Resonant Tunneling between Quantum Hall Edge States

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Resonant tunneling between fractional quantum Hall edge states is studied in the Luttinger liquid picture. For the $\nu = 1/3$ Laughlin parent state, the resonance line shape is a universal function whose width scales to zero at zero temperature. Extensive quantum Monte Carlo simulations are presented for $\nu = 1/3$ which confirm this picture and provide a parameter-free prediction for the line shape.

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It has been understood theoretically for over 30 years that an interacting one-dimensional electron gas should exhibit a novel non-Fermi-liquid phase, recently termed a "Luttinger liquid." Nevertheless, to date there has been scant, if any, clear experimental evidence for Luttinger liquid behavior. However, with the recent realization that the edge states in the fractional quantum Hall effect are Luttinger liquids [1], the time is finally ripe for direct confrontation with experiment. In this paper, in addition to describing the surprising universal scaling behavior expected for tunneling resonances in a Luttinger liquid, we calculate explicitly the universal resonance line shape appropriate for tunneling between $\nu = 1/3$ quantum Hall edges. A comparison of this universal line shape with experiment should provide strong evidence for Luttinger liquid behavior. Indeed, recent experiments by Webb and Milliken [2] on transport in a constricted $\nu = 1/3$ quantum Hall device find resonances which scale nicely and have a line shape which agrees quite well with our calculation.

In a beautiful series of articles, Wen [1] has shown that the gapless edge excitations of a fractional quantum Hall fluid are "chiral" Luttinger liquids. In contrast to the electrons in real one dimensional wires, which can be backscattered and localized by extraneous impurities, the electrons in a chiral Luttinger liquid move only in one direction, so that localization is completely unimportant. Backscattering can, however, occur in the vicinity of a point contact, and the Luttinger liquid correlations play a crucial role in determining the nature of tunneling and resonant tunneling between edges at low temperatures. This can be tested in an experiment which consists of measuring the two-terminal source-drain [3] conductance of the device shown in Fig. 1 as a function of a gate voltage which pinches off the channel, forming a point contact. When the point contact is wide, the conductance will be given by the quantized Hall conductance $G = \nu e^2/h$. As the channel is pinched off, the conductance will tend to decrease. However, there will typically be resonant transmissions at particular values of the gate voltage due to tunneling through localized states in the vicinity of the point contact. We show below that as the temperature is lowered below the bulk gap, these resonances should become sharper and have a universal shape. We present the results of extensive quantum Monte Carlo simulations which give the universal line shape expected for such resonances for the $\nu = 1/3$ state.

Recently, two of us have studied the problem of resonant tunneling in an interacting 1D electron gas—a Luttinger liquid [4,5]. A renormalization group (RG) analysis reveals that the resonance line shape at low temperatures is universal, depending only on the two-terminal conductance, $G = ge^2/h$, of the Luttinger liquid. The parameter g depends in a complicated way on the strength of the interactions between left- and right-moving electrons. However, in a quantum Hall edge state the electrons move only in one direction. Most strikingly, in this case g is a topological invariant controlled by the quantum Hall state in the bulk [1]. To see this, consider raising the chemical potential of the right movers relative to that of the left by an amount $\delta\mu$. This corresponds to applying a Hall voltage $V_H = \delta\mu/e$ and the resulting current is given by the quantized Hall coefficient as $I = \nu(e^2/h)V_H$. This immediately establishes the universal result within a given Hall plateau: $g = \nu$. *This remarkable fact makes the resonance line shape completely universal, model independent, and fully determined (up*

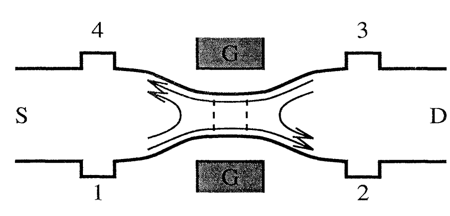

FIG. 1. Four-terminal Hall bar geometry with a narrow constriction between the source (S) and drain (D) formed by a lithographically patterned gate (G). The dotted lines represent two parallel tunneling paths for quasiparticles. Precisely on resonance, there is perfect edge transmission between the source and the drain, as indicated by the arrows. Away from resonance the edge channels are reflected at low temperatures.

to an overall temperature scale). The fractional quantum Hall regime ($g < 1$) is thus a far more promising place to observe pure Luttinger liquid behavior than in a single-channel quantum wire, where it is difficult to eliminate disorder and where the value of g is unknown. Chamon and Wen [6] have recently considered a theory of resonant tunneling between quantum Hall edge states which is valid in the limit in which the peak conductances of the resonances are much less than e^2/h, or in the tails of a stronger resonance. A similar theory for resonant tunneling in a Luttinger liquid has been developed by Furusaki and Nagaosa [7]. In contrast, the scaling theory presented below is valid over the entire width of the resonance for resonances whose peak conductance approaches the "perfect" value $\nu e^2/h$.

The analog of a weak impurity that causes backscattering in a 1D wire is a narrow constriction which brings the left and right movers close enough together to communicate via tunneling of Laughlin quasiparticles through a "weak link" as illustrated in Fig. 1. The analog of the two-impurity resonance geometry in a 1D wire considered by Kane and Fisher [5] would be two nearby, symmetric tunneling paths for quasiparticles [8]. For some value of magnetic field or gate voltage one will (randomly or intentionally) achieve the condition of destructive interference [9] which shuts off the interedge quasiparticle tunneling. This is the resonance (no backscattering) condition which will be manifested experimentally by the appearance of a two-terminal source-drain conductance [3] which peaks at a value that at low temperatures approaches the quantized value, $G = \nu e^2/h$. Away from resonance the quasiparticle tunneling causes current to leak from one edge to the other, thereby reducing the source-to-drain conductance (see Fig. 1). In fact, in the fractional quantum Hall effect the quasiparticle tunneling is expected to diverge upon cooling, driving the source-to-drain conductance all the way to zero in the zero temperature limit. At finite temperatures, this crossover is described by a universal scaling function, which we calculate below for $g = 1/3$ using quantum Monte Carlo simulations.

We begin our analysis by briefly reviewing the logic behind Wen's edge state theory. For simplicity we focus here on the primary Hall states with inverse filling factor ν^{-1} equal to an odd integer. In this case the edge state has only one branch.

Conservation of electron three-current j_μ permits us to introduce a fictitious gauge field a_μ via

$$j_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda. \qquad (1)$$

The bulk 2D electron gas is in an incompressible quantum Hall state with an excitation gap, which means that the low-energy, long-length-scale physics must be described by a massive theory. In (2+1)D the only massive gauge theory is the Chern-Simons theory which has (Euclidean) action [10] (ignoring irrelevant terms)

$$S_{\text{bulk}} = \frac{i}{4\pi\nu}\int a_\mu\partial_\nu a_\lambda\epsilon_{\mu\nu\lambda}\, d^2x\, d\tau. \qquad (2)$$

The coefficient ν^{-1} is uniquely fixed by the quantized Hall conductivity and specifies the number of zeros bound to the electrons in the Laughlin wave function [11–13].

Since the bulk is incompressible, the only low-lying excitations are distortions of the boundary shape which preserve the area of the incompressible liquid. Wen has shown that in the presence of a boundary, say at $y = 0$, an effective action for the edge state can be obtained as follows [1]: First integrate out a_τ in the bulk, which gives an incompressibility constraint on the electron density, $\epsilon_{ij}\partial_i a_j = 0$. Then solve the constraint in terms of a scalar field, $a_j = \partial_j\phi$. After an integration by parts the final Euclidean action for the edge state takes the form

$$S_{\text{edge}} = \frac{1}{4\pi g}\int dx\, d\tau(\partial_x\phi)(i\partial_\tau\phi + v\partial_x\phi). \qquad (3)$$

Here v is the velocity of the edge excitation, which is nonuniversal, and will depend on the details of the edge confining potential and the Coulomb interaction at the edge. The dimensionless parameter g, on the other hand, is *universal* and depends only on the quantum Hall state, $|g| = \nu$. As emphasized by Wen the requirement that the Hamiltonian associated with Eq. (2) be bounded below requires that v/g be positive.

This analysis neglects two principal effects of a long range Coulomb interaction, which have two principal effects. The first is a spreading away from the edges of the transport current carried by the condensate. This is unimportant because, as discussed below, it is only the total integrated current across the Hall bar and not the detailed spatial distribution which enters the final model [(Eq. (7)]. The second effect of the long range Coulomb interaction is to modify the dispersion and coupling of the two chiral modes. The latter is expected to be relevant, but only at very low temperatures (roughly $T < 10$ mK for a Hall bar of length 100 μm and width 1 μm) [1,14].

It follows from Eq. (1) that the one-dimensional electron density along the edge, $\rho(x)$, is given by $\rho = \partial_x\phi/2\pi$. An expression for the electron creation operator at the edge can be obtained by combining this with the fact that the momentum operator conjugate to ϕ is $\Pi_\phi = \rho/g$. Since adding an extra electron to the edge is equivalent to creating an "instanton" in ϕ, in which ϕ changes by 2π in the region near x, the electron creation operator at the edge is simply

$$\psi(x) \sim \exp\left[2\pi i\int^x \Pi_\phi(x')dx'\right] = e^{i\phi(x)/g}. \qquad (4)$$

A "vortex" or Laughlin quasiparticle at the edge is created simply by $e^{i\phi(x)}$, which carries fractional charge ge.

We now suppose that the right-moving and left-moving edges are coupled via a tunneling term, say at $x = 0$. The total action will then have the form $S_L[\phi_L] + S_R[\phi_R] + V(\phi_L, \phi_R)$. Lacking a specific model we cannot say whether quasiparticles or electrons will tunnel more easily. Presumably the fractionally charged quasiparticles see a lower barrier, but the matrix elements may compensate. Instead, we write down the most general form allowed by symmetry. Taking the weak link region to be at $x = 0$, we have

$$V = \sum_{m=1}^{\infty} v_m \exp\{im[\phi_L(x=0) - \phi_R(x=0)]\} + \text{c.c.}, \tag{5}$$

where the v_m are (complex) tunneling amplitudes. The term $m = 1$ represents the combined amplitudes for a quasielectron to tunnel from one edge to the other *or* a quasihole to tunnel in the opposite direction. These physically distinct processes lead to the same final state and hence add coherently to produce v_1. The term $m = 1/g$ corresponds to electron tunneling. We have no *a priori* knowledge of the v_m. Fortunately, for $g = 1/3$, all terms except v_1 are irrelevant, having a negative RG eigenvalue, $1 - gm^2$ [5]. Thus at low enough temperatures ($T \ll \Delta$, where $\Delta \sim 1$ K is the bulk excitation gap) and small enough v_1, the irrelevant variables $v_m, m > 1$ will flow to zero before $v_{\text{eff}} \sim v_1/T^{1-g}$ has grown large. Thus the RG flow will follow a *universal* trajectory away from the resonance fixed point ($v_1 = 0$) into the insulating fixed point.

At finite temperature the renormalization group flows will be cut off, and the system will end up somewhere along that universal trajectory. From this it follows that in the limit of low temperature, the conductance as a function of the resonance tuning parameter $\delta \sim |v_1|$ and the temperature will obey the scaling form

$$G(T, \delta) = \tilde{G}_g(c\delta/T^{1-g}). \tag{6}$$

The scaling function $\tilde{G}_g(X)$ is *universal* in the sense that it does not depend on microscopic details, but is a property of the universal trajectory connecting the two fixed points. Since $g = \nu$ in the quantum Hall effect, $\tilde{G}_g$ is completely determined by the theory. The parameter c is a nonuniversal dimensionful factor which sets the temperature scale. Demanding that the scaling form (6) matches onto the off-resonance conductance, which vanishes as $G(T) \sim T^{2/g-2}$, implies that the tails of the scaling function should decay like $X^{-2/g}$, or X^{-6} for $g = 1/3$.

Note that since $\delta \sim |v_1|$ and v_1 has both a real and an imaginary part, in general it will be necessary to tune two parameters to achieve resonance, $\delta = 0$. However, if the two quasiparticle tunneling paths in Fig. 1 have equal amplitude, then v_1 can be driven to zero with only a single parameter, such as the gate voltage. Varying the B field within the range of the $\nu = 1/3$ plateau can provide a second control parameter.

Though the general properties of the scaling function, such as the temperature dependence of the width and the exponent in the tails, are known the detailed shape of the scaling function has been calculated analytically [5] only for $g = 1/2$. This problem is idealy suited to Monte Carlo simulation, and we have explicitly computed $\tilde{G}(X)$, verified the predicted scaling behavior, and determined the entire scaling function for $\nu = 1/3$. Following Ref. [5] we note that the action is Gaussian in $\phi(x)$ for $x \neq 0$ and so we integrate out all degrees of freedom except $\phi(\tau) \equiv \phi_L(x = 0, \tau) - \phi_R(x = 0, \tau)$. This gives the action

$$S = \frac{1}{4\pi|g|} \sum_{i\omega_n} |\omega_n||\phi(\omega_n)|^2 + v_1 \int_0^\beta d\tau \cos\phi(\tau), \tag{7}$$

where we have retained only the single relevant operator. The total current integrated across the Hall bar I at $x = 0$ is given simply by $I = e\dot{\phi}/2\pi$. The finite frequency "two-terminal" conductance then follows from the Kubo formula,

$$G(\omega_n) = \frac{e^2}{2\pi h}|\omega_n|\langle|\phi(\omega_n)|^2\rangle. \tag{8}$$

A hard cutoff Λ is introduced by keeping only a finite number of Matsubara frequencies L (typically $L < 100$). We also simulated a dual version of the model in which the tunneling events are represented by a plasma of logarithmically interacting "charges" [5]. Essentially identical results were obtained in the two approaches.

In order to extract information about the temperature dependence of the dc conductance, analytic continuation to zero frequency is necessary. Though difficult to do exactly, this may be done with sufficient accuracy by fitting the finite Matsubara frequency data to a rational function [2/3] Padé form in order to extrapolate to $\omega = 0$.

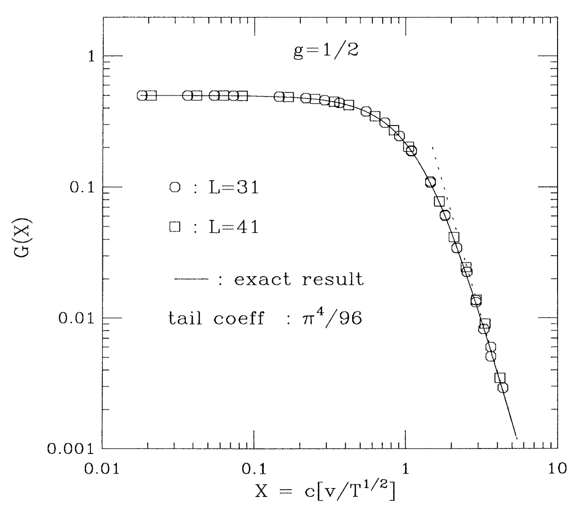

FIG. 2. Log-log plot comparing resonance scaling function for $g = 1/2$ obtained by Monte Carlo simulation for two different temperatures (squares and hexagons) and the exact solution (solid line). The dashed line is the asymptotic behavior predicted to decay as X^{-4}.

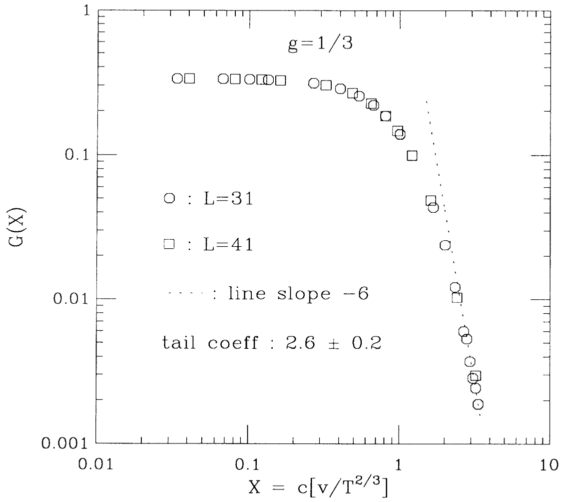

FIG. 3. Log-log plot of resonance scaling function for $\nu = 1/3$ fractional quantum Hall effect obtained by Monte Carlo simulation for two different temperatures. The dashed line is the asymptotic behavior predicted to decay as X^{-6}.

In order to check the code and to test our analytic continuation procedure, we compare our results to the exact solution which is available for $g = 1/2$ in Fig. 2. The solid line is the exact solution derived in Ref. [5], and the data points correspond to Monte Carlo simulations of G as a function of cv_1/T^{1-g} for different system sizes corresponding to $T = \Lambda/31\pi$ and $T = \Lambda/41\pi$. c is the single nonuniversal parameter which is adjusted to obtain the fit.

In Fig. 3 we display the results of our Monte Carlo simulation for $g = 1/3$. In this case the tails of the resonance are predicted to decay much faster, like X^{-6}. The data clearly scale. If the coefficient c in Eq. (7) is chosen such that the scaling function varies as $\tilde{G}(X) = g(1-X^2)$ for small X, this simulation allows us to determine that for large X $\tilde{G}(X) \sim KX^{-6}$ with $K = 2.6 \pm 0.2$.

It should be emphasized that this scaling behavior is to be expected for the fractional quantum Hall effect $\nu = 1/3$, but not for the integer effect $\nu = 1$ or higher-order fractions. In the integer case the edge state is equivalent to a noninteracting Fermi liquid, and at low temperatures the resonances should be temperature independent and Lorentzian. For all higher-order fractions in Laughlin's sequence, $1/\nu$ an odd integer, multiquasiparticle backscattering processes, v_m in (5) for $m > 1$, are also relevant and grow at low temperatures. Thus in these cases the conductance at the peak of the "resonance" ($v_1 = 0$) will decrease upon cooling, eventually killing completely the resonance in the zero temperature limit. The higher-order hierarchical quantum Hall fluids, such as $\nu = 2/3, 2/5, 2/7, \ldots$, have more than one branch of edge states [1,15], which complicates the analysis. Resonant tunneling between hierarchical edges will be considered in a subsequent paper.

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