New Upper Limits on the Tau-Neutrino Mass from Primordial Helium Considerations

A. D. Dolgov* and I. Z. Rothstein
The Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109
(Received 15 March 1993)

In this paper we reconsider recently derived bounds on MeV tau neutrinos, taking into account previously unaccounted for effects. We find that, assuming that the neutrino lifetime is longer than $\sim$ 100 sec, the constraint $N_{\text{eff}} < 3.6$ rules out ν_τ masses in the range $0.5 < m_{\nu_\tau} < 35$ MeV for Majorana and $0.3 < m_{\nu_\tau} < 35$ MeV for Dirac neutrinos. Given that the present laboratory upper bound is 31 MeV, our results imply an upper bound of 0.5 MeV for Majorana neutrinos and of 0.3 MeV for Dirac neutrinos.

PACS numbers: 98.80.Ft, 12.15.Ff, 14.60.Gh, 98.80.Cq

Despite a considerable experimental effort it is still unknown whether or not neutrinos have a nonzero mass. The following upper bounds are known [1]:

$$m_{\nu_e} < 10 \text{ eV}, \tag{1a}$$

$$m_{\nu_\mu} < 270 \text{ keV}, \tag{1b}$$

$$m_{\nu_\tau} < 31 \text{ MeV}. \tag{1c}$$

More stringent bounds can be determined from cosmological considerations. Assuming that there is no cosmological constant, the masses of light ($m_\nu \lesssim 100$ MeV) stable neutrinos are bounded by the Gerstein-Zeldovich limit [2] which can be written as [3]

$$m_\nu < (386 \text{ eV}) \left(\frac{9.78 \times 10^9}{t_U \text{ (yr)}} - h_{100} \right)^2 , \tag{1}$$

where t_U is the age of the Universe and $h_{100} = H/(100$ km/sec Mpc) is the dimensionless Hubble parameter. By most estimates $t_U > 12$ Gyr and $0.5 < h < 1$. Thus the cosmological upper bound for the neutrino mass (all weakly interacting flavors) is 40 eV. If there is a nonzero cosmological constant the bound is somewhat less restrictive, $m_\nu < 200$ eV [4]. These bounds do not apply if neutrinos are unstable with lifetimes smaller than the age of the Universe. It has been pointed out [5] that nucleosynthesis considerations can further constrain the mass of an unstable τ neutrino. Nucleosynthesis calculations, along with data on the light elements abundances, constrains the number of effective neutrino species contributing to the cosmic energy density N_ν, to be less than 3.3 [6]. In this paper we impose the constraint $N_\nu < 3.6$, as this is the 3σ bound. (Recent data also seems to indicate that this bound may be more appropriate [7].) This bound is a consequence of the fact that the rate at which the Universe cools depends on the total number of species contributing to the cosmological energy density which in turn determines the light element abundances. Naively one might think that the contribution from a heavy neutrino species with $m \gtrsim 1$ MeV could be neglected since its energy density was assumed to be Boltzmann suppressed. However, after a massive neutrino decouples and becomes nonrelativistic, its energy density grows relative to that of a massless species. Therefore, if the number density of a massive species at freeze out is on the order of the number density of a massless species, then the heavy species may have a greater effect on the light element abundance than a massless species.

In this paper we reconsider the most recent bounds on MeV neutrinos, which were derived in Ref. [8]. By using a more accurate treatment of the thermodynamics, as well as taking into account the change in the number density due to residual annihilations, we are able to strengthen the bound in [8]. We find that there is *no allowed window* between the experimental bound and the nucleosynthesis bounds, even when using the more conservative bound $N_\nu < 3.6$ (Ref. [8] used the bound $N_\nu < 3.4$).

In the standard calculations of the relic abundance of a particle species disappearing due to annihilation, the following two essential assumptions are made: The particles in question remain in kinetic equilibrium, and Boltzmann statistics are applicable. In this case the Boltzmann kinetic equation can be reduced to an ordinary differential equation. If the species considered is relativistic or nonrelativistic at the time of decoupling, the equation can be greatly simplified and relatively accurate analytical solutions can be found. However, for a species which is semirelativistic when it decouples, there is no known analytical approximation and one has to integrate the Boltzmann equation numerically. As such, we will present some of the details of the calculations.

If we are concerned with the abundance of species ψ then the Boltzmann equation is given by (we specialize to the case $\psi + 2 \to 3 + 4$)

$$\frac{dn_\psi}{dt} + 3Hn_\psi = \frac{g}{(2\pi)^3} \int C[f] \frac{d^3 p_\psi}{E_\psi}, \tag{2}$$

where $H = \dot{R}/R$ is the Hubble parameter. In this notation

$$n_\psi = g \int \frac{d^3 p}{(2\pi)^3} f_\psi(E, t), \tag{3}$$

0031-9007/93/71(4)/476(4)$06.00

$f_\psi(E,t)$ is the phase space density of species ψ, and g is the number of degrees of freedom for ψ. $C[f]$ is the collision term given by

$$\frac{g}{(2\pi)^3}\int C[f]\frac{d^3p_\psi}{E_\psi} = -\delta^4\left(\sum p\right)(2\pi)^4\int d\Pi_\psi d\Pi_2 d\Pi_3 d\Pi_4[|M_{\psi+2\to3+4}|^2\,(f_\psi f_2(1\pm f_3)(1\pm f_4)) - |M_{3+4\to\psi+2}|^2\,(f_3f_4(1\pm f_\psi)(1\pm 2))] \quad (4)$$

and

$$d\Pi_i = \frac{d^3p_i}{(2\pi)^3(2E_i)}. \quad (5)$$

The amplitude M is summed over initial and final spins. Assuming that the initial and final states are in kinetic equilibrium we may write $f_a = n_a\,\exp(-E/T)/n_a^{\rm EQ}$. Equation (4) may be written as

$$\frac{dn_\psi}{dt} + 3\frac{\dot R}{R}n_\psi = -g_\psi g_2\int d\Pi_\psi d\Pi_2\left[\sigma v_{\rm Moller}\,\exp[-(E_\psi+E_2)/T]\frac{(n_\psi n_2 - n_\psi^{\rm EQ}n_2^{\rm EQ})}{n_\psi^{\rm EQ}n_2^{\rm EQ}}\right]. \quad (6)$$

Following the notation of Ref. [9] we have defined

$$v_{\rm Moller} = \frac{F}{4E_\psi E_2} = [|\mathbf{v}_\psi - \mathbf{v}_2|^2 - |\mathbf{v}_\psi\times\mathbf{v}_2|^2]^{1/2}, \quad (7)$$

where F is the particle flux. In the nonrelativistic limit the Moller velocity reduces to the relative velocity.

It is convenient to introduce the dimensionless variable $x = m/T$ and the relative number density $r = n/n_0$, where m is the mass of annihilating particles and n_0 is conserved in a comoving volume. (Often the number density n is normalized to the entropy density s. This choice is convenient because s is usually conserved in the comoving volume. However, if a species is annihilating while it has considerable energy density, then when it drops out of equilibrium the entropy will *not* be conserved.) In terms of these quantities the Boltzmann equation takes the form:

$$\frac{dr}{dx} = \frac{n_0}{x(\dot T/T)}\langle\sigma v_{\rm Moller}\rangle(r^2 - r_{\rm EQ}^2). \quad (8)$$

It can be shown that [9]

$$\langle\sigma v_{\rm Moller}\rangle = \int_{4m^2}^\infty \frac{\sigma(s-4m^2)\sqrt{s}K_1(\sqrt{s}/T)}{8m^4TK_2^2(m/T)}. \quad (9)$$

In this equation K_1 and K_2 are the modified Bessel functions and $s = (p_\psi + p_2)^2$. The squared amplitude integrated over the final particle phase space, for the annihilation of two Majorana neutrinos with mass m_{ν_τ} into fermions with mass m_f is given by

$$\frac{1}{4}\sum_{\rm spins}\int d\Pi_3 d\Pi_4\,|M|_M^2 = 8\pi G_F^2\frac{w}{s}\left[(C_V^2+C_A^2)(m_{\nu_\tau}^2(m_f^2 - s/2)) + (C_A^2 - C_V^2)m_f^2(3m_{\nu_\tau}^2 - s/2)\right] + \frac{32}{12}\pi G_F^2\frac{w}{s^2}(C_V^2+C_A^2)\left[\frac{w^2}{2}(s/2 - m_{\nu_\tau}) + \frac{s^2}{4}(s+2m_f^2)\right]. \quad (10)$$

The analogous expression for the case of annihilating Dirac neutrinos is given by

$$\frac{1}{4}\sum_{\rm spins}\int d\Pi_3 d\Pi_4\,|M|_D^2 = 32\pi G_F^2\frac{w}{48}\left[(C_V^2+C_A^2)(s - m_f^2 - m_{\nu_\tau}^2 w^2/s^2)\right] + 4\pi G_F^2\frac{wm_f^2}{s}(C_V^2 - C_A^2)\,(s/2 - m_{\nu_\tau}^2). \quad (11)$$

We have defined $w = (s^2 - 4sm_f)$, where m_f is the mass of the final state particle, which for our purposes is the electron, or the other neutrinos species which we will take to be massless. In the nonrelativistic limit the cross section for the annihilation of Dirac neutrinos reduces to

$$\sigma_D v_{\rm rel} = \frac{G_F^2}{2\pi}m_{\nu_\tau}^2(1-z^2)^{1/2}(C_V^2+C_A^2)\left[1+\frac{\beta^2}{6}\left(5-2z^2+\frac{3}{1-z^2}\right)\right] + (C_V^2 - C_A^2)\left[\frac{z^2}{2}\left\{1+\beta^2\left(\frac{1}{2}+\frac{1}{2(1-z^2)}\right)\right\}\right], \quad (12)$$

where $z = m_f/M$, m_f is the decay product mass and β is the velocity in the center of mass frame. $v_{\rm rel} = 2\beta$ in the rest frame of the plasma. We include this expression because our result differs slightly from those stated previously in the literature [10–12].

To perform the numerical integration it is necessary to specify the function $n_0(T)$ as well as the function $T(t)$. In the simplest case the energy density is dominated by relativistic particles in thermal equilibrium $\dot T = -HT$ and $n_0 \propto T^3$. In what follows we take $n_0 = 0.181T^3$ which corresponds to the equilibrium number density of massless fermions with two helicity states. The Hubble parameter is given by the expression

$$H = \sqrt{8\pi\rho/3M_{\rm Pl}^2} = 1.66\sqrt{g_*}T^2/M_{\rm Pl}. \tag{13}$$

In this simple case all the quantities are defined in terms of the plasma temperature T and the numerical integration is straightforward. When there is a mixture of relativistic and nonrelativistic (or semirelativistic) species with a conserved number of nonrelativistic particles, one must substitute for g_* the expression

$$g_* = g_*^{\rm rel}(1 + \rho^{\rm NR}/\rho^{\rm rel}), \tag{14}$$

where $\rho^{\rm rel}$ and $\rho^{\rm NR}$ are the energy densities of relativistic and nonrelativistic species, respectively. If we neglect the energy exchange between the relativistic and nonrelativistic species the rate of the cooling would be the same, $\dot{T} = -HT$, and we have to solve kinetic equation (8) subject to (14). For the choice of normalization $n_0 = 0.181T^3$ Eq. (14) takes the form

$$g_* = g_*^{\rm rel} + 0.554\langle E\rangle r/T \tag{15}$$

with $\langle E\rangle$ being the average energy per particle for the massive fermions. The massive neutrinos will have a thermal distribution as long as they remain in kinetic equilibrium, which is true down to temperatures $T_0 \approx 2$ MeV. Below this temperature the distribution function in the phase space changes from $\exp\{-\sqrt{(m^2+p^2)}/T\}$ to $\exp\{-\sqrt{(m^2/T_0^2 + p^2/T^2)}\}$. This change in the distribution function will modify the Boltzmann equation. For large masses it is not expected that these changes will have any significant effect. We have calculated the effect of such changes and found them to be less the two percent for masses above 1 MeV. For masses $m < 1$ MeV, the neutrinos decoupled while they were still relativistic, so their number density will not be suppressed relative to a massless species. Thus in the case of $m < 1$ MeV, $r = 1$, and the massive neutrinos distribution in phase space is given by $\exp(-p/T)$, since the neutrinos will decouple while they are still relativistic. Given this distribution function the average energy is $\langle E\rangle \approx (m^2 + 0.414mT + 3.151T^2)^{1/2}$. This approximate expression is accurate up to 0.5% and was used in our numerical work for the case of light neutrinos. We should also point out that for smaller masses, $m = 1$–5 MeV the Boltzmann approximation is not accurate and the helium abundance will decrease by (5–10)% [13]. However, as we shall see, this will not affect our bounds.

Exchange of energy between massless and massive particles gives rise to a faster cooling. Covariant energy conservation demands that

$$\frac{\dot{T}}{T} \approx -H\left(1 + \frac{0.2r}{g_*^{\rm rel} + 0.42r\frac{m}{T}}\right) - \frac{0.14\dot{r}(\frac{3}{2} + \frac{m}{T})}{g_*^{\rm rel} + 0.42r\frac{m}{T}}. \tag{16}$$

This is valid if $\rho^{\rm NR} < \rho^{\rm rel}$ and the massive and massless particles are in thermal contact. If this inequality did not hold the cooling rate would be different, and Eq. (8) would not be valid since it was based on the assumption that the massive particles were in kinetic equilibrium.

We have numerically solved kinetic Eq. (8) for $g_*^{\rm rel} = 9$ and $g_*^{\rm NR}$ given by Eq. (15). This corresponds to the case of two light neutrinos and one heavy. Note that $g_*^{\rm NR}$ depends on the unknown function $r(T)$ which is determined from the solution of the kinetic equation. Our results for the freezeout number density of the massive Majorana and Dirac tau neutrinos are presented in Fig. 1. We then calculated the n/p ratio taking into account that r is not a constant but decreases due to annihilation of the heavy neutrinos. Furthermore, we used the exact expression for the average energy density of the heavy neutrinos by integrating the distribution function.

Bounds on the neutrino mass are derived by computing the net increase in helium production due to the presence of the heavy neutrino species. (The bounds will be effectively independent of the value of η chosen because the helium abundance is fairly insensitive to this parameter.) In accordance with the standard nucleosynthesis calculations almost all neutrons which survived down to the temperature $T_\gamma = 0.065$ MeV turn into ^{4}He. It is because of this fact that it is important to keep track of residual massive neutrino annihilation. The final neutron number density will be sensitive to the time between when the n-p reactions freeze out and the time when deuterium formation begins. This interval will depend on the energy density contribution from the heavy neutrinos.

By comparing the neutron to proton ratio calculated for two massless neutrinos and one neutrino with mass m_{ν_τ} at $T_\gamma = 0.065$ MeV, to the n/p ratio calculated for a variable number of massless neutrino species at the same temperature, we may bound the neutrino mass. If the n/p ratio yield for a given mass m_{ν_τ} exceeds the ratio calculated for 3.6 massless species, then that mass

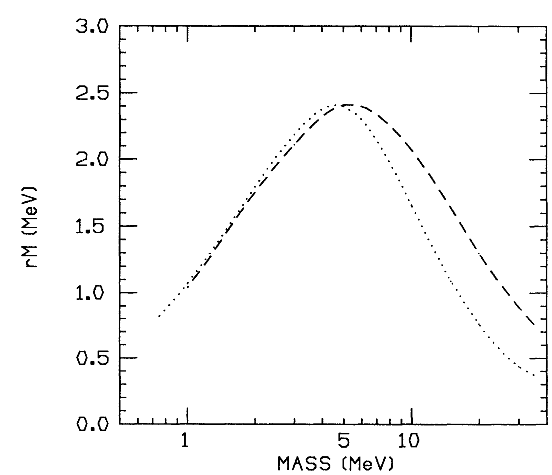

FIG. 1. rm for Majorana (dashed) and Dirac (dotted) neutrinos. rm for Dirac neutrinos does not include the contribution from the right-handed species.

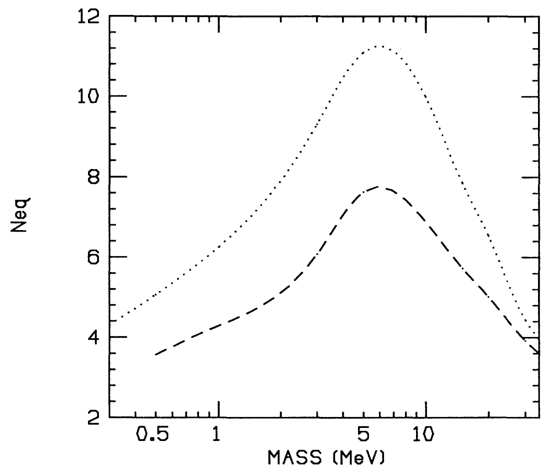

FIG. 2. Number of equivalent massless species for Majorana (dashed) and Dirac (dotted) neutrinos. The contribution from both chirality states of the Dirac neutrino are included.

is ruled out. For the case of Dirac neutrinos we assume that the right-handed species will be populated if $m_D > 0.74$ MeV [14]. From these considerations we find that, assuming the neutrino lifetime is longer than ~ 100 sec, the following range of masses are excluded (see Fig. 2):

$$0.5 < m_M < 35 \text{ MeV}, \tag{17}$$

$$0.3 < m_D < 35 \text{ MeV}. \tag{18}$$

Tau neutrinos with their mass in the region considered must be unstable in accordance with the Gerstein-Zeldovich limit. For our bounds to be valid the neutrino must not decay prior to primordial nucleosynthesis. Since the characteristic temperature scale is near 0.1 MeV the lifetime should be larger than or of the order of 100 sec. For lifetimes shorter than 1 sec, there will be no bound for Majorana neutrinos because the decay will keep the neutrino in thermal equilibrium (this is assuming that the neutrino decays only into particles present in the standard model). However, for Dirac neutrinos there will be a small region of excluded masses near 1 MeV, as the contribution to the energy density from such a neutrino will be twice that of a massless Weyl species.

One could deduce bounds on both m_{ν_τ} and on τ_{ν_τ} accounting for the decay of ν_τ in the kinetic equation governing both the number density of ν_τ and the n/p ratio. The final n/p ratio depends not only the lifetime of ν_τ but also on the type of particles in the final state. A considerable effect associated with the decay might emerge from the distortion of the electron neutrino spectrum if there is a decay into ν_e. This is analogous to the distortion of the spectrum of the electron neutrinos due to electron-positron annihilation in the standard scenario at the level about 1% found recently [15,16]. The size of the effect in the case of massive ν_τ annihilation or decay should be bigger because the (hypothetical) mass of ν_τ is assumed to be larger than $m_e = 0.5$ MeV. We hope to take all these effects into account in the subsequent publication.

The authors would like to thank Terry Walker for corroborating our results on the neutron to proton ratio for the case of massless neutrinos, as well as Dima Dolgov for assistance with numerical work. The authors would also like to thank M. Turner for useful discussions and for pointing out an error in the original manuscript. This work was supported by the following Grants No. NSF-AST-9296246 and No. NSF-PHY-9296020.

Note added.—At the time of writing, the authors became aware of similar work done by H. Kang, M. Kawasaki, A. Kernan, R. Sherrer, G. Steigman, and T. Walker, which was presented at Neutrino '92 (Ohio State Report No. OSU-TA-11/92). These authors also consider the effects of decaying neutrinos.

* Permanent address: Institute of Theoretical and Experimental Physics, 113259, Moscow, Russia.

[1] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992). ARGUS Collaboration, in "Second Workshop on Tau Lepton Physics," The Ohio State University, September 1992 (unpublished).
[2] S.S. Gerstein and Ya. B. Zeldovich, Pis'ma Zh. Eksp. Teor. Fiz. **4**, 258 (1966) [JETP Lett. **4**, 174 (1966)]; R. Cowsik and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972).
[3] A.D. Dolgov, in Proceedings of the XXII International Conference on High Energy Physics, Leipzig, 1984 (unpublished).
[4] R.A. Syunaev and Ya. B. Zeldovich, Pis'ma Astron. Zh. **6**, 451 (1980).
[5] E. Kolb and R. Scherrer, Phys. Rev. D **25**, 1481 (1982).
[6] T. Walker *et al.*, Astrophys. J. **376**, 51 (1991).
[7] T. Walker, Ohio State Report No. OSU-TA-1/93 (to be published).
[8] E. Kolb, M.S. Turner, A. Chakravorty, and D.N. Schramm, Phys. Rev. Lett. **67**, 533 (1991).
[9] P. Gondolo and G. Gelmini, Nucl. Phys. **B360**, 145 (1991).
[10] G. Kane and I. Kani, Nucl. Phys. **B277**, 525 (1986).
[11] E. Kolb and K. Olive, Phys. Rev. D **33**, 1202 (1986); **34**, 2531(E) (1986).
[12] M. Srednicki, R. Watkins, and K. Olive, Nucl. Phys. **B310**, 693 (1988).
[13] A.D. Dolgov and K. Kainulainen, Minnesota Report No. TPI-MINN-92-T (to be published).
[14] G.M. Fuller and R.A. Malaney, Phys. Rev. D **43**, 3136 (1991).
[15] A.D. Dolgov and M. Fukugita, Pis'ma Zh. Eksp. Teor. Fiz. **56**, 129 (1992) [JETP Lett. **56**, 123 (1992)]; Phys. Rev. D **46**, 5378 (1992).
[16] M.S. Turner and S. Dodelson, Phys. Rev. D **46**, 3372 (1992); B.D. Fields, S. Dodelson, and M.S. Turner, *ibid.* **47**, 4309 (1993).