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Topological Transitions in Berry's Phase Interference Effects

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We consider a topological transition which is an abrupt variation from π to zero of Berry's phase, in the case when it is a phase difference of interfering waves. It manifests itself in a steplike current–magnetic field and current–gate voltage characteristics predicted for in-plane magnetoresistance of rings in noncentrosymmetric materials. Transition points occur at external magnetic fields equal to momentum-dependent effective magnetic fields for different tunneling channels of a quasi-one-dimensional ring. Similar effects due to the angular anisotropy of electron g factor are considered.

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The concept of Berry's phase [1] and its generalizations [2–4] has been a subject of considerable interest during recent years in different areas of modern physics [5–7]. The topological phases are connected to the cyclic evolution of physical systems in which the state of a system or the parameters it is described by return to the original values after an evolution. These topological phases are observable by interference: The system whose parameters were changed is recombined with another that was separated from the first one at an earlier time and whose parameters were varied in a different way. The familiar example of the topological phase factor is an extra phase acquired by the spin wave function in addition to a standard dynamical phase in a magnetic field with a constant value and a direction which follows adiabatically a closed trajectory. Recently Aronov and the present author [8] have demonstrated the existence of the spin-orbit Berry's phase, which arises from the adiabatical variation of the electron momentum in low dimensional or reduced symmetry conductors. The electron spin in these systems is influenced by the momentum-dependent effective magnetic field. For an electron moving in a ring the momentum and the corresponding magnetic field follow a closed path, which implies the topological phase effect.

Spin Berry's phase is proportinal to a solid angle subtended by a magnetic field in a space. A special case occurs when a circuit around which the magnetic field is transported is confined to a plane. Actually this case was studied by Herzberg and Lonquet-Higgins [9] in connection with a sign change of eigenfunctions of real symmetric matrices around a degeneracy. In the present paper I discuss such a situation from a completely different point of view. As it was considered by Berry [1], for fermions the topological phase is equal to the odd number of π if a magnetic field trajectory circuit is confined to a plane and encloses the point of degeneracy (the point at which the magnetic field is zero). Therefore, for an experiment in which the difference in the dynamical phase of recombining waves is compensated, the interference is destructive. Now imagine that the environment of the interference experiment is suddenly changed and the plane curve which is subtended by the magnetic field during an evolution does not encircle the degeneracy any more. This situation can be easily achieved by applying a static in-plane magnetic field whose value is more than an amplitude of the magnetic field rotating within the same plane. Then, Berry's phase acquired by a fermion wave function drops from the odd number of π to zero and the character of the interference immediately changes from destructive to constructive. This phenomenon is considered here as a topological transition. I demonstrate that such a transition naturally arises in the in-plane magnetoresistance of rings in noncentrosymmetric materials and predict the steplike current–magnetic field, current–gate voltage and current–uniaxial strain characteristics. Finally, I deal with Berry's phase due to the angular anisotropy of the electron g factor, leading to similar effects.

Before considering the topological transitions in mag-

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netoresistance of rings I will describe this effect for a model example of the spin evolution in the in-plane magnetic field. Consider the time-dependent Hamiltonian

$$\mathcal{H} = g\mu\Big[\sigma_x(B_0\cos\omega t + B_1) + \sigma_y B_0 \sin\omega t\Big] \quad (1)$$

which describes the spin $\frac{1}{2}$ motion in a magnetic field having a static and an alternating component within a plane (x, y). In Eq. (1) σ_i are the spin Pauli matrices, g is the electron g factor, μ is the Bohr magneton, and ω is a frequency of the rotating field. A natural basis for $\mathcal{H}(t)$ consists of $|n_+(t)\rangle$ and $|n_-(t)\rangle$ that satisfy $\mathcal{H}(t)|n_\pm(t)\rangle = E_\pm(t)|n_\pm(t)\rangle$. The energies $E_\pm(t)$ are

$$E_\pm(t) = \pm\hbar\sqrt{\omega_0^2 + \omega_1^2 + 2\omega_1\omega_0\cos\omega t}, \quad (2)$$

$\omega_i = \frac{g\mu B_i}{\hbar}$, and the corresponding eigenstates have a form

$$|n_\pm(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \frac{E_\pm}{\hbar(\omega_1+\omega_0\exp -i\omega t)}\end{pmatrix}. \quad (3)$$

If we write the normalized state $|\Psi(t)\rangle$ evolving according to Schrödinger's equation $i\hbar\partial|\Psi(t)\rangle/\partial t = \mathcal{H}(t)|\Psi(t)\rangle$ in a form $|\Psi(t)\rangle = \sum_\pm a_\pm(t)\exp\left[-\frac{i}{\hbar}\int E_\pm(t)dt\right]|n_\pm(t)\rangle$, we obtain the following system of equations for $a_\pm(t)$:

$$\frac{\partial a_+}{\partial t} = -a_+D_{++} - a_-D_{+-}\exp\frac{i}{\hbar}\int_0^t(E_+ - E_-)dt, \quad (4)$$

$$\frac{\partial a_-}{\partial t} = -a_-D_{--} - a_+D_{-+}\exp\frac{i}{\hbar}\int_0^t(E_- - E_+)dt.$$

Here $D_{ij} = \langle n_i|\frac{\partial}{\partial t}|n_j\rangle$. The coefficients D_{ij} are given by

$$D_{++} = D_{--} = \frac{i}{2}\omega\omega_0\frac{\omega_1\cos\omega t + \omega_0}{\omega_1^2 + \omega_0^2 + 2\omega_1\omega_0\cos\omega t}, \quad (5)$$

$D_{-+} = D_{+-} = -D_{++}$. If $a_i(t=0) = \delta_{ij}$ and a state mixing during the evolution within a time interval $[0, t_1]$ is negligible the system would remain in an eigenstate of $\mathcal{H}(t)$ to a good approximation, and we obtain

$$a_j(t) \cong a_j(0)\exp\left(-\int D_{jj}dt\right) \quad (6)$$

for $t \in [0, t_1]$. Using Eq. (5) and substituting a variable $z = \exp i\omega t$ we can express the integral in the exponent of Eq. (6) $I = \int_0^{\frac{2\pi}{\omega}} D_{jj}(t)dt$ for a cyclic motion as

$$I = \frac{\omega_0}{4}\oint_{|z|=1}\frac{\omega_1(z^2+1) + 2\omega_0 z}{\omega_1\omega_0(z^2+1) + (\omega_1^2+\omega_0^2)z}\frac{dz}{z}. \quad (7)$$

The integral on the right of Eq. (7) is taken along the unit circle enclosing two isolated singularities for $\omega_0 \neq \omega_1$ (one more singularity is outside the unit circle) and one isolated singularity $z_0 = 0$ for the case $\omega_0 = \omega_1$. Taking a sum of residues within a circle we have $I = \pi i$ if $\omega_1 < \omega_0$, $I = \frac{\pi i}{2}$ if $\omega_1 = \omega_0$, and $I = 0$ if $\omega_1 > \omega_0$.

The value of integral I determines an extra phase in addition to the dynamical phase during a cycle of evolution, which is Berry's phase φ_B by definition. Equation (7) gives a mathematical argument for the existence of a topological transition from destructive ($\varphi_B = \pi$) to constructive ($\varphi_B = 0$) interference in Berry's phase coherent experiment. The result coincides with a consideration from the geometric point of view. If $\omega_0 < \omega_1$ the magnetic field trajectory circuit does not enclose the degeneracy and is not seen from it. If $\omega_0 > \omega_1$ the degeneracy is encircled and the measure of a view of the circuit is $\pm 2\pi$. Berry's phase for spin $\frac{1}{2}$ is a half of an angle that trajectory subtends at degeneracy and we have $\varphi_B = 0$ or $\varphi_B = \pm\pi$ that is in complete agreement with [1]. The intermediate case $\omega_0 = \omega_1$ turns out to be very interesting. It can also be interpreted from the topological point of view. Degeneracy is crossed by the trajectory circuit. Staying at degeneracy we cannot see the trajectory looking at one of the half planes and be able to see the whole circuit looking to another side. So, the measure of a view of a circuit is $\pm\pi$ and the result $\frac{\pi}{2}$ for the spin $\frac{1}{2}$ in Eq. (7) is obvious. For each rotation of the alternating magnetic field by 2π rad the wave function rotates by $\frac{\pi}{2}$ rad. Unfortunately this fine mathematical result seems to have low chances for being detected in a real experiment. In a physical reality we cannot force two values to be exactly equal, and only the fact of a transition from $\varphi_B = \pi$ to $\varphi_B = 0$ can be measured.

Now we are to prove that the state mixing for our system can be really negligible and to find out what are the conditions to be met in this situation. Applying the time-dependent perturbation theory to the system of equations (4) we obtain that, say, for $a_+(0) = 1, a_-(0) = 0$ the probability P_{+-} to find a system in a state $|n_-(t)\rangle$ at the time $t = t_1$ is given by

$$P_{+-}^{(t_1)} = \left|\int_0^{t_1} D_{-+}\exp\frac{i}{\hbar}\int_0^t(E_+ - E_-)d\tau\right|^2. \quad (8)$$

If the probability $P_{+-} \ll 1$ the state mixing is unimportant; the system remains in a state $|n_+(t)\rangle$ and the concept of Berry's phase is applicable. This condition has the simplest form in the case $\omega_1 = 0$ when D_{+-} and $E_\pm$ are time independent. In this situation the state mixing is negligible if $\omega \ll \omega_0$ and this is the adiabatical condition which was required for validity of consideration in [8]. If $E_\pm$ and D_{+-} do depend on time, the conventional condition of the adiabatic approximation applicability and the possibility to neglect the mixing of states is

$$\max\left|\frac{\hbar D_{+-}(t)}{E_+(t) - E_-(t)}\right| = \frac{\omega\omega_0}{2(\omega_1 - \omega_0)^2} \ll 1. \quad (9)$$

For our purposes this restriction turns out to be too strong. Indeed, even for the most crucial case $\omega_1 = \omega_0$, in which the adiabatic approximation breaks down for $t = \frac{\pi}{\omega}$, the condition $P_{+-} \ll 1$ has a form

$$\left|\int_0^{y_1} dy\exp\left(8i\frac{\omega_0}{\omega}\int_0^y dx|\cos x|\right)\right|^2 \ll 1, \quad (10)$$

which easily follows from (2), (5), and (8). Using the representation of the integrand in a form of Fourier series with Bessel functions as coefficients,

$$\exp \pm iz\sin\phi = J_0(z) + 2\sum_{k=1}^{\infty} J_{2k}(z)\cos 2k\phi + 2i\sum_{k=0}^{\infty} J_{2k+1}(z)\sin(2k+1)\phi, \qquad (11)$$

we see that for one cycle of magnetic field rotation $y_1 = \pi$ the condition (10) is satisfied if $z \gg 1$. Moreover, for $\frac{\omega_0}{\omega} \simeq 10$ Eq. (10) is satisfied for any y_1 within the interval $[0,\pi]$ including the vicinity of $y = \frac{\pi}{2}$, which is the point of degeneracy. For a sufficiently large number of cycles the condition of nonessential state mixing always breaks, regardless of how slow the alternating field rotates. Likely, the interesting effects take place within a low number of cycles, and in this situation mixing is actually negligible if rotation is slow:

$$\frac{\omega_0}{\omega} \gg 1. \qquad (12)$$

The cases $\omega_0 > \omega_1$ and $\omega_0 < \omega_1$ are more secure in a sense that the state mixing can always be prevented by satisfying the condition (9). So, for slow enough rotation (12) the concept of Berry's phase can be applied at any value of the static magnetic field, and the topological transition is to be rather sharp.

Consider a quasi-one-dimensional ring of radius r defined electrostatically in the two-dimensional electron gas (2DEG) of a semiconductor heterostructure. The effects on rings induced by Berry's phase were discussed by Loss, Goldbart, and Balatsky in connection with persistent currents [10] and by Stern [11] in connection with mesoscopic conductivity and motive forces in a ring. The effect of spin-orbit Berry's phase [8] was studied for Aharonov-Bohm geometry in the tunneling of electrons traversing a ring coupled to current leads. Spin-orbit topological phase results in destructive interference, negative magnetoresistance and produces the shift of Aharonov-Bohm oscillations. In the absence of the spin-orbit effects the oscillations of transparancy of the normal metal rings were studied in detail by Buttiker, Imry, and Azbel [12].

Here I demonstrate that spin-orbit interaction manifests itself in topological transitions leading to steplike current–magnetic field dependence in the in-plane magnetoresistance of rings. The effective electron Hamiltonian of 2DEG includes the kinetic energy, Zeemann's term in a static magnetic field, and the linear in momentum $\boldsymbol{p}$ term, describing the spin-orbit splitting of electron states at $p \neq 0$. If the normal to the heterostructure interface $z \parallel (111)$, the Hamiltonian takes the form

$$\mathcal{H}_e = \frac{p_x^2 + p_y^2}{2m} + \hbar\omega_s\sigma_x + \hbar\beta\left[\boldsymbol{\sigma}\times\boldsymbol{p}\right]_z, \qquad (13)$$

where m is the effective mass, ω_s is the Larmor frequency, and β is the spin-orbit splitting coefficient. We assume that a static in-plane magnetic field affects only the electron spin; as for magnetic length $L_B \gg a$, a is the quantum well width and the orbital effects are absent. Consider first, for simplicity, the case of a one-channel ring. If the electron wavelength $\lambda = \hbar/p$ is much less than the circumference of a ring $L = 2\pi r$ (p is the momentum along the ring) an electron can be described quasiclassically. In the zeroth order it is a classical rotator with the frequency $\Omega = \frac{p}{mr}$, and the wave function is $\Psi(\varphi,t) = \Psi_r(\varphi,t)\,\chi(\varphi,t)$, $\Psi_r(\varphi,t)$ is the rotator wave function, and χ is a spinor. Inserting $\Psi(\varphi,t)$ into the time-dependent Schrödinger equation with the Hamiltonian (13) in the first order to $\hbar$ we obtain

$$i\hbar\frac{\partial\chi}{\partial t} = \left[\sigma_x\left(\hbar\omega_s + \hbar\beta mr\Omega\cos\Omega t\right) + \sigma_y\hbar\beta mr\sin\Omega t\right]\chi, \qquad (14)$$

which is exactly Eq. (1): $\hbar\omega_s$ corresponds to $g\mu B_1$ and $\hbar\beta mr\Omega$ replaces $g\mu B_0, \Omega = \omega$. Actually, the equivalence between the systems described by (1) and (14) is obvious, because the effective field is transported around a circle while the electron momentum subtends a closed trajectory in a ring. Therefore, considering the electrons in a ring described by (13), we can apply the concept of Berry's phase and all the results obtained for Eq. (1). If the upper and the lower branches traversed by an electron coming from the lead are symmetric, the dynamical phase factors obtained in these branches are equal to each other. Consequently, the phase difference between the amplitudes interfering at the second junction of a ring to a lead is equal to Berry's phase [13]. For the strong coupling limit [14], in which the junction is completely transparent for electrons and a wave of unit amplitude coming from the lead is transmitted into the two branches with equal amplitudes $\frac{\sqrt{2}}{2}$, the transparancy of a ring is zero for $\varphi_B = \pm\pi$ and equal to unity for $\varphi_B = 0$. Therefore the conductance is zero in the former and is equal to $\frac{e^2}{2h}$ in the latter case. It undergoes a topological transition and changes its value from 0 to $\frac{e^2}{2h}$ at the static magnetic field equal to the amplitude of the momentum-dependent effective magnetic field. Increasing the static magnetic field we change the character of the interference from destructive to constructive in more and more channels, and the topological transitions for these channels result in steplike current–magnetic field characteristics. The points of transitions are determined by the values of momentum in the channels and, consequently, by the potential confinement of a ring.

For instance, if a static magnetic field is applied along the x direction and a current lead is parallel to the direction y, the value of momentum of the incident electron coming to a ring is $p_n = \left[2(m\beta\hbar)^2 + 2m\varepsilon \pm 2\sqrt{(m\beta\hbar)^4 + 2(m\beta\hbar)^2 m\varepsilon + (m\hbar\omega_s)^2}\right]^{\frac{1}{2}}$, where the signs $+$ and $-$ correspond to the opposite spin projections and

$\varepsilon = E_F - E_n$ is the difference between the Fermi energy E_F and the energy of size quantization E_n. The strong coupling limit implies nearly the adiabatic junction of a lead to a ring, which is passed by an electron without spin flip and the mixing of different channels. The confinement potential of the leads and of the branches of the ring is assumed to be the same, and therefore, the value of the effective spin-orbit magnetic field for the nth channel is determined by p_n. For a given energy the spin up and spin down electrons are influenced by different effective fields and each channel manifests itself in two steps on the conductance curve. The energy E_n, which describes p_n, is determined by the confinement of the 2D layer and by the confinement of a ring with leads within this layer. Its form may vary between a harmonic oscillator and a rectangular square well potential, and the spacing of the topological transition points may be different.

The condition of the slow enough rotation of the effective field, according to (12) and (14), is $\hbar\beta mr \gg 1$ and the rings of a sufficiently large size can satisfy it. The upper limit of the circumference of a ring is the mean free path, because we study the ballistic motion of electrons in order to avoid the state mixing due to the elastic scattering [15]. Thus, we also neglect the spin-flip processes in a ring and the spin up and down electrons traverse the ring independently.

The estimations show that for an InAs ring of 5 μm radius and 60 nm width ($m = 0.023m_0$, $g = -15$, the spin-orbit coefficient $\hbar^2\beta = 6 \times 10^{-10}$ eV cm [16], the 2D electron density is taken as $n_s = 10^{12}$ cm^{-2}, and $E_F = 0.1$ eV) four levels within the first level of size quantization in 2DEG may contribute to the conductance and eight transition points may be observable (due to the spin splitting) within the interval of magnetic fields 1–2 T. Additional experimental possibilities are coming from the dependence of the spin-orbit coefficient and effective magnetic field on the uniaxial strain or on the external electric field [in our geometry both applied parallel to (111), perpendicular to the plane of a ring]. Correspondingly, the magnetic field can be kept constant, and the transitions can be investigated for a dependence of the conductance on the gate voltage or on the deformation.

The described topological transitions can take place not only on rings in noncentrosymmetric materials, but also in multiconnected samples of any symmetry due to the angular anisotropy of the electron g factor. The effective electron Hamiltonian in this case has a form

$$\mathcal{H} = \frac{p^2}{2m} + g\mu\boldsymbol{\sigma}\boldsymbol{B} + g_1(\boldsymbol{\sigma}\boldsymbol{p})(\boldsymbol{p}\boldsymbol{B}) + g_2(\boldsymbol{\sigma} \times \boldsymbol{p})_z(\boldsymbol{p} \times \boldsymbol{B})_z. \qquad (15)$$

The two last terms can be derived, for instance, for Dirac electrons using the Foldy-Wouthuyzen transformation [17], if we take into account the terms of order to $(\frac{1}{c})^3$. For free electrons the momentum-dependent fields of this type are always negligible in comparison with a field proportional to a conventional g value. However, the promising situation occurs in the semiconductor solid solutions or in layered structures formed by the compounds with the opposite signs of the electron g factor. Several solid solutions have a g value close to zero within some interval of concentrations of ingredients [18]. In this case the amplitude of the effective field determined by g_1 and g_2 can be comparable with one described by g. Representing the electron in a ring as a classical rotator, we obtain that the spin motion for $\boldsymbol{B} \parallel x$ is governed by Eq. (1) with $gB_1 = \left(g + \frac{(g_1-g_2)}{2\mu}p^2\right)B, gB_0 = \frac{(g_1+g_2)}{2\mu}p^2B, \omega = \frac{2p}{mr}$. Varying the electron momentum [19] we change the amplitude of the alternating field in comparison with a static one and get the topological transitions due to angular anisotropy of the electron g factor. A solid solution is characterized by a smaller mean free path than a pure material, and the ring of a smaller radius is to be chosen. The condition of the negligible mixing of states needed for applicability of Berry'phase concept, $\omega \ll \frac{g\mu B_0}{\hbar}$, can be preserved by increasing the value of magnetic field B. Note that the present effect results in the steps of conductance of a ring, when the linear in the in-plane momentum effective magnetic field is absent.

In summary, a concept of a topological transition is introduced. This transition is a sudden change of phase difference of interfering waves connected with Berry's phase. It results in a steplike conductance characteristic for the in-plane magnetoresistance of rings.

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[13] As for the allowed asymmetry of branches, the estimation providing that the main contribution to the dynamical phase is due to the kinetic energy gives the difference in the lengths of branches δl less than the electron wavelength. Under the experimental conditions discussed in this paper $\delta l \sim 5$ nm.
[14] In the weak coupling limit the small transmission ampli-

tude η from the lead into a branch makes the vacuumlike interference unimportant and the main effect is the resonant tunneling with the width of the resonant states proportional to η. The resonant levels undergo a shift due to Berry's phase, but such an effect is much less pronounced than one for a strong coupling limit [8].

[15] It should be noted that in these conditions we are really interested in one-two cycles of rotation, i.e., in the vacuum interference effects. To consider more cycles is meaningless due to the breaking of the conditions like (14) as well as to the state mixing because of a scattering.

[16] For the 10 nm GaAs quantum well the spin-orbit coefficient is 3.5×10^{-10} eV cm (G.E. Pikus, V.A. Marushak, and A.N. Titkov, Fiz. Tekh. Poluprovodn. **22**, 185 (1988) [Sov. Phys. Semicond. **22**, 115 (1988)]); for the $In_{0.6}Ga_{0.4}As$ 15 nm well it is 3×10^{-10} eV cm [B. Das, D.C. Miller, S. Datta, R. Reitenberger, W.P. Hong, P. Bhattacharya, J. Singh, and M. Jaffe, Phys. Rev. B **39**, 1411 (1989); S.I. Dorozhkin, Phys. Rev. B **41**, 3235 (1990)]. The chosen value is an estimation for the 10 nm InAs well.

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[19] The Fermi level can be easily varied in such a manner that the new channels should not contribute to the conductance.