Excitation of Multiphonon States in Relativistic Heavy Ion Collisions

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One- and two-step Coulomb excitation cross sections of the double-phonon giant dipole resonance states are calculated in a semiclassical formalism. The contribution of nuclear interaction is shown to be small. The multipole sum rules are used to evaluate corresponding nuclear matrix elements; the widths of final and intermediate states are taken into account. The recent experimental data are discussed and compared to our calculations.

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With the excitation energy increasing, nuclear collective motion reflects to a greater extent the general properties of nuclear matter rather than the specific features of individual nuclei. It explains the growing interest to the excitation and decay of collective modes high in continuum. Fast charged particles, especially heavy ions, producing a strong electromagnetic pulse in a peripheral collision with nuclei, seem to be an adequate tool for this purpose (for a review see [1] and references therein).

The excitation of giant resonances (GR) in heavy-ion collisions was first observed in cosmic-ray experiments [2,3]. Since the pioneer work [4] several experiments have aimed at the investigation of this process. Because of the huge excitation cross sections for large-Z nuclei, the possibility of excitation of multiphonon GR states was suggested [5]. While the cross sections for the Coulomb excitation of GR can be as large as several barns, those for double-phonon states are smaller by a factor 10–100. The identification of these states by gamma-decay techniques is feasible [5], but the cross sections are even smaller by another factor 10–100. Successful experiments on the Coulomb excitation of multiphonon states have been reported recently [6–9] with a clear resonance identified as the double giant dipole resonance (GDR).

The theory of Coulomb excitation of GR is considered to be well understood [1,5,10,11]. However, the calculation for multiphonon excitation requires new assumptions so that the comparison with coming data will serve as a more stringent test. In this Letter we explore the key ingredients of the theory. We calculate the excitation of single and multiphonon GR with a semiclassical formalism [11] justified [1] due to the short wavelength of ions and perturbative character of the electromagnetic interaction. The conclusions drawn could be helpful for future experimental studies.

Let us consider the excitation of a projectile in a collision with a target with charge Z_T at an impact parameter b. The first order amplitude for the transition $i \to f$, $\omega_{fi} = \omega$, is given by ($\hbar = 1$)

$$a_{fi} = -i \int_{-\infty}^{\infty} e^{i\omega t}\, [\rho_{fi}(\mathbf{r})\, \phi(\mathbf{r},\ t) - (\mathbf{v}/c) \cdot \mathbf{j}_{fi}\, \phi(\mathbf{r},\ t)]\, d^3r\, dt\ , \tag{1}$$

where ρ_{fi} and $\mathbf{j}_{fi}$ are the charge density and current matrix elements, respectively, $\phi(\mathbf{r},\ t)$ is the Lienard-Wiechert potential,

$$\phi(\mathbf{r},\ t) = Z_T e \gamma\ [(b-x)^2 + y^2 + \gamma^2 (z - vt)^2]^{-1/2}\ , \tag{2}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ and v is the projectile velocity.

At relativistic energies, the magnetic interaction in Eq. (1) is important even for pure electric excitations. In the long wavelength approximation, $\omega r/c \ll 1$, performing the multipole expansion of the potential (2), the time integrals in Eq. (1) are expressed in terms of the modified Bessel functions K_n. Using the continuity equation we obtain for the electric dipole excitations (higher multipoles are treated similarly)

$$a_{fi}^{(E1)} = -i \sqrt{\frac{8\pi}{3}}\ \frac{Z_T e}{vb}\ \xi\ \left\{ [D_{fi}^{(-1)} - D_{fi}^{(1)}]\ K_1(\xi) + i \frac{\sqrt{2}}{\gamma}\ D_{fi}^{(0)}\ K_0(\xi) \right\}\ , \tag{3}$$

where $\xi = \omega b / \gamma v$ and

$$D_{fi}^{(m)} = \int r\ Y_{1m}(\hat{\mathbf{r}})\ \rho_{fi}(\mathbf{r})\ d^3r \tag{4}$$

is the dipole matrix element for the nuclear excitation. The magnetic interaction is responsible for the factor $1/\gamma$ in the term $m = 0$ in Eq. (3) which suppresses at relativistic energies the longitudinal (with respect to the beam axis) excitation so that only the transverse ($m = \pm 1$) components are important. When the long wavelength approximation is not valid the matrix elements (4) are to be replaced by the nonapproximated ones [12]; the other factors do not change [11].

The amplitude for a two-step excitation to a state $|2\rangle$ is given by the sum over intermediate states $|1\rangle$,

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$$a_{20}^{2\mathrm{nd}} = -\sum_{1} \int_{-\infty}^{\infty} dt\ e^{i\omega_{21}t}\ V_{21}(t) \int_{-\infty}^{t} dt'\ e^{i\omega_{10}t'} V_{10}(t') , \tag{5}$$

where $V_{fi}(t)$ is the matrix element inside brackets of the integrand of Eq. (1). It is convenient to express this amplitude in a symmetric form

$$a_{20}^{2\mathrm{nd}} = \frac{i}{2\pi} \sum_{1} \int \frac{dq}{q+i0} a_{21}(\omega_{21}-q) a_{10}(\omega_{10}+q). \tag{6}$$

Because of strong nuclear absorption, the total cross sections for Coulomb excitation are usually obtained by integrating over impact parameters from a minimum value b_{min} which should be considered as a phenomenological parameter, i.e.,

$$\sigma^{C} = 2\pi \int_{b_{\mathrm{min}}}^{\infty} db\ b\ |a_{fi}|^2 . \tag{7}$$

To evaluate the matrix element (4), we consider the conventional sum rule derived without exchange and velocity-dependent corrections,

$$\sum_{f} \omega_{fi} |D_{fi}^{(m)}|^2 = \frac{3}{4\pi}\ \frac{1}{2m_N}\ \frac{NZ}{A}\ e^2. \tag{8}$$

The right-hand side S_D of (8) does not depend on the choice of the state $|i\rangle$. (This dependence is weak even if exchange terms are taken into account.) If the sum rule for the ground state $|0\rangle$ is saturated by the GDR $|1\rangle$, we obtain for the reduced matrix element $(1||D||0)^2 = S_D/\omega_{10}$. For the state $|1\rangle$ as an initial one, the sum (8), under similar assumption, is exhausted by (i) "down" transition $1 \to 0$ which has negative transition energy $-\omega_{10}$, and (ii) "up" transitions to the double GDR states $|2;L\rangle$ with angular momenta $L = 0, 2$ and excitation energies ω_{21}^{L}. After simple algebra, we obtain for the up transition

$$(2;L||D||1)^2 = 2\frac{S_D}{\omega_{21}^{L}} = 2\frac{\omega_{10}}{\omega_{21}^{L}} (1||D||0)^2. \tag{9}$$

This relation includes, in addition to the stimulated radiation factor of 2, the ratio of frequencies which, according to the data, is larger than 1. The generalization for higher order processes and other multipoles is straightforward.

The assumption of saturation ignores the widths of the resonances. The presence of widths might also be important for the total cross sections since, due to the "adiabatic cutoff" [1], the amplitudes (3) might vary strongly with ω. We assume that the spreading of the GR is mostly due to the coupling to the background of complicated states. The stationary superpositions are

$$|f\rangle = C_{\lambda}^{(f)}\ |\lambda\rangle + \sum_{\nu} C_{\nu}^{(f)} |\nu\rangle , \tag{10}$$

where $|\lambda\rangle$ is a collective state and $|\nu\rangle$ are complicated many-particle–many-hole states. If the resonance component dominates as it should be for the one-body multipole operator, we find the first-order amplitude $a_{fi}^{(\lambda)} \simeq [C_{\lambda}^{(f)}]^*\ a_{\lambda}^{1\mathrm{st}}(\omega_{fi})$ where $a_{\lambda}^{1\mathrm{st}}$ stands for the original amplitude of Eq. (1). The fragmentation of the collective mode is described [13] by the strength function

$$\mathcal{F}_{\lambda}(\omega) = \sum_{f} |C_{\lambda}^{(f)}|^2\ \delta(\omega - \omega_{fi}) , \tag{11}$$

which is usually taken as the Breit-Wigner (BW) function with the centroid and width parameters ω_λ and Γ_λ. The direct probabilities are then given by

$$P_{\lambda}^{1\mathrm{st}}(\omega) = \mathcal{F}_{\lambda}(\omega)\ |a_{\lambda}^{1\mathrm{st}}(\omega)|^2 . \tag{12}$$

The total probability to excite the double phonon state is

$$P(\omega) = \sum_{f} |a_{fi}^{1\mathrm{st}} + a_{fi}^{2\mathrm{nd}}|^2\ \delta(\omega - \omega_{fi}) \equiv P^{1\mathrm{st}}(\omega) + P^{2\mathrm{nd}}(\omega) + P^{\mathrm{int}}(\omega). \tag{13}$$

Retaining collective components both for the intermediate and final states and assuming the shape $\mathcal{F}_1(\omega)$ of the single phonon peak, we obtain from Eq. (6)

$$a_{fi}^{2\mathrm{nd}} = [C_{2}^{(f)}]^* \int d\omega' \mathcal{F}_1(\omega') \int \frac{i}{2\pi} \frac{dq}{q+i0} a_{21}(\omega_{fi} - \omega' - q) a_{10}(\omega' + q) \equiv [C_{2}^{(f)}]^*\ \tilde{a}_{20}(\omega_{fi}) . \tag{14}$$

Therefore, the second-order excitation is given by

$$P^{2\mathrm{nd}}(\omega) = \sum_{f} |a_{fi}^{2\mathrm{nd}}|^2\ \delta(\omega_{fi} - \omega) = \mathcal{F}_2(\omega) |\tilde{a}_{20}(\omega)|^2 ; \tag{15}$$

the interference term in (13) is derived analogously.

Collective modes can be excited also by nuclear interaction. For near-grazing collisions we may assume that the projectile interacts with the target via vibrational fluctuations of the optical potential (see, e.g., [14]). It allows us to relate the nuclear matrix element to that for the electromagnetic transition. Treating the relative motion for high energy collisions in the eikonal approximation [15], the nuclear excitation cross section can be expressed similar to (7),

$$\sigma_{\lambda}^{N} = 2\pi \int db\ b\ \mathcal{P}_{\lambda}(b) , \tag{16}$$

$$\mathcal{P}_{\lambda} = \left(\frac{4\pi^2}{\hbar^2 v^2}\right)^2 \left(\frac{3}{4\pi}\ ZeR_c\right)^{-2} B(E\lambda; \lambda \to 0)\ \exp\{-2\ \mathrm{Im}\chi(b)\} \sum_{\mu} |F_{\lambda\mu}(\omega_\lambda,\ b)|^2 . \tag{17}$$

Here R_c is a (uniform) charge distribution radius in the ground state of the projectile, $B(E\lambda)$ stands for the reduced probability of vibrational excitation, and $F_{\lambda\mu}(\omega_\lambda, b)$ is given by the integral along the straight trajectory,

$$F_{\lambda\mu}(\omega_\lambda, b) = \frac{i^{\mu}}{(2\pi)^2} \int dZ\ e^{i\omega_\lambda Z/v} R \frac{dU_{\mathrm{opt}}}{dR} Y_{\lambda\mu}(\theta, 0), \tag{18}$$

with $R = \sqrt{b^2 + Z^2}$ and $\cos\theta = Z/R$. At energies higher than 100 MeV/nucleon we can assume that the optical potential is pure imaginary being related to the nucleon-nucleon cross sections and to the ground state matter distribution which gives [15]

$$2 \,\mathrm{Im}\, \chi(b) = \sigma_{NN} \int dZ \int d^3r \; \rho_T(\mathbf{R} - \mathbf{r}) \; \rho_P(\mathbf{r}) \, . \qquad (19)$$

The nuclear excitation probability (17) is peaked at grazing impact parameters.

We have calculated the cross sections for the reactions ^{136}Xe+^{208}Pb at 0.69 GeV/nucleon and ^{209}Bi+^{208}Pb at 1 GeV/nucleon measured recently at GSI [7,8]. Cross sections (in mb) for the Coulomb excitation of the isovector GDR and the isoscalar and isovector giant quadrupole resonance (IVGDR, ISGQR, and IVGQR) in ^{136}Xe are given in Table I. We have assumed that they are located at 15.3, 12.3, and 24 MeV, have widths 4.8, 4, and 7 MeV, and exhaust 100%, 70%, and 80% of the corresponding sum rules, respectively [16]. We used $b_{\mathrm{min}}^{<} = 1.2(A_P^{1/3} + A_T^{1/3})$ fm=13.3 fm as a lower limit guess and $b_{\mathrm{min}}^{>}$=15.6 fm suggested by the parametrization [17] as an upper limit (numbers inside parentheses). The parametrization [18] yields an intermediate value for this quantity. Various angular momentum components are shown separately. In the calculation of the last column the widths of the GR are taken into account with the BW strength function $\mathcal{F}_\lambda$ which increases the cross sections by about (10–20)% (see also [10]). The experimental value [7] 1110±80 mb for the GDR is much smaller which made the authors of [7] claim that the GDR absorbs only 65% of the sum rule (this number apparently contradicts to the systematics of data for real monochromatic photons [16,19]). Using this value, our result reduces to 1613 (1183) mb which seems to prefer the upper value, $b_{\mathrm{min}}^{>}$. The numbers in parentheses are also in rough agreement with the data [7] for the ISGQR and IVGQR.

For the nuclear excitation of the ISGQR in the same reaction (excitation of isovector modes is suppressed by a factor $[(N - Z)/A]^2$), we found $\sigma^N = 5.3$ mb with a deformation parameter $\beta R = 0.7$ fm for ^{136}Xe. In the calculations we used σ_{NN}=40 mb and Fermi density distributions with $\rho_0 = 0.17$ fm^{-3}, $R = 5.6$ (6.5) fm, and $a = 0.65$ (0.65) fm for Xe (Pb). The nuclear contribution is weak due to the poor overlap in impact parameter between the last two terms of Eq. (17).

For the double GDR state we have calculated (see Table II) the direct (for $L = 2$, since $L = 0$ states cannot be Coulomb excited [1]) and two-step probabilities.

The inclusion of the widths of the final (GDR × GDR) and the intermediate (GDR) state again increases the cross sections by (10–20)%. For the position and width of the GDR × GDR state we took $E = 28.3$ MeV and $\Gamma = 7$ MeV, respectively [7], which corresponds to $\omega_{10} = 15.3$ MeV and $\omega_{21}^L = 13$ MeV, for both $L = 2$ and $L = 0$. For the direct excitation we assumed that the resonance would exhaust 20% of the ISGQR sum rule if the missing strength of the ISGQR could be located at the double dipole energy due to the (quadrupole-dipole-dipole) anharmonic coupling. Microscopic calculations [20] give much smaller value than this overestimated upper boundary of the direct excitation. It means that the two-step process dominates.

The excitation of the $L = 2$ state is much stronger than that of $L = 0$. The total cross section for the double GDR state (excluding the direct mechanism) is equal to 182 (90) mb. The experimental value [7] is 215 ± 50 mb. As stated above, the nuclear contribution to the (direct) excitation of the double phonon state is small (1.1 mb using 20% of the $L = 2$ sum rule at $\beta R = 0.1$ fm). Contrary to the single phonon case, the appropriate value of b_{min} for the double GDR experiment [7] is $b_{\mathrm{min}}^{<}$=13.3 fm.

We also compare our results with the cross sections of 4.7±0.4 b for the single GDR and 770 ± 220 mb [8] for the double GDR excitation in a ^{208}Pb target by ^{209}Bi projectiles at 1 GeV/nucleon. Using $E_1 = 13.5$ MeV, $\Gamma_1 = 4$ MeV, $E_2 = 27$ MeV, and $\Gamma_2 = 6$ MeV for the GDR and the GDR × GDR in ^{208}Pb, respectively, we find $\sigma_1 = 5234$ b and $\sigma_2 = 692$ mb for $b_{\mathrm{min}} = b_{\mathrm{min}}^{<}$= 14.2 fm. The parametrization [17] with $b_{\mathrm{min}} = b_{\mathrm{min}}^{>}$=16.97 fm would lead to smaller cross sections $\sigma_1 = 4130$ mb and $\sigma_2 = 319$ mb. We found the ratio $(P_{m=+1} + P_{m=-1})/P_{m=0} = 9.4$ for the GDR excitation in the experiment [8]. They quote the value 28 in their calculations and fit the gamma-ray angular distribution according to this value. We think that this could somewhat change the value of σ_2 extracted in [8].

With the same formalism we find the excitation cross sections 19.2 mb (with $b_{\mathrm{min}}^{<}$=13.3 fm) and 117 mb (with $b_{\mathrm{min}}^{>}$=14.2 fm) for the three phonon states in the experiments [7] and [8], respectively. The anharmonic effects, suggested in [7] to explain the large excitation of double GDR, are expected to be small since the mixing of single

TABLE I. Cross sections (in mb) for the Coulomb excitation of the IVGDR, ISGQR, and IVGQR in ^{136}Xe incident on ^{208}Pb at 0.69 GeV/nucleon. The cross sections in the last column are calculated with the widths of the states taken into account. The values outside (inside) parentheses use $b_{\mathrm{min}}^{<(>)}$=13.3 (15.6) fm.

	$m = \pm 2$	$m = \pm 1$	$m = 0$	σ_{total}	σ_{width}
IVGDR	···	949 (712)	264 (201)	2162 (1630)	2482 (1820)
ISGQR	90 (64)	8.4 (6.09)	14.3 (10.6)	211 (150)	241 (169)
IVGQR	29.7 (25.6)	6.1 (5.46)	14 (12.4)	84.1 (74.5)	102 (93)

TABLE II. Cross sections (in mb) for the Coulomb excitation of the double GDR in ^{136}Xe incident on Pb at 0.69 GeV/nucleon. The cross sections in the last column are calculated with the widths of the states taken into account. The values outside (inside) parentheses use $b_{\min}^{<(>)}$=13.3 (15.6) fm.

Double phonon state	$m = \pm 2$	$m = \pm 1$	$m = 0$	σ_{total}	σ_{width}
L=0 (two-step)	...	...	22.8 (10.7)	22.8 (10.7)	28.4 (13.3)
L=2 (two-step)	23.3 (11.2)	13.4 (6.6)	51.4 (26.8)	124.8 (62.4)	154 (77)
L=2 (direct—20% of SR)	3.27 (2.85)	0.86 (0.77)	2.12 (1.88)	10.3 (9.12)	11.8 (10.8)

and double phonon states is forbidden by the angular momentum and parity. The main anharmonic effect, apart from the weak coupling of the double GDR with $L = 2$ to GQR, is the interacting-boson-model-like scattering of dipole phonons which splits $L = 0$ and $L = 2$ states but hardly changes excitation and decay properties.

Another important question is related to the width of the multiphonon states. Early estimates [5,21] predicted the widths scaling with the phonon number n as $r_n \equiv \Gamma_n/\Gamma_1 = n$ due to the Bose factor $\sqrt{n}$ in the spreading matrix elements $V_{n\nu}$. This is seen clearly from the "standard" model [13] which gives $\Gamma_n^{(s)} = 2\pi\langle V_{n\nu}^2\rangle/d$ where d is the background level spacing. As was discussed in the different context in [22], the phonons do not decay independently being coupled via common channels in a fragmentation interval characterized by the intrinsic energy scale a. Then the effective width is expected to be proportional to $\sqrt{a\Gamma_n^{(s)}}$. It would correspond to the scaling $r_n = \sqrt{n}$ apparently preferred by the data. In this case the assumption [8] of the same branching ratios for gamma deexcitation of the single and double GDR would be inadequate leading to the cross section σ_2 overestimated by a factor $\sqrt{2}$. This problem remains a challenge for the understanding of the damping mechanism and onset of chaos. We hope to address it more in detail elsewhere [23].

In conclusion, we have calculated cross sections for the excitation of multiphonon states in relativistic heavy-ion collisions. The nuclear contribution to the cross sections is small for large-Z nuclei. At this stage, we have used a phenomenological sum rule approach assuming the saturation by the fragmented collective GR. The GDR $\times$ GDR excitation probability varies as b^{-4} and therefore it is more sensitive to the value of $b_{\min}$ than that of the single GDR ($\propto b^{-2}$). The lower value of $b_{\min}^{<}$ allows one to reproduce the double GDR excitation cross sections. The contradiction still remains concerning the low experimental cross section [7] of the single GDR in ^{136}Xe. The systematic study of multiphonon excitations for various combinations projectile + target is highly desirable. From a theoretical point of view, the sharp cutoff at $b = b_{\min}$ oversimplifies the complicated description of the simultaneous action of electromagnetic and nuclear forces in near-grazing collisions of the extended quantum objects. The microscopic analysis of the problem is under progress.

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[1] C.A. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
[2] V.K. Balasubrahmanyan, C.J. Crannell, F.A. Hagen, J.F. Ormes, and M.J. Ryan, in *Proceedings of the 12th International Conference on Cosmic Rays* (Hobart, Tasmanian, 1972).
[3] X. Artru and G.B. Yodh, Phys. Lett. **40B**, 43 (1972).
[4] H.H. Heckman and P.J. Lindstrom, Phys. Rev. Lett. **37**, 56 (1976).
[5] G. Baur and C.A. Bertulani, Phys. Lett. B **174**, 23 (1986); Nucl. Phys. **A482**, 313 (1988); Proceedings of the International School on Heavy Ion Physics, Erice, Italy, October 1986, edited by R.A. Broglia and G.F. Bertsch (Plenum, New York, unpublished).
[6] W. Llope, Ph.D. dissertation, SUNY at Stony Brook, 1992; (private communication).
[7] R. Schmidt *et al.*, Phys. Rev. Lett. **70**, 1767 (1993).
[8] J. Ritman *et al.*, Phys. Rev. Lett. **70**, 533 (1993); J. Ritman, Ph.D. thesis, SUNY at Stony Brook, May 1993.
[9] T. Aumann *et al.*, Phys. Rev. C **47**, 1728 (1993).
[10] W. Llope and P. Braun-Munzinger, Phys. Rev. C **45**, 799 (1992).
[11] A. Winther and K. Alder, Nucl. Phys. **A319**, 518 (1979).
[12] K. Alder and A. Winther, *Coulomb Excitation* (Academic Press, New York, 1965).
[13] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1; *ibid.* (Benjamin, Reading, MA, 1975), Vol. II.
[14] N. Austern and J.S. Blair, Ann. Phys. (N.Y.) **33**, 15 (1965).
[15] R.J. Glauber, in *Lecture Notes in Physics*, edited by W.E. Britin (Interscience, New York, 1959), Vol. I.
[16] B.L. Berman, At. Data Nucl. Data Tables **15**, 319 (1975).
[17] S. Kox *et al.*, Phys. Rev. C **35**, 1678 (1987).
[18] C.J. Benesh, B.C. Cook, and J.P. Vary, Phys. Rev. C **40**, 1198 (1989).
[19] H. Emling (private communication).
[20] V.Yu. Ponomarev and V.V. Voronov, Phys. Lett. B **279**, 1 (1992).
[21] S. Mordechai *et al.*, Phys. Rev. C **41**, 202 (1990).
[22] V.V. Sokolov and V.G. Zelevinsky, Ann. Phys. (N.Y.) **216**, 323 (1992).
[23] C.A. Bertulani and V.G. Zelevinsky (to be published).