Electron Spectral Function of an Interacting Two Dimensional Electron Gas in a Strong Magnetic Field

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The electron spectral function of a 2D interacting electron system in a strong magnetic field is found to exhibit an energy gap at arbitrary filling fractions. In general, this gap is larger than the energy cost associated with a charge e excitation. At $\nu=\frac{1}{2}$ a numerical estimate of the tunneling gap is $\Delta_{\rm tun}=(0.3\pm0.1)e^2/\epsilon l_B$ for the Coulomb interaction.

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Two dimensional electron systems in a strong magnetic field exhibit many interesting properties [1–3]. In this work, we investigate the electron spectral function and show that it can have an energy gap larger than that of the charge e excitations above the ground state. In particular, the gap in the spectral function can be finite even for compressible states, as has been observed in recent tunneling experiments [4,5]. The tunneling gap appears to be a general feature of 2D electron systems in strong magnetic fields regardless of whether the electrons are in a compressible or incompressible state. The gap in the electron spectral function is a pure interaction effect and, in general, depends on the electron density. Other explanations of the tunneling gap relying on the picture of a Fermi liquid theory for the $\nu=\frac{1}{2}$ state and the classical Coulomb gap can be found in Refs. [6,7].

We first present a simple argument to show that the energy gap in the electron spectral function and that associated with the charge e excitations are in general different due to a selection rule. For simplicity, we consider the electron system on a $2D$ sphere of radius R [8]. The results for the planar geometry should be obtained by taking properly the radius R to the infinity limit. Let N_ϕ denote the number of the magnetic flux quanta piercing the sphere and N_e the total number of electrons. We assume that all electrons are spin polarized, that the filling fraction $\nu<1$ (i.e., $N_e<N_\phi+1$), and that the cyclotron frequency ω_c is the largest energy scale in the problem. By definition, the energy gap for a charge e excitation is given by

$$\Delta=E_0(N_e+1)+E_0(N_e-1)-2E_0(N_e)\,, \tag{1}$$

where $E_0(N_e)$ is the ground state energy of the N_e electron system. If Δ is finite in the thermodynamic limit, the electron system is in an incompressible state; otherwise the electrons form a compressible liquid.

The electron system on the sphere has an SO(3) rotation symmetry. The electron states can be labeled by the angular momentum quantum numbers L^2 and L_z. The single electron states in the first Landau level form a representation of the rotation group with angular momentum $L=N_\phi/2$ [8]. Let $c_m^\dagger$ $(m=-N_\phi/2,\ldots,N_\phi/2)$ denote the electron operator that creates an electron in the first Landau level with $L_z=m$. Let us insert an electron at the north pole with $c_{N_\phi/2}^\dagger\equiv c^\dagger(0)$. The electron spectral function can be written as

$$\begin{aligned} f(\epsilon)&=f_e(\epsilon)+f_h(-\epsilon)\,,\\ f_e(\epsilon)&=\sum_n|\langle n|c^\dagger(0)|0\rangle|^2\delta(\epsilon_n-\epsilon)\,,\\ f_h(\epsilon)&=\sum_n|\langle n|c(0)|0\rangle|^2\delta(\epsilon_n-\epsilon)\,, \end{aligned} \tag{2}$$

where $|0\rangle$ is the ground state and ϵ_n denotes the energy of the eigenstate $|n\rangle$ measured from the ground state energy. If the ground state is uniform (i.e., it carries zero angular momentum), then $c(0)|0\rangle$ and $c^\dagger(0)|0\rangle$ carry angular momentum $L=N_\phi/2$. Only the excited states with angular momentum $L=N_\phi/2$ can have finite overlap with the states created by the electron operators. Therefore, the energy gap in the electron spectral function is given by

$$\Delta_{\rm tun}=E(N_e+1,N_\phi/2)+E(N_e-1,N_\phi/2)-2E_0(N_e,0)\,, \tag{3}$$

where $E(N_e,L)$ is the lowest energy for states with N_e electrons and total angular momentum L. $\Delta_{\rm tun}$ is the energy gap that is directly measured in the tunneling experiments [4,5]. Comparing (1) and (3), we conclude that, for a uniform ground state, the energy gap for charge e excitations and that occurring in the electron spectral function can be different.

For real systems, the gap in the spectral function can be much larger than the gap for the charge e excitations. Let us consider the $\nu=m/(2m+1)$ state as an example. The electron operator creates a state with m charge e/m quasiparticles located at the same point. Such a state is definitely not the lowest energy state and has angular momentum $L=N_\phi/2$. Therefore the electron spectral function has a substantial overlap with this high energy state. The energy of the latter is of the order of the Coulomb energy $e^2/\epsilon l_B$, where l_B is the magnetic length. Because of the conservation of the angular momentum, a localized electron can hardly decay into separated quasiparticles. Thus we expect the electron spectral function to exhibit a gaplike structure. This gap should be of the

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order of the Coulomb energy. According to the above picture, the gap should not be sensitive to m. In particular, we expect the gap to remain of the order of the Coulomb energy in the limit $m \to \infty$, in which case the $\nu = m/(2m+1)$ FQH state approaches the $\frac{1}{2}$ metallic state.

To confirm the existence of the gap and to understand the dependence of the tunneling gap on the filling fraction, the shape of the spectral function, etc., we evaluated numerically the electron spectral function on a sphere for finite systems [9]. The Coulomb interaction is parametrized by the pseudopotentials [8] which, for simplicity, are chosen to be those of the planar geometry. The energies are measured in units $e^2/\epsilon l_B$ and the background charge is not included. The spectral functions are calculated by the method of Ref. [10]. Note that we mainly concentrate here on the spectral functions for singlet ground states in order to avoid large finite size effects.

The spectral functions of the electron and the hole are shown in Fig. 1 for $N_e = 9, N_\phi = 16$ ($\nu = \frac{1}{2}$). In the language of the "Fermi" liquid theory of the $\frac{1}{2}$ state, the nine pseudoparticles form a closed shell filling of $L = 0, 1, 2$ states on the sphere. The numerical results illustrate the angular momentum selection rule discussed above, as only states with $L = N_\phi/2 = 8$ have a finite matrix element with states created by electron operators $c_m^\dagger$ and c_m.

In the case where an electron is added to the system, most spectral weight is concentrated on the lowest excited state (with $L = N_\phi/2 = 8$). The density profiles of this low lying state (with $L_z = 8$) are plotted in Fig. 2 as a function of the polar angle θ. We also show a density profile of the state created by $c^\dagger(0)$. We find that the above two states are similar and it is consistent with a single peak structure. For the hole case, the spectral function has two main peaks at low energies. Correspondingly, the density profile of this hole state is more complex.

Here we comment on the case $L \neq 0$ where the ground state is degenerate. Each degenerate ground state has an inhomogeneous charge density. We investigated numerically several $L \neq 0$ ground states and calculated the spectral functions for each degenerate L_z state and for the average spectral function by taking the trace over the degenerate states. As expected, the charge density of the $L_z = L$ ($L_z = -L$) state is higher (lower) on the north hemisphere. Therefore the energy cost to insert a hole (electron) at the north pole is less than that associated with the insertion of a hole (electron) into the uniform singlet state. As a consequence, the onset of the tunneling gap in the spectral function is reduced substantially. However, we infer from the study of finite systems with nearly half filling that the energy gap between the dominant peaks agrees within $\pm 10\%$ with the value calculated in the system of closest size with $L = 0$. The degeneracy of the ground state in a finite size system results from commensurability effects (e.g., between the number of electrons and the underlying shell structure). Both the degeneracy and the charge inhomogeneity of the ground state suggest the occurrence of defects due to the incommensurability. The electron tunneling into the defect definitely lowers the onset of the energy gap as we have seen in our calculation. To estimate the tunneling gap in the thermodynamic limit, we restrict ourselves to the $L = 0$ state as we assume that the ground state is unique and homogeneous in this limit.

Next, we calculated $\Delta_{\text{tun}}(N_e, N_\phi)$ [for (N_e, N_ϕ) with singlet ground state]. $\Delta_{\text{tun}}(N_e, N_\phi)$ is plotted as a function of $N_e/(N_\phi+1)$ [in the thermodynamic limit $N_e/(N_\phi+1) \to \nu$]. The results are shown as circles in Fig. 3.

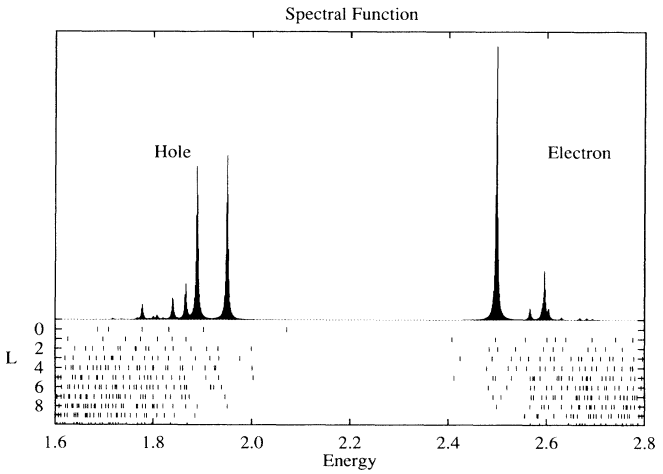

FIG. 1. Spectral functions for electrons (higher energy part) and holes (lower energy part) for $N_e = 9$, $N_\phi = 16$, $\nu = \frac{1}{2}$. Energy is measured from the highest energy state of the hole state. The lower part of the figure shows energies of the final states classified by the angular momentum.

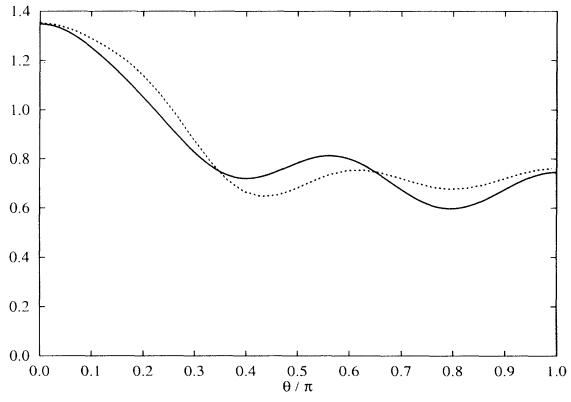

FIG. 2. Solid line: density profile of the final state, $N_\phi = 16$, $N_e = 10$, $L_z = N_\phi/2 = 8$, with lowest energy as a function of a polar angle θ on the sphere. Dotted line: same as the solid line but the state is given by an insertion of one particle at $L_z = 8$ (north pole) to the ground state of the $N_\phi = 16$, $N_e = 9$ system (normalized).

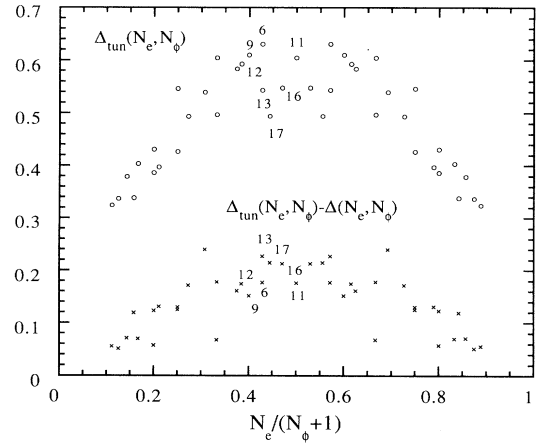

FIG. 3. Energy gap $\Delta_{tun}(N_e,N_\phi)=E(N_e+1,N_\phi/2)-2E(N_e,0)+E(N_e-1,N_\phi/2)$ for finite systems (circles), and $\Delta_{tun}(N_e,N_\phi)-\Delta(N_e,N_\phi)=E(N_e+1,N_\phi/2)-E_0(N_e+1)+E(N_e-1,N_\phi/2)-E_0(N_e-1)$ (crosses). (Near $\frac{1}{2}$ states, N_ϕ are shown in the figure.)

Since the background charge is not included in our calculations, Δ_{tun} includes the charging energy. The charging energy is proportional to $1/\sqrt{N_\phi}$ and vanishes in the thermodynamic limit. However, for a finite system, the latter has a sizable contribution and the calculated Δ_{tun} gives an upper bound to the tunneling gap. To estimate the contribution of the charging energy, we calculated $\Delta_t=\Delta_{tun}(N_e,N_\phi)-\Delta(N_e,N_\phi)$. Δ_t gives a lower bound to the tunneling gap, since in the thermodynamic limit $\Delta(N_e,N_\phi)$ approaches zero for the compressible state and remains finite for the incompressible state. Δ_t does not contain the charging energy and the difference between Δ_{tun} and Δ_t is the charging energy plus the incompressibility gap. The results for Δ_t are shown as crosses in Fig. 3. The numerical results suggest that in the thermodynamic limit there is a finite energy gap in the electron spectral function for all filling fractions. The true value of the gap is between Δ_{tun} and Δ_t calculated for finite systems. For example, the expected tunneling gap for the $\frac{1}{2}$ state is estimated to be about 0.3 ± 0.1. However, since the systems considered are small, we cannot eliminate a large system size dependence. This is especially true for the compressible states. The numerical calculation cannot determine whether the gap is a real gap or a pseudogap [6]. The energy gap in the spectral function has the expected particle-hole symmetry and appears to have a maximum near $\nu=\frac{1}{2}$. At low density, the interaction between electrons is weak and we expect the gap to be proportional to the square root of the electron density.

It is instructive to understand the above results within the framework of the "Fermi liquid" theory of the $\frac{1}{2}$ state proposed in Ref. [11]. It was shown that the low energy dynamical properties of the $\frac{1}{2}$ state are described by the Fermi liquid theory of the pseudoparticles, each of which corresponds to a bound state of an electron with two flux quanta. The ground state is a uniform compressible state. Inserting an electron into the system not only creates a pseudoparticle but also a vortex of circulating currents $\mathbf{j}\propto\hat{\mathbf{z}}\times\mathbf{r}/r^2$ centered at the pseudoparticle position.

On the sphere, the state created by the electron operator corresponds to a collective pseudoparticle excitation with an angular momentum $L=N_\phi/2$. Such an angular momentum can be converted into a planar momentum p through the relation $L=pR$, where R is the radius of the sphere. We find $p=N_e^{1/2}k_F$, where k_F is the Fermi momentum of the pseudoparticle. Therefore the collective pseudoparticle excitation carries a *huge* momentum, i.e., much larger than the Fermi momentum. Such a large momentum cannot originate in the localized pseudoparticle which can carry at most a momentum of order k_F but rather arises from the circulating currents of the vortex. This complicated collective excitation with large momentum associated with the insertion of an electron is responsible for the gaplike structure in the electron spectral function.

Within the standard Fermi liquid theory (on a sphere), what is the minimum energy state with angular momentum $N_\phi/2$ (or, equivalently, a planar momentum $p=N_e^{1/2}k_F$)? One candidate is the state in which the electrons rotate uniformly around the z axis. It has an energy $3k_F^2/2m^*=3E_F$, where m^* is the effective mass of the pseudoparticle. E_F is of order of the Coulomb energy since the effective mass is generated by the Coulomb interaction. If the pseudoparticle had only short range interactions, the uniform rotating state would indeed be the lowest energy state at fixed angular momentum. However, because of the long range interactions between the pseudoparticles, there might be other states with lower energy. The uniform rotating state (UR) gives us an upper bound on Δ_{tun}:

$$\Delta_{tun}\leq 2k_F^2/m^*. \tag{4}$$

Another candidate is the state obtained by creating a localized electron at position r (PV state). In the pseudoparticle picture, this state has, at first sight, a logarithmically divergent energy due to the associated vortex. Below we show that a vortex can have a finite energy when combined with a localized pseudoparticle as a consequence of the long range interactions. Note that the density in the PV state is peaked near the localized pseudoparticle (see Fig. 2). If the PV state has the lowest energy among the $L=N_\phi/2$ states, the density peak cannot diffuse away.

Our numerical results for the spectral function of the electron and the associated density profile calculations suggest that most weight in the electron spectral function is carried by the PV state. The fact that the PV state has finite energy indicates the existence of the gaplike structure in the spectral function. On the other hand, the

spectral function for $\epsilon<0$ and the wave function of lowest energy for the hole (Fig. 1) are complicated. We do not have a simple picture. (See note added below.)

A rough estimate of the mean energy cost to instantaneously insert an electron into the ground state $|\text{g.s.}\rangle$ is

$$E=\frac{\langle \text{g.s.}|\psi_e(r)H\psi_e^\dagger(r)|\text{g.s.}\rangle}{\langle \text{g.s.}|\psi_e(r)\psi_e^\dagger(r)|\text{g.s.}\rangle}. \tag{5}$$

The relation between the electron and the pseudoparticle is given by

$$\psi_e^\dagger(r)=\psi^\dagger(r)e^{i\tilde{\phi}\int d^2r'\varphi(r-r')\rho(r')}, \tag{6}$$

where $\tilde{\phi}=2$ for the $\nu=\frac{1}{2}$ state. $\varphi(r)$ denotes the azimuthal angle (in an arbitrary geometry) and $\rho(r)$ the electron (or the pseudoparticle) density operator at point r. A physical interpretation of (6) is that the electron operator creates a localized pseudoparticle at r and a vortex centered at the pseudoparticle position.

For simplicity, we consider a planar geometry and assume that the electron is created at the origin $r=0$. As in Ref. [11] we rewrite the Hamiltonian (5) in terms of the pseudoparticle operators by introducing an additional gauge field $a(r)$ given by $a(r)=\tilde{\phi}\int d'r^2\nabla\varphi(r-r')\rho(r')$. The gauge field leads to long range interaction between the pseudoparticles. The average energy (5) consists of the following divergent terms: (i) a contribution which arises from the vortex current (its energy density is proportional to $[\nabla\varphi(r)]^2$ and leads to the usual logarithmically divergent energy); (ii) a contribution which represents the long range interaction between the vortex current and the added pseudoparticles; and (iii) a term which describes the long range interaction between the pseudoparticles. The above three terms turn out to cancel each other. The finite contribution of the average energy can be written as $\langle \text{g.s.}|\Psi\tilde{H}\Psi^\dagger|\text{g.s.}\rangle$ where the effective Hamiltonian $\tilde{H}$ is given by

$$\begin{aligned}\tilde{H}=&H_0+V-\tilde{\phi}\int d^2r\int d^2r'J(r)\nabla\varphi(r-r')\delta\rho(r')\\&+\frac{\tilde{\phi}^2}{2m^*}\int d^2r\int d^2r'\delta\rho(r)\delta\rho(r')\nabla\varphi(r)\nabla\varphi(r')\\&-\pi\rho_0\frac{\tilde{\phi}^2}{m^*}\int d^2r\int d^2r'\delta\rho(r)\delta\rho(r')\ln|r-r'|,\end{aligned} \tag{7}$$

where $\delta\rho\equiv\rho-\rho_0-\delta(r)$. Physically, we can say that the localized pseudoparticle allows the vortex to acquire a finite energy in contrast to the usual situation. Conversely, the vortex of circulating currents sustains the localized pseudoparticle in its original position and makes it difficult for the excess charge density to diffuse away.

We remark that we have ignored the effects of impurities in the above calculations. In the presence of the impurities, the translation and rotation symmetries are broken, and we expect that there is only a pseudogap in the electron spectral function [6].

The tunneling gap observed in experiments is at $\nu=\frac{1}{2}$, $2\Delta_1=0.15e^2/\epsilon l_B$, and $2\Delta_2=0.45e^2/\epsilon l_B$ for the onset and the peak positions of the tunneling current, respectively (see Ref. [5]). These values are comparable with our estimate $\Delta_{\text{tun}}\sim 0.3e^2/\epsilon l_B$. We ignored the finite thickness of the electron wave function in the z direction which has the effect of reducing the gap.

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Note added.—After completion of this work and submission of the original manuscript, B. I. Halperin pointed out to us the possible occurrence of $L=N_\phi/2$ angular momentum states with very low energies. This is specific to the compressible states. Within the standard Fermi liquid theory, the $L=N_\phi/2$ state always has a finite energy. However, because of the singular gauge interaction in the "Fermi liquid theory" of the $\frac{1}{2}$ state, one may be able to construct an $L=N_\phi/2$ state with zero energy. The existence of this zero energy state is beyond the standard Fermi liquid theory and suggests some non-Fermi-liquid behaviors in the $\frac{1}{2}$ state. Although finite size calculations do not provide an evidence for the existence of such zero energy states, we cannot exclude their occurrence in a very large system. We believe that the dominant contribution to the electron spectral function originates from the PV state. Therefore our estimate of the tunneling gap should be comparable with the main peaks observed in the experiments for arbitrary filling fractions.

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