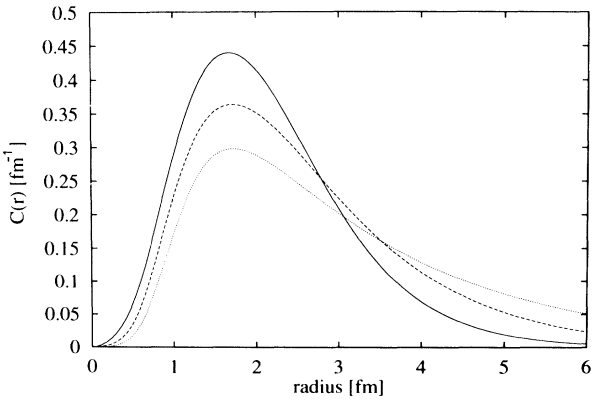

FIG. 2. The correlation function for the deuteron (dotted curve), triton (dashed curve), and the α particle (solid curve) based on the Bonn B potential.

ties for ^{3}He and ^{4}He are also shown in [27]; however, they are based on a severely truncated six-component Y wave function.

In conclusion, we have calculated for the first time for various existing realistic NN forces the α-particle ground state energy by solving the $4N$ Y equations precisely. Depending on the d-state probability in the deuteron, as generated by the various NN forces, the α-particle binding energy is within 1–5 MeV from the experimental value. It appears that an improved knowledge of the deuteron wave function itself is needed to decide which of the potentials is closer to reality. That knowledge might be inferred from electron scattering and nucleon-deuteron scattering at higher energies, where the obscuring rescattering effects diminish. In view of the relatively small defects in the binding energy in comparison to the total potential energy and the exciting agreement of $3N$ scattering observables with the data, one can say that this simple dynamical picture of unperturbed NN forces is already a good representation of reality.

This work was supported financially by the Deutsche Forschungsgemeinschaft (H.K.). The numerical calculations were performed on the NEC SX3 of the Universität Köln and the CRAY Y-MP of the Höchstleistungsrechenzentrum in Jülich.

[1] W. Glöckle, H. Witała, and Th. Cornelius, Nucl. Phys. **A508**, 115c (1989).
[2] W. Glöckle, H. Kamada, and H. Witała, Second Chinese-German Symposium on Medium Energy Physics, Bochum, 1992 (to be published).
[3] H. Witała and W.Glöckle, Nucl. Phys. **A528**, 48 (1991).
[4] D. Hüber, H. Witała, and W. Glöckle, Few-Body Syst. (to be published).
[5] For a recent review, see B. F. Gibson, Nucl. Phys. **A543**, 1c (1992).
[6] I. Sick (private communication).
[7] W. Tornow (private communication).
[8] J. Carlson, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A401**, 59 (1983) ; R. B. Wiringa, Nucl. Phys. **A401**, 86 (1983).
[9] R. B. Wiringa, Phys. Rev. C **43**, 1585 (1991).
[10] J. Carlson, Phys. Rev. C **38**, 1879 (1988); Nucl. Phys. **A508**, 141c (1990).
[11] O. A. Yakubovsky, Sov. J. Nucl. Phys. **5**, 937 (1967).
[12] H. Kamada and W. Glöckle, Nucl. Phys. **A548**, 205 (1992); H. Kamada and W. Glöckle, Phys. Lett. B **292** , 1 (1992).
[13] P. Grassberger and W. Sandhas, Nucl. Phys. **B2**, 181 (1967); E. O. Alt, P. Grassberger, and W. Sandhas, Phys. Rev. C **1**, 85 (1970).
[14] W. Glöckle, *The Quantum Mechanical Few-Body Problem* (Springer-Verlag, Heidelberg, 1983).
[15] N. W. Schellingerhout, J. J. Schut, and L. P. Kok, Phys. Rev. C **46**, 1192 (1992).
[16] J. Lomnitz-Adler, V. R. Pandharipande, and R. A. Smith, Nucl. Phys. **A361**, 399 (1981).
[17] H. Kameyama, M. Kamimura, and Y. Fukushima, Phys. Rev. C **40**, 974 (1989); M. Kamimura and H. Kameyama, Nucl. Phys. **A508**, 17 (1990).
[18] Y. Akaishi, in *Models and Methods in Few-Body Physics*, edited by L. S. Ferreira *et al.*, Lecture Notes in Physics Vol. 273 (Springer, Berlin, 1986), p. 324.
[19] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D **17**, 768 (1978).
[20] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[21] M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).
[22] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[23] R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968); B. D. Day, Phys. Rev. C **24**, 1203 (1981).
[24] R. B. Wiringa (private communication).
[25] P. Goldhammer, Phys. Rev. C **29**, 1444 (1984).
[26] J. A. Tjon, Phys. Lett. **56B**, 217 (1975).
[27] B. Doyle, B. Goulard, and G. Cory, Phys. Rev. C **45**, 1444 (1992).