ΔI=4 Bifurcation in a Superdeformed Band: Evidence for a C_4 Symmetry

S. Flibotte,[1,*] H. R. Andrews,[2] G. C. Ball,[2] C. W. Beausang,[3] F. A. Beck,[1] G. Belier,[1] T. Byrski,[1] D. Curien,[1] P. J. Dagnall,[3] G. de France,[1] D. Disdier,[1] G. Duchêne,[1] Ch. Finck,[1] B. Haas,[1] G. Hackman,[4] D. S. Haslip,[4] V. P. Janzen,[2,4] B. Kharraja,[1] J. C. Lisle,[5] J. C. Merdinger,[1] S. M. Mullins,[4] W. Nazarewicz,[6,7,†] D. C. Radford,[2] V. Rauch,[1] H. Savajols,[1] J. Styczen,[8] Ch. Theisen,[1] P. J. Twin,[3] J. P. Vivien,[1] J. C. Waddington,[4] D. Ward,[2] K. Zuber,[8] and S. Åberg[6,9]

[1] *Centre de Recherches Nucléaires, Institut National de Physique Nucléaire et de Physique de Particules–Centre National de la Recherche Scientifique/Université Louis Pasteur, F-67037 Strasbourg Cedex, France*

[2] *AECL Research, Chalk River Laboratories, Chalk River, Ontario, Canada K0J 1J0*

[3] *Oliver Lodge Laboratory, University of Liverpool, Liverpool L69 3BX, United Kingdom*

[4] *Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1*

[5] *Schuster Laboratory, University of Manchester, Manchester M139PL, United Kingdom*

[6] *Joint Institute for Heavy-Ion Research, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831*

[7] *Department of Physics, University of Tennessee, Knoxville, Tennessee 37996*

[8] *Institute of Nuclear Physics, Cracow, Poland*

[9] *Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden*

(Received 15 July 1993)

The moment of inertia of the yrast superdeformed band in ^{149}Gd exhibits an unexpected bifurcation at high rotational frequency. States differing by four units of angular momentum show an energy shift of about 60 eV. This indicates the remnant of a new quantum number associated with the fourfold rotational symmetry.

PACS numbers: 21.10.Re, 21.60.Fw, 23.20.Lv, 27.60.+j

The identification of rotational bands in atomic nuclei about forty years ago led to the conclusion that nuclei may possess a deformed shape [1]. In most cases spin sequences with ΔI=2, connected with $E2$ transition matrix elements, form the rotational bands. This implies R invariance (C_2 symmetry); i.e., the intrinsic Hamiltonian is invariant under a rotation of 180° around an axis perpendicular to the symmetry axis and the signature, $r = \exp(\pi i I) = \exp(\pi i \alpha)$, is a good quantum number [2]. In some nuclei there are indications of a further reduction of symmetry as the rotational spectra are consistent with reflection asymmetric shapes which in the extreme limit means ΔI=1 rotational band sequences with alternating parity [3].

In this Letter we report on experimental evidence for the presence of a *higher* symmetry in the rotational spectra. The yrast superdeformed band in ^{149}Gd [4,5] shows an unexpected staggering. At high rotational frequencies the ΔI=2 rotational band is perturbed and two ΔI=4 rotational sequences emerge with an energy splitting of about 120 eV. This feature suggests the remnant of a quantum number associated with an invariance of the Hamiltonian under a rotation of 90° around the rotation axis (C_4 symmetry). A classical analogy of this effect is a C_4 bifurcation in nonlinear dynamics [6].

The yrast superdeformed band of ^{149}Gd has been studied using the Eurogam [7] multidetector array. In its phase I configuration, the Franco-British γ-ray spectrometer sited at the Daresbury Nuclear Structure Facility consisted of 44 large-volume germanium detectors. Each individual Ge crystal was surrounded by a bismuth germanate (BGO) Compton-suppression shield. In the present experiment, the ^{149}Gd residual nucleus was populated via the ^{124}Sn(^{30}Si,5n)^{149}Gd reaction at a bombarding energy of 158 MeV. The target consisted of a stack of two tin foils of 0.5 mg cm^{-2} thickness enriched to 98% in ^{124}Sn. The event trigger required at least six unsuppressed Ge detectors to fire. With this experimental setup, approximately 1×10^9 events were acquired during a 3-d time period. After unpacking the higher-fold events, there were a total of $\sim 3\times10^9$ fourfold Compton-suppressed γ-ray coincidences in the data set.

The high-energy part of the γ-ray spectrum corresponding to the yrast superdeformed band in ^{149}Gd is

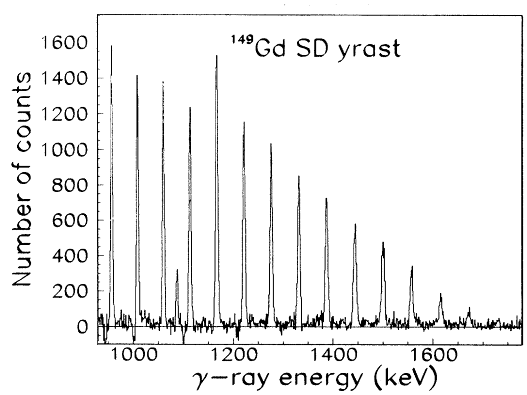

FIG. 1. Partial γ-ray spectrum corresponding to the yrast superdeformed band in ^{149}Gd obtained by summing a number of combinations of three-dimensional energy windows set on fourfold coincidence events. A small fraction of the corresponding double-gated spectrum has been subtracted.

presented in Fig. 1 and the γ-ray energies are listed in Table I. The analysis of fourfold and fivefold coincidence events [8] produces extremely clean spectra which require little background subtraction. This was essential for a very precise determination of γ-ray energies. The background spectra corresponding to the n-fold data were generated by selecting the same energy windows from $(n-1)$-fold γ-ray coincidences. The superdeformed γ-ray peaks in the final spectrum were fitted with the well-established GF2 computer code [9] which is based on the standard least squares analysis [10]. The experimental uncertainties on the transition energies quoted in Table I take into account both the statistical errors from the coincidence spectrum and its associated background spectrum. In this Letter, we are principally interested in the energy differences between two consecutive transitions and small uncertainties in the calibration coefficients will not affect our physical results. Furthermore, strongly contaminated transitions were avoided in the gating procedure. For example, the band member at 1167 keV has not been selected since this γ ray is very close in energy with a transition belonging to the normal-deformed level scheme [11] in the spin region where the superdeformed band decays. It is therefore natural to observe, in the final one-dimensional spectrum of Fig. 1, an apparently larger intensity for this transition compared with the intensities of other band members. The new γ-ray transition at 1730 keV represents the highest rotational frequency ($\hbar\omega = E_\gamma/2$) observed so far in a discrete superdeformed band in the $A \sim$150 mass region.

TABLE I. Measured γ-ray transition energies, energy differences between two consecutive transitions, and dynamical moment of inertia of the yrast superdeformed band in ^{149}Gd.

E_γ (keV)	ΔE_γ (keV)	$\Im^{(2)}$ ($\hbar^2$ MeV^{-1})
617.8(1)	46.4(1)	86.2(2)
664.2(1)	47.6(1)	84.0(2)
711.8(1)	47.9(1)	83.5(2)
759.7(1)	48.4(1)	82.6(2)
808.1(1)	49.0(1)	81.6(2)
857.1(1)	49.6(1)	80.6(2)
906.7(1)	50.4(1)	79.4(2)
957.1(1)	51.6(1)	77.5(2)
1008.7(1)	52.0(1)	76.9(2)
1060.7(1)	53.1(1)	75.3(2)
1113.8(1)	53.4(2)	74.9(3)
1167.2(2)	54.6(2)	73.3(3)
1221.8(1)	54.7(1)	73.1(2)
1276.5(1)	55.5(1)	72.1(2)
1332.0(1)	55.6(1)	71.9(2)
1387.6(1)	56.6(1)	70.7(2)
1444.2(1)	56.3(2)	71.0(3)
1500.5(2)	57.3(3)	69.8(3)
1557.8(2)	57.9(4)	69.1(4)
1615.7(3)	56.4(5)	70.9(6)
1672.1(4)	57.8(9)	69.2(11)
1729.9(8)		

Assuming the theoretical spin assignments of Ragnarsson [12,13], this γ ray is emitted by a state having an angular momentum of $I = (139/2)\hbar$.

The deduced $\Im^{(2)}$ dynamical moment of inertia of the yrast superdeformed band in ^{149}Gd is plotted in Fig. 2 as a function of rotational frequency. This quantity, related to the curvature of the excitation energy as a function of spin, can be extracted from the energy difference between two consecutive quadrupole transitions in the band by $\Im^{(2)}(I) = 4\hbar^2/\Delta E_\gamma(I)$ where $\Delta E_\gamma(I) = E_\gamma(I+2) - E_\gamma(I)$. The dynamical moment of inertia therefore does not depend on the knowledge of the spin I but only on the measured γ-ray energies. The $\Im^{(2)}$ and ΔE_γ values are also listed in Table I. In the rare-earth region, the gross behavior of $\Im^{(2)}$ associated with the occupancy of high-N intruder orbitals is rather well understood [14,15]. For example, the configuration $\pi 6^2 \nu 7^1$ assigned to this band not only reproduces the absolute magnitude but also the slope of $\Im^{(2)}$ as a function of $\hbar\omega$. However, as seen in Fig. 2, the moment of inertia exhibits an anomalous staggering at frequencies greater than $\hbar\omega = 0.49$ MeV. The amplitude of the oscillations is weak but definitely outside the experimental error bars so that the staggering effect certainly originates from a perturbation of the superdeformed energy levels. At the highest observed frequencies ($\hbar\omega \geq 0.79$ MeV) the oscillation pattern becomes irregular and it may invert but the irregularity could also be due to an accidental degeneracy with another band having the same spin and parity. It should be pointed out that the observed $\Im^{(2)}$ oscillations cannot be an artifact related to the nonlinearity of the detector electronics because (i) the yrast superdeformed bands in ^{151}Tb and ^{152}Dy studied with the same experimental device do not show the effect and (ii) there were some hints of $\Im^{(2)}$ staggering in a study of ^{149}Gd performed [5] with a different γ-ray spectrometer.

The effects of the perturbation on ΔE_γ can be ex-

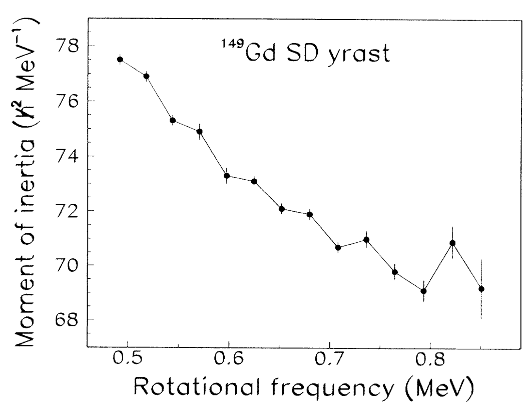

FIG. 2. Dynamical moment of inertia of the yrast superdeformed band in ^{149}Gd corresponding to the γ-ray energy region displayed in Fig. 1. The solid line joining the points is intended to guide the eye.

perimentally determined by comparing the $\Delta E_\gamma(I)$ values with a smooth reference calculated with the help of the expression $\Delta E_\gamma^{\rm ref}(I) = [\Delta E_\gamma(I+2) + 2\Delta E_\gamma(I) + \Delta E_\gamma(I-2)]/4$. It should be pointed out that the quantities $\Delta E_\gamma(I)$ and $\Delta E_\gamma^{\rm ref}(I)$ are not totally independent; their difference can be expressed by $[E_\gamma(I-2)-3E_\gamma(I)+3E_\gamma(I+2)-E_\gamma(I+4)]/4$. Knowing the uncertainties on the γ-ray energies, it is then straightforward to calculate the experimental uncertainty on $\Delta E_\gamma(I)-\Delta E_\gamma^{\rm ref}(I)$. As shown in Fig. 3, the absolute value of $\Delta E_\gamma(I)-\Delta E_\gamma^{\rm ref}(I)$ in the region between $\hbar\omega \simeq 0.50$ and 0.75 MeV corresponds on average to 230 ± 71 eV, with alternating signs. In order to take into account the correlations between the data points, the uncertainty on the magnitude of the oscillations has been determined with a Monte Carlo technique assuming that the γ-ray peaks have Gaussian shapes with centroids and standard deviations quoted in Table I. The pattern revealed by Fig. 3 can be produced by a similar alternating effect in the γ-ray transition energies, the E_γ values being alternately shifted up and down by 115 ± 36 eV. The superdeformed energy levels are consequently separated into two sequences with the spin values $I, I+4, I+8, \ldots$ and $I+2, I+6, I+10, \ldots$, respectively. The $\Delta I=2$ spin states are, however, still connected with strong $E2$ transition matrix elements, the levels being, for example, alternately pushed up and down by 58 ± 18 eV relative to their unperturbed positions. The observed oscillations are 3.2 standard deviations away from zero, which corresponds to a confidence limit of 0.999. Considering the fact that these superdeformed states are at an excitation energy of $\sim$20 MeV above the ground state, the observed shifts correspond to a 10^{-6} perturbation on the energy levels.

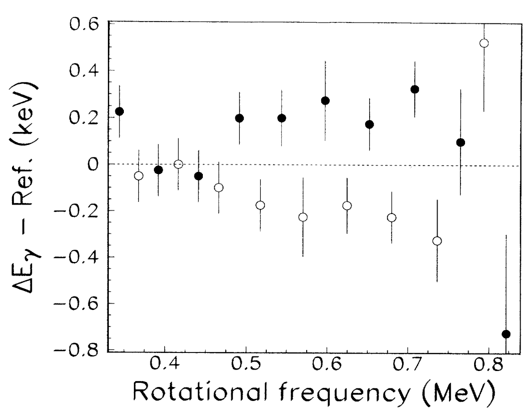

FIG. 3. Energy differences ΔE_γ between two consecutive γ-ray transitions of the superdeformed band in ^{149}Gd as a function of rotational frequency after subtraction of a smooth reference given by $\Delta E_\gamma^{\rm ref}(I) = [\Delta E_\gamma(I+2) + 2\Delta E_\gamma(I)+\Delta E_\gamma(I-2)]/4$. Filled and empty symbols refer to different values of α_4. The staggering effect sets in just above $\hbar\omega$=0.4 MeV, i.e., just after alignment of the N=6 proton pair.

Because of the regularity of the perturbation over many transitions the phenomenon causing it is unlikely to be of chaotic origin. Thus explanations based on the interaction with the states in the first well or the coupling to the fission channel [16] are less probable. The presence of the two regular ΔI=4 families in a rotational band suggests an explanation based on a fourfold rotational symmetry. For the quantal system which is invariant under a rotation of $\pi/2$ about the x axis (i.e., invariant with respect to the point group C$_4$), a good quantum number appears, $r_4 \equiv \exp(\frac{1}{2}\pi i I)$. By introducing the r_4-exponent quantum number, α_4, defined by means of $r_4 = \exp(\frac{1}{2}\pi i \alpha_4)$, the following relation holds:

$$I = \alpha_4 \ (\mathrm{mod} 4), \tag{1}$$

where α_4 can take the values {0,1,2,3} (even A) and $\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\}$ (odd A). Excitations with the same α_4 are expected to form one family of states. In the presence of the C$_4$-type perturbation the rotational energy can be written as

$$E(I) = \tilde{E}(I) + (-1)^{\frac{1}{2}[\alpha_4 - \alpha]} C_0 , \tag{2}$$

where $\tilde{E}(I)$ is a smooth function of I and C_0 is the magnitude of the perturbation. Examples of molecular rotations in the presence of C$_4$ symmetry can be found in Ref. [17].

In atomic nuclei there have been a few examples indicating an approximate fourfold rotational symmetry. For instance, the 4p-4h (four-particle–four-hole) band in ^{16}O built upon the $I^\pi = 0_2^+$ level at 6.0494 MeV [18] can be understood in terms of four alpha particles in a plane forming the kite shape [19], i.e., close to the (C$_4$-invariant) square configuration. For this band, the parameter C_0 =194 keV. Examples of $\Im^{(2)}$ oscillations have been pointed out by Peker *et al.* [20] in the octupole $K^\pi=0^-$ bands of ^{236}U (C_0 =361 eV) and ^{238}U (C_0 =46 eV). However, in all these cases the $\Im^{(2)}$ oscillations are not as regular as in ^{149}Gd and they take place over much shorter spin sequences. The C_0 values for ^{149}Gd, ^{236}U, and ^{238}U are 3 orders of magnitude smaller than that for the kite-shaped ^{16}O suggesting that the former cases are associated with dynamical effects.

Hamamoto, using the particle-plus-rotor-model description, has pointed out [21] that a strongly triaxial system with irrotational moments of inertia ($\gamma = -30°$), for which the two principal moments of inertia, $\Im_2$ and $\Im_3$, are equal and smaller than $\Im_1$, is invariant with respect to C$_4$. However, according to calculations [14] the superdeformed bands in the $A \sim 150$ mass region are fairly rigid with respect to γ and the moments of inertia differ from irrotational ones because of weak pairing correlations at superdeformed shapes.

A possible origin of the fourfold symmetry manifestation could be the coupling to the hexadecapole field, suggested in Ref. [20]. The yrast superdeformed band in ^{149}Gd is expected [14] to have a large β_4 deformation of

$\sim$0.08 which changes with rotational frequency, indicating softness to the hexadecapole distortion. In the limit of large angular momentum, the hexadecapole phonon becomes aligned along the axis of rotation (x axis) producing a perturbation invariant with respect to $R_1(\pi/2)$. While two components of this field, namely, $Q^4_{0_x}$ and $Q^4_{2_x}$, lead to a small renormalization of the quadrupole fields with M_x=0 and 2, the $Q^4_{4_x}$ interaction is the first nontrivial one [22]. Such a perturbation can cause a small (second order) splitting between states with different α_4 values. Unfortunately, due to the smallness of the effect it is difficult to confirm this scenario by microscopic calculations.

Several observations regarding the ΔI=4 effect in ^{149}Gd can be drawn. First, there is evidence that the staggering effect in $\Im^{(2)}$ may also be present in other superdeformed bands [5,23] based on configurations involving one N=7 neutron but is definitely absent in bands based on two N=7 neutrons (e.g., yrast superdeformed bands in ^{151}Tb and ^{152}Dy). Second, the staggering sets in at a rotational frequency slightly above $\hbar\omega$=0.4 MeV at which the pair of N=6 protons becomes aligned [14]. This suggests that the effect could either be associated with the polarization of the superdeformed ^{152}Dy core by three aligned high-N holes, or by the mutual proton-neutron interaction between the N=6 and N=7 valence holes, or both. Third, it is interesting to note that the K^π=0$^-$ bands in ^{236,238}U discussed by Peker *et al.* also involve the N=6 protons and N=7 neutrons. These observations suggest that the microscopic origin of the bifurcation is linked with these high-N orbitals.

In summary, we have observed bifurcation of the dynamical moment of inertia at high spin in the yrast superdeformed band of ^{149}Gd. It is related to states differing by four units of angular momentum having the same perturbation and indicates evidence of a quantum number associated with the fourfold rotational symmetry. The observed perturbation is extremely small and only the analysis of fourfold and fivefold coincidences has allowed the determination of the γ-ray transition energies with the precision required to reveal a systematic effect.

Eurogam is funded jointly by the SERC (U.K.) and IN2P3 (France). We gratefully thank R. K. Bhaduri, D. Boosé, I. Hamamoto, I. Ragnarsson, and N. Rowley for stimulating discussions. Additional support from NSERC (Canada), AECL Research, the U.S. Department of Energy (through Contracts No. DE-FG05-87-ER40361 and No. DE-FG05-93ER40770), and the Swedish Natural Science Research Council is gratefully acknowledged.

* Present address: Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1.

† Permanent address: Institute of Theoretical Physics, Warsaw University, Warsaw, Poland.

[1] A. Bohr, Phys. Rev. **81**, 134 (1951).
[2] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
[3] I. Ahmad and P. A. Butler, Annu. Rev. Nucl. Part. Sci. (to be published).
[4] B. Haas *et al.*, Phys. Rev. Lett. **60**, 503 (1988).
[5] B. Haas *et al.*, Phys. Rev. C **42**, R1817 (1990).
[6] I. M. Pavlichenkov, Phys. Rep. **226**, 173 (1993).
[7] F. A. Beck, Prog. Part. Nucl. Phys. **28**, 443 (1992); C. W. Beausang *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **313**, 37 (1992).
[8] S. Flibotte *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **320**, 325 (1992).
[9] D. C. Radford, in Proceedings of the 1989 International Nuclear Physics Conference, Sao Paulo, Brazil, August 1989 (unpublished).
[10] P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969).
[11] S. Flibotte *et al.*, Nucl. Phys. **A530**, 187 (1991).
[12] I. Ragnarsson, Nucl. Phys. **A557**, 167c (1993).
[13] B. Haas *et al.*, Nucl. Phys. **A561**, 251 (1993).
[14] W. Nazarewicz, R. Wyss, and A. Johnson, Nucl. Phys. **A503**, 285 (1989).
[15] T. Bengtsson, I. Ragnarsson, and S. Åberg, Phys. Lett. B **208**, 39 (1988).
[16] J. F. Berger *et al.*, Phys. Rev. C **41**, R2483 (1990).
[17] V. M. Krivtsun, D. A. Sadovskii, and B. I. Zhilinskii, J. Mol. Spectrosc. **139**, 126 (1990).
[18] F. Ajzenberg-Selove, Nucl. Phys. **A460**, 1 (1986).
[19] W. Bauhoff, H. Schultheis, and R. Schultheis, Phys. Rev. C **29**, 1046 (1984).
[20] L. K. Peker *et al.*, Phys. Rev. Lett. **50**, 1749 (1983).
[21] I. Hamamoto, Phys. Lett. B **193**, 399 (1987).
[22] S. Åberg and W. Nazarewicz, ORNL Physics Division Annual Report, 1993 (unpublished).
[23] C. W. Beausang *et al.*, Phys. Rev. Lett. **71**, 1800 (1993).