Transverse Tau Polarization in Decays of the Top and Bottom Quarks in the Weinberg Model of *CP* Nonconservation

D. Atwood,[1] G. Eilam,[2] and A. Soni[3]
[1]*Department of Physics, Stanford Linear Accelerator Center, Box 4349 Stanford, California 94309*
[2]*Department of Physics, Technion, Haifa, Israel*
[3]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973*
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We show that the transverse polarization asymmetry of the τ lepton in the decay $t \to b\tau\nu$ is extremely sensitive to *CP* violating phases arising from the charged Higgs boson exchange in the Weinberg model of *CP* nonconservation. Qualitatively, the polarization asymmetries are enhanced over rate or energy asymmetries by a factor of $\approx m_t/m_\tau \sim 100$. Thus for optimal values of the parameters the method requires $\approx 10^4$ top pairs to be observable rather than 10^7 needed for rate or energy asymmetries. We also examine τ polarization in b decays via $b \to c\nu\tau$ and find that it can also be very effective in constraining the *CP* violation parameters of the extended Higgs sector.

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Because of its mass scale the top quark represents a unique probe for addressing the long-standing issue of *CP* violation. While the Fermilab Tevatron is expected to produce enough top quarks to establish its existence, the hadronic and e^+-e^- colliders under construction and being proposed should facilitate studies related to *CP*. In this context, the search for optimal experimental strategies is clearly important and requires extended phenomenological studies.

In the standard model (SM) *CP* violation effects in the top quark are too small to be observable [1]. However, it is difficult to see why the Kobayashi-Maskawa [2] (KM) phase should be the only source for *CP* nonconservation. Extensions of the SM almost invariably lead to additional *CP* violating phases. Indeed the observed baryon asymmetry in the Universe is often used to argue that additional sources of *CP* beyond the KM phase are a necessity. We are therefore motivated to continue our investigation of *CP* nonconservation caused in the production [3] and decays [4,5] of the top quark in one popular extension of the SM [6], namely, the Weinberg model [7] (WM). This model leads to large, possibly observable dipole moments [8] and rather sizable partially integrated rate asymmetry [4] (PIRA) in the semileptonic modes $t \to b\tau\nu_\tau$. In this work we show that the transverse polarization of the τ in these semileptonic transitions is extremely sensitive to *CP* violation effects due to the extended Higgs sector. Thereby, with optimal values of the parameters in the WM, requiring only a few thousand top quark pairs rather than $\gtrsim 10^7$ needed for the rate, the triple correlation, or the energy asymmetries [4,5,9]. Thus the polarization effects become accessible not only to the Superconducting Super Collider and the CERN Large Hadron Collider, where the estimates are for 10^7–10^8 top pairs/yr, but also to an electron-positron linear collider with an anticipated rate of about 10^4/yr. Clearly this approach can only be used if the top detectors are capable of measuring the τ polarization.

We have also examined the effectiveness of τ polarization as a possible signal of *CP* violation in semileptonic b decays due to the extended Higgs sector. Our finding is that one of the transverse polarization asymmetries is useful in b decays and it can also lead to stringent constraints on the parameters of the Higgs model with about 10^6 B mesons.

The semileptonic decay of the top quark proceeds through W exchange and Higgs boson exchange graphs [see Figs. 1(a) and 1(b)]. The *CP* violating phase is in the Higgs boson exchange. Since $m_t > m_W$, the dominant contribution of the W graph is from on shell W bosons. Thus Fig. 1(a) possesses both dispersive and absorptive pieces, and indeed both can have significant resonance enhancement, as we shall see below [1,4,5]. The absorptive piece of Fig. 1(a) interferes with the *CP* violating Higgs phase to contribute to observables (e.g., rate asymmetry) that are *CP* odd, T_N even [10]. Here T_N denotes naive time reversal where the spins and momenta of all particles are reversed but initial and final states are not interchanged. On the other hand observables such as momentum triple correlations that are *CP* odd, T_N odd receive contributions from the dispersive part of Fig. 1(a).

Let us now qualitatively try to understand why the transverse polarization is much more sensitive compared

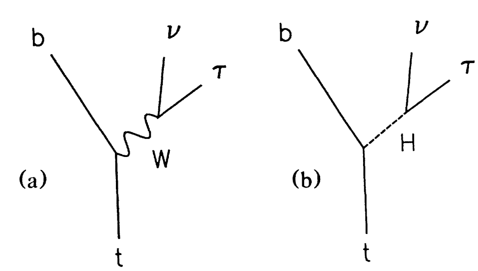

FIG. 1. (a) The Feynman diagram for $t \to b\tau^+\nu_\tau$ through W^+ exchange (i.e., the SM mechanism). (b) The Feynman diagram for $t \to b\tau^+\nu_\tau$ through H^+ exchange. For b decays via $b \to c\tau\nu$, replace $t \to b$ with $b \to c$ in the two diagrams.

to energy or rate asymmetries. Recall that PIRA (or energy asymmetry) goes as [11] m_τ^2/m_H^2. Here one power of m_τ originates from the Yukawa couplings at the $H\tau\nu_\tau$ vertex. The second power of m_τ comes from the trace over the lepton loop resulting from the interference between the W and the Higgs boson: $\mathrm{Tr}[\gamma^\mu(\not p_\tau+m_\tau)(1-\gamma_5)\not p_\nu]=4m_\tau p_\nu^\mu$. The m_τ in this trace can be avoided provided we do not sum over the spins of the τ. Thus we arrive at CP violating transverse polarization asymmetries of the τ that are larger by $\approx m_t/m_\tau \approx 100$ compared to PIRA requiring therefore $\approx 10^2$ fewer top pairs.

Recall that the Weinberg model [7] consists of three Higgs doublets enabling it to possess a CP violating Higgs sector without the flavor changing neutral currents. It therefore has two charged Higgs states $H_1^\pm$ and $H_2^\pm$. For simplicity we will assume that $H_1^\pm$ has a much higher mass than $H_2^\pm$ (m_H) so that we need to consider effects due to $H_2^\pm$ only. The Lagrangian coupling H_2 to quarks and leptons is given [12] by

$$L=\frac{g}{\sqrt{2}}H_2^+\left\{\bar{u}_i\frac{m_{dj}}{m_W}U_dP_Rd_jV_{ij}^{\mathrm{KM}}+\bar{u}_i\frac{m_{ui}}{m_W}U_uP_Ld_jV_{ij}^{\mathrm{KM}}+\bar{\nu}_i\frac{m_{li}}{m_W}U_lP_Rl_j\delta_{ij}\right\}+\mathrm{H.c.}, \tag{1}$$

where

$$U_l=-\frac{c_1s_2s_3+c_2c_3e^{i\delta}}{s_1s_2},\qquad U_u=\frac{c_1c_2s_3-s_2c_3e^{i\delta}}{s_1c_2},\quad U_d=\frac{s_1s_3}{c_1}, \tag{2}$$

and V^{KM} is the Kobayashi-Maskawa matrix. We denote $s_i=\sin(\theta_i)$ and $c_i=\cos(\theta_i)$, where θ_i and δ are parameters of the Higgs potential. The CP violation which we consider is proportional to combinations of these couplings such as $V_{ul}\equiv\mathrm{Im}(U_u^*U_l)$.

In the rest frame of the τ lepton, let us define a reference frame where the momentum of the top quark is in the $-x$ direction, the y direction is defined to be in the decay plane such that the y component of the b momentum is positive, and the z axis is defined by the right-hand rule. In the limit that the τ mass is small, the WM can give rise to two kinds of CP violating polarization asymmetries. These are

$$A_Y=\frac{\tau^+(\uparrow)-\tau^+(\downarrow)+\tau^-(\uparrow)-\tau^-(\downarrow)}{\tau^+(\uparrow)+\tau^+(\downarrow)+\tau^-(\uparrow)+\tau^-(\downarrow)},\qquad A_Z=\frac{\tau^+(\uparrow)-\tau^+(\downarrow)-\tau^-(\uparrow)+\tau^-(\downarrow)}{\tau^+(\uparrow)+\tau^+(\downarrow)+\tau^-(\uparrow)+\tau^-(\downarrow)}, \tag{3}$$

where for A_Y (A_Z) the arrows indicate the spin up or down in the direction y (z). While both A_Y and A_Z are CP odd, being T_N even, A_Y needs an interaction phase; A_Z is odd under T_N and does not need an interaction phase [13]. Thus A_Y (A_Z) is proportional to the absorptive (dispersive) part of Fig. 1(a).

For our analysis of top decay, we will ignore the masses of the b quark and the τ lepton. Also let us define $s=2p_\nu\cdot p_\tau$, $t=2p_\tau\cdot p_b$, $u=2p_b\cdot p_\nu$, $y_W=\Gamma_W^2/m_t^2$, $\lambda=s/m_t^2$, and $x_i=m_i^2/m_t^2$ for $i=b$, τ, H, and W. The angle θ is defined to be the angle between the W momentum and the τ momentum in the W rest frame. Consider now the case of the CP violating polarization in the x-y plane. This gives rise to a polarization asymmetry in the direction $V=(sp_b-tp_\nu)/\sqrt{stu}$, which lies in the x-y plane. In the τ rest frame if ψ_b is the azimuthal angle of the b quark momentum from the x axis and ψ_ν is the angle of the ν, then V defined above is a unit vector at angle $\frac{1}{2}(\psi_b+\psi_\nu-\pi)$ in the x-y plane. Note, however, that since the τ rest frame is highly boosted with respect to the top quark rest frame, the b and ν are close to the $-x$ direction so V will be very close to the y axis. The differential asymmetry is thus given by [using, here and in the relevant formulas to follow, $B(W\to\tau\nu)=1/9$]:

$$dA_Y=\frac{9g^2}{32\pi^2}V_{ul}\frac{(1-\lambda)^2\sqrt{x_\tau\lambda y_Wx_W}\sin\theta}{(1-x_W)^2(1+2x_W)(x_H-\lambda)[(\lambda-x_W)^2+y_Wx_W]}d\lambda\,d\cos\theta. \tag{4}$$

Integrating this over θ and λ using the narrow resonance approximation the result for the total asymmetry is

$$A_Y=\frac{9}{64}g^2V_{ul}\frac{\sqrt{x_\tau x_W}}{(1+2x_W)(x_H-x_W)}. \tag{5}$$

The CP violating polarization asymmetry in the z direction is proportional to the real part of the resonant W propagator. The differential asymmetry is

$$dA_Z=-\frac{9g^2}{32\pi^2}V_{ul}\frac{\sqrt{x_l}\sin\theta(\lambda-x_w)(1-\lambda)\sqrt{\lambda}}{(1-x_W)^2(1-2x_W)[(\lambda-x_W)^2+x_Wy_W](x_H-\lambda)}d\lambda\,d\cos\theta. \tag{6}$$

Integrating this over λ and θ one obtains the total asymmetry

$$A_Z=-\frac{9}{64\pi}g^2V_{ul}\frac{\sqrt{x_l}}{(1-x_W)^2(1+2x_W)x_H}f(x_W,y_W,x_H), \tag{7}$$

where f is the integral

$$f(x_W,y_W,x_H)=\int_0^1 \frac{(\lambda-x_W)x_H(1-\lambda)\sqrt{\lambda}}{[(\lambda-x_W)^2+x_Wy_W](x_H-\lambda)}d\lambda\,. \tag{8}$$

Note that due to the dependence on the real part of the resonance propagator, as s passes through x_W, the net polarization changes sign. There is, therefore, a partial cancellation of the polarization as defined above. If, however, the invariant mass of the $\tau\nu$ system can be experimentally determined within a few GeV, this information may be taken into account by weighting events where $s\le x_W$ with -1 and events where $s\ge x_W$ with $+1$. The total asymmetry defined in this way, A_Z', is thus given by

$$A_Z'=-\frac{9}{64\pi}g^2V_{ul}\frac{\sqrt{x_l}}{(1-x_W)^2(1+2x_W)x_H}f'(x_W,y_W,x_H)\,, \tag{9}$$

where f' is the integral as Eq. (11) except $\lambda-x_W$ is replaced by $|\lambda-x_W|$. Note that whereas f is smooth as $y_W\to 0$, f' grows logarithmically since in this case the resonant enhancement is not canceled.

Of course the polarization of the τ is not directly observable and must be inferred from the decay distributions of the τ. Let us define the sensitivity S of a method of measuring the polarization of the τ so that given N_τ τ leptons, the error in the measurement of the polarization, ΔP, is given by $\Delta P=(S\sqrt{N})^{-1}$. In a study [14], the decay modes $\pi\nu$, $2\pi\nu$, $3\pi\nu$, $e\nu\bar{\nu}$, and $\mu\nu\bar{\nu}$ are considered as polarization analyzers. The sensitivity which could be obtained in an ideal detector is found to be $S=0.25$ if one considers only the mode $2\pi\nu$; if one combines information from all four decay modes, $S=0.35$. Thus, the error in measuring a polarization asymmetry A, given N_t top quarks, is given by $\Delta A=[S\sqrt{NB(t\to\tau\nu b)}]^{-1}$.

In [4] the measurement of CP violation in this model is considered in $t\to\tau\nu b$ without using polarization information from the τ. CP violation is manifested by a difference in the distribution of τ leptons in $\cos\theta$ leading to an asymmetry in the partially integrated rate defined through the quantity α_+ as

$$\alpha_+=\frac{\Gamma_f(t\to b\tau^+\nu_\tau)-\Gamma_f(\bar{t}\to b\tau^-\nu_\tau)}{\Gamma_f(t\to b\tau^+\nu_\tau)+\Gamma_f(\bar{t}\to b\tau^-\nu_\tau)}\,, \tag{10}$$

where Γ_f is the rate for the decay into a state where the angle between the t momentum and the τ momentum in the W rest frame is greater than $\pi/2$. The value of this asymmetry is

$$\alpha_+=\frac{9\sqrt{2}}{4\pi}\frac{G_Fm_\tau^2r_{WH}V_{ul}}{(2+r_{Wt})(1-r_{WH})}\,, \tag{11}$$

where $r_{WH}=m_W^2/m_H^2$ and $r_{Wt}=m_W^2/m_H^2$.

All these quantities are proportional to V_{ul}. As in [4] we now consider what the maximum value of V_{ul} is that is consistent with experimental observations. It turns out that the one constraint comes from $\tau\to l\nu_l\nu_\tau$ decays where a 2% violation of lepton universality is not excluded by experiment [15]. We therefore impose

$$\frac{\Gamma(\tau\to\mu\nu_\mu\nu_\tau)_{W+H}-\Gamma(\tau\to\mu\nu_\mu\nu_\tau)_W}{\Gamma(\tau\to\mu\nu_\mu\nu_\tau)_W}\le 0.02\,, \tag{12}$$

where the subscript $H+W$ means the total rate including both H and W exchange while the subscript W means only the W exchange (i.e., the SM). This gives rise to the condition

$$\frac{m_\mu^2|U_u|^2|U_l|^2}{M_H^2}\left|\frac{m_\tau^2|U_u|^2|U_l|^2}{4m_H^2}-2\right|\le 0.02\,. \tag{13}$$

Another constraint comes from the nonobservation of $b\to s\gamma$. This decay in the WM is calculated in [16]. Following this paper we take $B(b\to s\gamma)\le 8.5\times10^{-4}$. In addition perturbative limits on the charged-Higgs-boson–fermion couplings imply $|U_u|m_t$, $|U_d|m_b$, $|U_l|m_\tau$ $\le 4\pi m_W/g\sqrt{2}$.

The result thus obtained is $V_{ul}=10^3$. Using this value of V_{ul}, Table I shows (see the second column) the value of each of the asymmetries α_+, A_Y, A_Z, and A_Z' for $m_t=150$ GeV and $m_H=200$ and 300 GeV. We then show how many top quarks are needed to observe these asymmetries where we take the sensitivity to τ polarization $S=0.25$ which is obtained using only the $2\pi\nu$ mode. We also calculate the restriction which may be placed on V_{ul} using 10^6 top quarks (see the last column). We see that the polarization asymmetries place much more stringent restrictions than α_+, being 1 to 2 orders of magnitude more effective.

CP violating τ-lepton polarization asymmetries may also be considered in semileptonic decays of B mesons to τ via $B\to\tau+\nu+X$. Now the signal will be largely proportional to V_{ld} [17]. Since for semileptonic B decays the spectator model works quite well it is sufficient for our purpose to study the free quark decay, $b\to\tau\nu c$. In this case, the W is far off shell so that the asymmetries which depend on interaction phases are higher order in the coupling compared to asymmetries which do not require such a phase. Thus, polarization asymmetries in the x-y plane and energy asymmetries of the τ lepton are expected to be much smaller than the polarization asymmetry in the z direction. The value of this asymmetry may be readily calculated as in the case of the top quark, giving

$$A_Z=\frac{8\pi}{35}\left(V_{ld}-\frac{m_c^2}{m_b^2}V_{ul}\right)\frac{\sqrt{u_\tau}}{u_H}\frac{J(u_\tau,u_c)}{I(u_\tau,u_c)}\,, \tag{14}$$

where $u_i=m_i^2/m_b^2$ for $i=\tau,c$ and H, I, and J are the kinematic integrals:

$$I(u_c,u_\tau)=12\int_{r_0}^1(r-u_c-u_\tau)(1-r)^2\lambda^{1/2}(r,u_\tau,u_c)\frac{dr}{r}\,,$$
$$J(u_c,u_\tau)=\frac{105}{16}\int_{r_0}^1 r^{-3/2}(1-r)^2\lambda(r,u_\tau,u_c)dr\,. \tag{15}$$

TABLE I. Here the value for the asymmetries (second column) above are the maximum consistent with the restrictions described above (i.e., using $V_{ul}=10^3$). N_t is the number of top quarks required to see the asymmetry, given in the second column, taking into account the sensitivity to τ polarization using only the $2\pi\nu$ decay mode. $V_{ul}^{\max}$ is the restriction on V_{ul} that may be attainable with 10^6 top quarks. In all cases $m_t=150$ GeV, the upper number in each case is for $m_H=200$ GeV and the lower is for $m_H=300$ GeV.

Asymmetry	Value (10^{-3})	N_t	$V_{ul}^{\max}$
α_+	2.8	3.2×10^6	1800
	1.4	13×10^6	3600
A_Y	161	5.6×10^3	75
	65	3.4×10^4	184
A_Z	96	1.6×10^4	126
	29	1.7×10^5	412
A_Z'	540	4.9×10^2	22
	220	3.0×10^3	55

Here $r_0=(u_c^{1/2}+u_\tau^{1/2})^2$ and $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ca$. Evaluating the above expression we find that $A_Z=1.2\times10^{-4}V_{ld}$ for $m_H=200$ GeV and $A_Z=5.2\times10^{-5}[V_{ld}-(m_c^2/m_b^2)V_{ul}]$ for $m_H=300$ GeV. We thus note that asymmetries from b decay will primarily put restrictions on the quantity V_{ld} because the factor m_c^2/m_b^2 suppresses the contribution of the term with V_{ul} by about an order of magnitude. Returning to the discussion of the existing experimental bounds on the Weinberg model, we find that there is little restriction on V_{lb} from existing experiments; in fact it may be as large as 1.3×10^4. Even if V_{lb} is an order of magnitude less it would require about 10^5 B's (assuming the same efficiency of 0.25 for detecting the τ polarization) to show up as a nonvanishing effect. At a B factory, given 10^8 b quarks, under ideal condition (using just the $2\pi\nu$ decay mode to determine the polarization of the τ), at 1σ one may put a restriction of $V_{ld}\le 16$ if $m_H=200$ GeV and $V_{ld}\le 37$ for $m_H=300$ GeV. We thus see that it takes about 100 times more B's than top quarks to attain a similar reach on the CP parameters, i.e., V's, of the extended Higgs model [18,19].

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[18] Needless to say, our emphasis in this paper is on the theoretical issue(s). Experimental determination of τ polarization in the environment of future colliders dealing with top quark decays may be considerably more complicated than the LEP studies mentioned above (Ref. [14]). In view of the fact that two neutrinos are being lost, full reconstruction will be very difficult. It is conceivable that directional information on the τ will be crucially needed. Correlation between the b jet and some of the decay products of the τ, e.g., the ρ, could also prove useful in capitalizing most of the sensitivity being discussed here.
[19] In passing we note that the CP asymmetries considered in this work, as well as in Refs. [4] and [5], receive almost all of their contributions from the transverse part of the W propagator. There is universal agreement on the form of this transverse piece. The discussions in the recent literature concerning the form of the longitudinal piece of the W propagator [see, e.g., J. Liu, Phys. Rev. D **47**, R1741 (1993); M. Nowakowski and A. Pilaftsis, University of Mainz Report No. MZ-TH-92-16 (to be published); D. Atwood *et al.*, Report No. TECHNION-PH-92-39 (to be published)] are not relevant to the dominant effects being discussed in this work.