Current Echoes

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(Received 19 March 1993)

A new echo phenomenon is suggested, which we call the current echo. A disordered one-dimensional tight-binding conductor subject to two very short voltage pulses is considered. While the current response following the first pulse decays due to scattering off the disorder, a delayed current pulse is predicted following the second voltage pulse, its delay being equal to the temporal separation of the two voltage pulses. This prediction is illustrated for an ensemble of finite, disordered chains.

PACS numbers: 72.20.Ht, 42.50.Md, 73.20.Dx, 76.60.Lz

Echoes are fascinating phenomena in transient coherent spectroscopy. Spin echoes have been known for a long time [1], while photon echoes were first reported in the sixties [2] for the case of optical excitation of two-level absorbers. Only recently photon echoes in intrinsic semiconductors have been demonstrated [3]. Both phenomena have been widely applied to the study of phase destroying mechanisms. Common to these phenomena is the excitation of an ensemble of dipoles with resonance frequencies distributed over a certain interval. When excited by a pulse at time $t = 0$, the macroscopic response decays on a time scale given by the reciprocal width of the distribution. A second pulse, arriving after a delay τ, then interacts nonlinearly with the ensemble of the (reversibly) dephased oscillators such that at time $t = 2\tau$ the individual oscillators are again in phase and an echo is emitted.

In this Letter we propose a new echo phenomenon, which we call the current echo. We consider a disordered one-dimensional conductor. A short voltage pulse induces a current in the direction of the external field, which decays due to elastic scattering off the disorder after a characteristic elastic scattering time $\tau_{\rm el}$. We will show that a second voltage pulse, arriving after a delay time $\tau > \tau_{\rm el}$, is able to produce a current pulse at time $t = 2\tau$ after the first pulse. We first introduce the notion of current echoes by considering the simple model case of a two-site conductor. Next we present current echo traces obtained from a model calculation on the basis of an ensemble of disordered one-dimensional tight-binding conductors. Finally, we discuss the observability of current echoes in experiments on realistic systems.

We consider a single-band tight-binding system described by

$$H = \sum_{ij} T_{ij} c_i^\dagger c_j - e\mathbf{E}(t) \cdot \sum_i \mathbf{R}_i c_i^\dagger c_i, \qquad (1)$$

where T_{ij} is the Hamiltonian matrix with diagonal disorder, and $\mathbf{E}(t)$ is the applied field pulse; $\mathbf{R}_i$ is the position vector of site i placed on a lattice with lattice constant a, and $c_i^\dagger, c_j$ are electron operators.

The essential features of a current echo can already be demonstrated for an ensemble of chains each consisting of just two levels 1 and 2 (see Fig. 1). Diagonalizing the first term of the Hamiltonian yields a two-level system in analogy to a spin-1/2 system, or to the photon echo situation for the case of two-level absorbers. In the new basis the Hamiltonian reads

$$H = \frac{\hbar\omega_0}{2} c_u^\dagger c_u - \frac{\hbar\omega_0}{2} c_l^\dagger c_l - eaE(t)(c_l^\dagger c_u + \text{H.c.}),$$
$$\hbar\omega_0 = 2T_{12}. \qquad (2)$$

The equation of motion for the density matrix of a single chain in the new basis, $\rho_{\alpha\beta} = \langle c_\alpha^\dagger c_\beta \rangle$ (with α, $\beta = u, l$), reads $\dot{\mathbf{S}} = \mathbf{\Omega} \times \mathbf{S}$ with a Bloch-vector $\mathbf{S}$ defined by

$$\begin{aligned} S_1 &= \rho_{ul} + \rho_{lu}, \\ S_2 &= i(\rho_{ul} - \rho_{lu}), \\ S_3 &= \rho_{uu} - \rho_{ll}, \end{aligned} \qquad (3)$$

and $\hbar\mathbf{\Omega} = (-2eaE, 0, \hbar\omega_0)$. This description is equivalent to the usual spin-vector (Bloch-vector) formalism for the spin and photon echo, respectively. In the spin and photon echo situations one is interested in the polarization given by S_1 [4]. The classical echo situation is characterized by the following pulse sequence: a $\pi/2$ pulse (pulse 1) arrives at $t = 0$ at the system being in its ground state ($S_3 = -1$), followed by a π pulse (pulse 2) at $t = \tau$. Pulse 1 moves $\mathbf{S}$ into the equatorial 1-2 plane, where, due to the inhomogeneously distributed eigenfrequencies, the phase coherence of the individual systems becomes lost. Pulse 2 inverts $\mathbf{S}$ in the equatorial plane, such that after another interval $\tau \to 2\tau$ the vectors $\mathbf{S}$ of all the two-level systems move again in phase; the re-

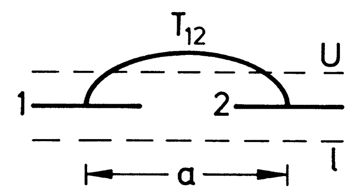

FIG. 1. The two-level system. Levels 1 and 2 are coupled by T_{12}, yielding upper (u) and lower (l) eigenstates.

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alignment of all components S_1 then leads to the emission of an echo signal. Here, in contrast, we are interested in the components S_2, which are proportional to the current $j_{12} = ieaT_{12}(\rho_{12} - \rho_{21})/\hbar = ieaT_{12}(\rho_{ul} - \rho_{lu})/\hbar = eaT_{12}S_2/\hbar$. The realignment of all $\mathbf{S}$ vectors implies also that all components S_2 are in phase at time $t = 2\tau$, thus producing an echo surge in the macroscopic current.

Within the two-level system approach there is a one-to-one correspondence of spin echo, photon echo, and current echo. However, this correspondence is lost if the two-level systems are mutually coupled. For an adequate description of this latter situation it is also more appropriate to resort to perturbation theory with respect to the external fields. Generally, at least third-order perturbation theory is needed to describe echo phenomena [5].

The treatment of current echoes from mutually coupled levels is based on the tight-binding Hamiltonian in Eq. (1) with $i, j = 1, \ldots, N$, where $N > 2$. The current is given by

$$\mathbf{j}(t) = e\langle\dot{\mathbf{R}}\rangle = i\frac{e}{\hbar}\sum_{ij}\mathbf{R}_{ij}T_{ij}\rho_{ij}\ , \tag{4}$$

with $\mathbf{R}_{ij} = \mathbf{R}_j - \mathbf{R}_i$, and $\rho_{ij} = \langle c_i^\dagger c_j\rangle$ is the density matrix in site representation. Using an obvious matrix notation, the time dependence of ρ_{ij} is given by

$$\partial_t\rho = \frac{i}{\hbar}L\rho + \frac{i}{\hbar}e\mathbf{E}\cdot\mathbf{M}(\rho), \tag{5}$$

where $L\rho = [T, \rho]$ and $[\mathbf{E}\cdot\mathbf{M}(\rho)]_{ij} = \mathbf{E}\cdot\mathbf{R}_{ij}\rho_{ij}$.

We will now illustrate the appearance of a current echo by numerically calculating the current $j^{(122)}(t)$ in third order in the external field. Assuming $\delta(t)$-like voltage pulses, and concentrating on the contribution from the pulse sequence (1, 2, 2), we find from Eqs. (4) and (5)

$$\mathbf{j}^{(122)}(t) = \frac{1}{2}\left(\frac{e}{\hbar}\right)^4\Theta(t-\tau)\Theta(\tau) \times \mathrm{Tr}\left\{\mathbf{M}(T)e^{i(t-\tau)T/\hbar}\mathbf{E}^{(2)}\cdot\mathbf{M}\Big(\mathbf{E}^{(2)}\cdot\mathbf{M}\Big(e^{i\tau T/\hbar}\mathbf{E}^{(1)}\cdot\mathbf{M}(\rho(0^-))e^{-i\tau T/\hbar}\Big)\Big)e^{-i(t-\tau)T/\hbar}\right\}, \tag{6}$$

where $\rho_{ij}(0^-) = \langle 0|c_i^\dagger c_j|0\rangle$ and $|0\rangle$ is the equilibrium ground state.

The numerical calculations are performed for a linear chain consisting of N sites with nearest-neighbor coupling T_{ij} (i, j nearest neighbors), and random site energies T_{ii} drawn from a rectangular distribution function of width W. The average is taken over an ensemble of 1000 chains. For this one-dimensional system $[M(B)]_{ij} = (i-j)aB_{ij}$. The lowest eigenstate of T has been taken as the equilibrium ground state $|0\rangle$. The calculation of $\mathbf{j}^{(122)}(t)$ from Eq. (6) involves a number of transformations from the eigenbasis of T to the site basis and back. Figure 2 shows a third-order current trace for times larger than the pulse delay time and for an ensemble of chains with $N = 5$ sites each. The disorder is given by a width $W = 0.65J_0$. The nearest-neighbor coupling $T_{ij} = J_0$ has been taken as the unit of energy. The first-order current has already decayed in the time domain shown. The current trace, which oscillates around zero, consists of two parts. There is a background for all times shown; its amplitude decreases with the number of disordered chains in the ensemble (an average over 1000 chains is shown in Fig. 2). In contrast, the large amplitude centered at $t = 2\tau$ does not depend on the size of the ensemble. This is the current echo. It is interesting to note that the current attains a maximum negative value at exactly $t = 2\tau$.

The appearance of a current echo can be understood in the following way. The Hamiltonian matrix T can be diagonalized yielding closely spaced eigenvalues. Initially the system is in its ground state, represented by a certain number of its lowest lying states. The first pulse at $t = 0$ induces pair excitations between these occupied states and unoccupied states (e.g., between $|E_\alpha\rangle$ and $|E_\gamma\rangle$). These pairs are mutually interrelated, since $|E_\alpha\rangle$ is also coupled to $|E_\delta\rangle$, etc. The situation at hand therefore reminds one of quantum beats in an $N \times M$ system. In optical spectroscopy [6], it is known that for large N and M, a total decay of the nonlinear signal takes place without any occurrence of echoes or quantum beats.

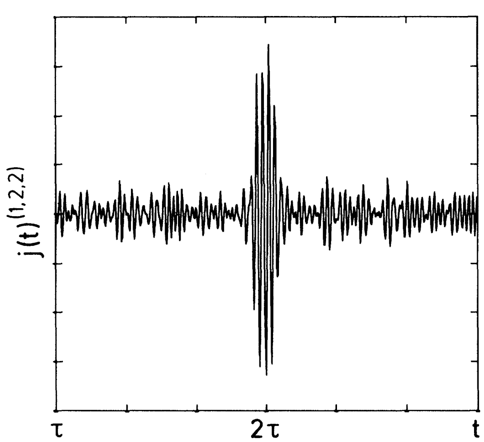

FIG. 2. Nonlinear current response $j^{(122)}$ as a function of time, after the second voltage pulse, for an ensemble of 1000 linear disordered chains having $N = 5$ sites each. Note that the current oscillates around zero.

However, in our situation, and for weak disorder, the electric field predominantly couples those states which are nearest neighbors in energy, provided the length of the chain is sufficiently large. These pairs of states can be viewed as an ensemble of isolated two-level systems, and the preceding discussion applies. Their contribution to the nonlinear signal is the current echo. For even larger chains, and/or larger disorder, the chains break up into smaller subchains due to localization, and the above reasoning applies to each of these subchains as well. Besides excitations between pairs of eigenstates, the pulse sequence also produces excitations involving more than just two levels. Their contribution to the current, however, is suppressed by the configurational average. The background noise seen in Fig. 2 is a remainder of these signals and can be further reduced by considering more (> 1000) chains in the averaging procedure. The fast oscillations in the current peak around $t = 2\tau$ are due to the fact that the average level spacing in our calculations is larger than the width W of the diagonal disorder. For the opposite case (i.e., the average level spacing is smaller than W), a single echo peak without any additional oscillations will occur. This situation can be achieved by enlarging N [7]. Note that the sign of the echo peak is negative since the echo current at time $t = 2\tau$ is generally negative, i.e., reversed with respect to the first-order current.

In essence, the current echo represents a delayed current response of the system to a succession of two short voltage pulses. It is a nonlinear coherent effect and relies on the presence of static disorder.

So far we have neglected any irreversible phase relaxation due to interaction with dynamical degrees of freedom, e.g., interactions with phonons. They take place on a time scale given by the phase relaxation time τ_{deph}. In order to be able to observe current echoes, the elastic scattering time τ_{el} has to be sufficiently smaller than the dephasing time τ_{deph}. These two times define upper and lower limits for the observation of current echoes. Suitable candidates for the experimental demonstration of current echoes have to fulfill the following requirements: nearest-neighbor coupling, sufficiently long dephasing times τ_{deph}, and controllable disorder to achieve short enough elastic scattering times τ_{el}. We propose semiconductor heterostructures, such as coupled quantum wells or superlattices, as possible systems, which can be prepared to meet the above requirements [8]. These structures are characterized by intraband dephasing times as long as 7 ps [9]; this gives us an upper limit of the order of some ps. The lower limit τ_{el} depends on the interwell coupling and on the nature of the disorder. Both the interwell coupling and, to a certain degree, the disorder can be tailored at will in semiconductor heterostructures. As a final ingredient, sufficiently short voltage pulses are needed. Such pulses could be supplied by the electrical THz pulses most recently demonstrated in optical experiments on semiconductor heterostructures [10].

In conclusion, we have given theoretical evidence for a new kind of echo phenomenon, which we call the current echo. The demonstration is based on an ensemble of finite, disordered, one-dimensional tight-binding conductors. We suggest to verify the existence of current echoes experimentally by applying THz voltage pulses to suitably designed semiconductor heterostructures. Finally, we suggest that once the existence of current echoes has been demonstrated, their decay, being due to dephasing interactions, could be used to determine the inelastic transport scattering time τ_{deph} independently of the much smaller elastic scattering time τ_{el} in these structures.

The idea for this work originated from discussions with Stefan Schmitt-Rink. Discussions with R. Zimmermann are also gratefully acknowledged.

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