Non-Fermi-Liquid Behavior in Quantum Critical Systems

Junwu Gan and Eugene Wong

*Department of Physics, The University of British Columbia, 6224 Agricultural Road,
Vancouver, British Columbia, Canada V6T 1Z1*

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The problem of an electron gas interacting via exchanging transverse gauge bosons is studied using the renormalization group method. The long wavelength behavior of the gauge field is shown to be in the Gaussian universality class with a dynamical exponent $z = 3$ in dimensions $D \geq 2$. This implies that the gauge coupling constant is exactly marginal. Scattering of the electrons by the gauge mode leads to non-Fermi-liquid behavior in $D \leq 3$. The asymptotic electron and gauge Green's functions, interaction vertex, specific heat, and resistivity are presented.

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The observation of unconventional normal state properties in the high T_c cuprates has stimulated great interest in 2D models possessing a low energy non-Fermi-liquid (NFL) fixed point and a Fermi surface [1,2]. It has been realized that NFL behavior would follow naturally if the electrons or quasiparticles experience long range or singular interactions. Unfortunately, long range interactions generally do not survive screening in the presence of a large Fermi sea and the low energy physics is again a Fermi liquid, unless they arise from the critical fluctuating mode at a phase transition where the mass of the mode is tuned to zero.

An exception is the system of an electron gas interacting via exchanging transverse gauge bosons like photons [3]. The interaction cannot be screened because gauge invariance prevents the photon from acquiring a mass, provided gauge invariance is not spontaneously broken. However, if the gauge field is the regular electromagnetic one, the effects due to its coupling to the electrons are suppressed by the fine structure constant $(1/137)$ and the ratio of the Fermi velocity to the speed of light v_F/c, thus practically unobservable. Recently, the same problem appeared again in the study of the half filled Landau level [4] and in the context of strongly correlated systems [5–8]. The local correlation such as eliminating double occupation induces strong phase fluctuations which may be described by gauge fields in the long wavelength limit. In this case, the effects of the gauge interaction are usually not suppressed. In fact, it has been suggested that the gauge interaction is probably an essential element of an effective theory of high T_c superconductivity [5–8]. Although singular behaviors, signaling breakdown of the Fermi-liquid theory, have been seen in several physical quantities for this system [3–10], and even some suggestions have been made about the low energy fixed point [11], its nature still remains unclear. In this Letter, we reexamine this problem using the renormalization group (RG) method and derive a scaling solution of the low energy fixed point.

We consider the following Hamiltonian:

$$H = \int d^D r \, \psi^\dagger(\mathbf{r}) \left[\frac{1}{2m}(-i\nabla - g\mathbf{A})^2 - \mu \right] \psi(\mathbf{r}) + \frac{1}{2} \int d^D r \left[\left(\frac{\partial \mathbf{A}}{\partial t} \right)^2 + (\nabla \times \mathbf{A})^2 \right], \tag{1}$$

where ψ and $\psi^\dagger$ are electron annihilation and creation operators with the spin index neglected, μ is the chemical potential, and $\mathbf{A}$ is the transverse vector potential in the Coulomb gauge. We do not include the scalar potential since it is going to be screened. We set the photon velocity $c = 1$ and consider $v_F \sim c$. The coupling constant g is considered to be less than 1 but not too small so that the effects of the gauge interaction become observable at a temperature where other effects, such as impurity scattering, have not taken over yet. We are interested in the low energy and long wavelength behavior of the system. That is, we shall scale the frequency ν_n and the momen-

tum q of the gauge field as well as the frequency ω_n of the electrons to zero. But the electron momentum k is scaled to the Fermi wave vector k_F. This problem is similar to the quantum critical phenomenon (QCP) considered by Hertz [12]. The only difference is that in QCP one has to adjust a relevant parameter to land on the critical point, while for the gauge interaction, the $T = 0$ criticality is guaranteed by the gauge invariance.

To determine the low energy and long wavelength behavior of the gauge field, we integrate out the electrons and expand the result in powers of gauge field $\mathbf{A}$,

$$S_{\text{eff}}(A) = S_{\text{eff}}^{(2)}(A) + \sum \Gamma^{(3)} A^3 + \sum_{\bar{q},\bar{q}_1,\bar{q}_2} \Gamma^{(4)}_{\alpha\beta\lambda\gamma}(\bar{q}, \bar{q}_1, \bar{q}_2) A_\alpha(\bar{q}) A_\beta(\bar{q}_1) A_\lambda(\bar{q}_2) A_\gamma(-\bar{q} - \bar{q}_1 - \bar{q}_2) + \cdots, \tag{2}$$

where we have introduced a short hand notation $\bar{q} = (\mathbf{q}, \nu_n)$. The A^3 term has been studied before [13]. The quadratic part of the effective action is

$$S_{\text{eff}}^{(2)}(A) = \sum_{\mathbf{q},\nu_n} \left(q^2 + \frac{\gamma|\nu_n|}{q} \right) \left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right)$$
$$\times A_\alpha(\bar{q}) A_\beta(-\bar{q}), \tag{3}$$

where $\gamma \sim g^2 k_F^2$. We have dropped the ν_n^2 term in the original action since it is irrelevant under the following scaling which preserves the form of (3):

$$q \to sq, \qquad \nu_n \to s^3 \nu_n, \quad \text{for} \quad s \to 0. \tag{4}$$

Obviously, the dynamical exponent is $z = 3$. To see the effects of interactions, we simply count scaling dimen-

sions. Under the scaling (4), the gauge field scales as $A \sim s^{-(5+D)/2}$. The A^3 interaction is marginal if $\Gamma^{(3)}$ scales as $s^{(3-D)/2}$. From the Hall effect study [13], it has been known that $\Gamma^{(3)}$ vanishes faster than s under the scaling (4). Thus, it is irrelevant in dimensions $D \geq 2$ (transverse modes exist only in $D \geq 2$). Similarly, the A^4 interaction $\Gamma^{(4)}$ is marginal if it scales as $\Gamma^{(4)} \sim s^{1-D}$ and so on. The $\Gamma^{(4)}$ term includes three diagrams (Fig. 1). We now verify that $\Gamma^{(4)}$ is nonsingular as $\bar{q} \to 0$ and furthermore that the constant term vanishes as required by gauge invariance. Letting the external frequencies go to zero and then taking the limit $\mathbf{q}_i \to 0$, the leading term in $D = 3$ is

$$\Gamma_{\alpha\beta\lambda\gamma}^{(4)}(0) = \frac{g^4}{8m^2} \delta_{\alpha\gamma}\delta_{\beta\lambda} \sum_{\mathbf{k}} \left[n_F'(\epsilon_{\mathbf{k}}) + \frac{2k^2}{3m} n_F''(\epsilon_{\mathbf{k}}) + \frac{k^4}{15m^2} n_F'''(\epsilon_{\mathbf{k}}) \right] = 0, \tag{5}$$

where $n_F(\epsilon)$ is the Fermi-Dirac function and the primes denote derivatives. The result (5) also holds in other dimensions. Thus, we reach the conclusion that $\Gamma^{(3)}$, $\Gamma^{(4)}$, and all interactions in the effective gauge action are irrelevant [12] because higher order terms are even more suppressed under the scaling (4).

An immediate consequence of the irrelevance of all corrections to the Gaussian action (3) is that we can derive the asymptotic form of the specific heat. Integrating out the gauge field in (3), the free energy is

$$F(T) = \frac{1}{\beta} \sum_{\mathbf{q},\nu_n} \ln \left(q^2 + \frac{\gamma|\nu_n|}{q} \right) \sim T^{1+D/3} \int_0^{q_c^3/\gamma T} dx\, x^{D/3-1} \int_0^\infty \frac{dy}{e^y - 1} \tan^{-1}\left(\frac{y}{x} \right), \tag{6}$$

where we have an upper cutoff for the q integration at q_c, of order k_F, and the frequency integration at q_c^3/γ which has been sent to infinity due to the convergence of the integration. It is easy to see that $F(T) \sim T^2 \ln T$ in $D = 3$ and $F(T) \sim T^{1+D/3}$ in $D < 3$. The corresponding specific heat is $C \sim T \ln T$ in $D = 3$ [3], and $C \sim T^{D/3}$ in $D < 3$, respectively. Further corrections have higher powers in T. These results are consistent with the general scaling analysis since the scaling form of the specific heat is uniquely determined by the dynamical exponent and the dimensionality.

Another consequence is that the coupling constant g is exactly marginal; its beta function vanishes identically. This follows from the Ward identity which stipulates that the vertex renormalization factor Z_1, in the standard quantum electrodynamics notation, is equal to the electron wave function renormalization factor Z_2 which represents the magnitude of the Fermi surface discontinuity. Thus, the renormalization of the coupling constant g is solely determined by the gauge field wave function renormalization factor Z_3, which remains equal to 1 because all corrections due to the interactions are irrelevant. One also finds a vanishing beta function of g by imposing RG invariance on the specific heat [14].

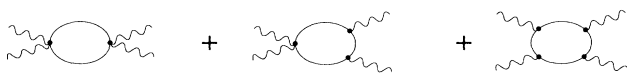We now turn to the behavior of the electrons. In cal-

FIG. 1. The fourth order interaction vertex of the effective gauge action. The internal lines represent the noninteracting electron Green's function.

culating the electron self-energy, the photon propagator is given by (3). We do not need to include further photon self-energy corrections because they are irrelevant. When analytically continued to the real frequency, the electron self-energy is $\Sigma(\mathbf{k}, i\omega_n = \omega + i0^+) = \Sigma'(\mathbf{k},\omega) + i\Sigma''(\mathbf{k},\omega)$. To the lowest order, we find

$$\Sigma''(k_F,\omega) = -\frac{g^2 v_F}{8\pi} \gamma^{D/3-1} |\omega|^{D/3}. \tag{7}$$

The real part of the self-energy is given by the Kramers-Kronig relation,

$$\Sigma'(k_F,\omega) = \frac{2\omega}{\pi} \int_0^\Omega d\epsilon\, \frac{\Sigma''(k_F,\epsilon)}{\epsilon^2 - \omega^2}$$
$$\simeq -\frac{g^2 v_F}{4\pi^2} \gamma^{D/3-1} \omega^{D/3} \int_0^{\Omega/\omega} \frac{dx}{\pi} \frac{x^{D/3}}{x^2 - 1}, \tag{8}$$

where $\Omega \sim q_c^3/\gamma$ is the frequency cutoff.

Let us first concentrate on $D = 3$. From (8), we have

$$Z_2(k_F,\omega) = \left[1 - \frac{\partial \Sigma'(k_F,\omega)}{\partial \omega} \right]^{-1} \simeq 1 - \frac{g^2 v_F}{4\pi^2} \ln\left(\frac{\Omega}{\omega} \right). \tag{9}$$

The physics of this logarithmic term is similar to the well known infrared catastrophe [15]. Because of the critical nature of the gauge field, the electrons near the Fermi surface are dressed by a cascade of damped photons. Technically, Z_2 given by (9) is reliable for $\omega > \Omega e^{-4\pi^2/g^2 v_F}$. In order to find Z_2 for $\omega \to 0$, we use the standard RG method and first obtain

$$\eta = \frac{d\ln Z_2}{d\ln\omega} = \frac{g^2 v_F}{4\pi^2}. \tag{10}$$

Bearing in mind that the coupling constant g stays unchanged, the leading logarithmic series is summed up by integrating (10) over the range $[\omega, \Omega]$ to give

$$Z_2(k_F, \omega) \sim \omega^\eta, \quad G''(k_F, \omega) \sim \frac{1}{\omega^{1-\eta}}. \quad (11)$$

The spectral density G'' has a power law divergence, removing all remnant characters of the quasiparticle and destroying the Fermi liquid.

In $D < 3$, the situation is less transparent. From (8), the leading self-energy is $\Sigma'(k_F, \omega) \sim \Sigma''(k_F, \omega) \sim \omega^{D/3}$, with an exponent $D/3 < 1$. As the energy is lowered, the electron Green's function is dominated by the effect of the self-energy. The crucial question is then whether or not more singular terms will appear as $\omega \to 0$ in higher order calculations, such as $\omega^{[1-n(3-D)/3]}$ in the nth order. For the following reason, we do not expect these kinds of terms. The exact marginality of the coupling constant g in all dimensions means that there should be *no* infrared divergence. This does not contradict the appearance of the logarithmic divergence in the electron self-energy at $D = 3$ which is purely due to the infrared catastrophe, indicating that each electron at the Fermi surface is accompanied by an infinite number of soft photons. The total energy of these photons is finite. If higher singular powers were generated in high order calculations for $D < 3$ and we still tried to interpret them as the infrared catastrophe, it would imply a divergent total energy of the accompanying photons which is unphysical. The reasonable expectation as suggested by Polchinski [11] is that once we have included the new term $\omega^{D/3}$ in the electron Green's function, there will be neither infrared divergence responsible for the renormalization of g nor infrared catastrophe (which occurs only in $D = 3$ but cancels out in gauge invariant quantities). This is partially supported in the direct evaluation of the first crossing diagram of the electron self-energy [16]. By finding an asymptotic solution of the Dyson equations (this approach is physically sensible because g is exactly marginal), we verify this expectation. Specifically, we shall prove that the full gauge propagator is given by (3), and the full electron propagator and the irreducible gauge interaction vertex have the following asymptotic forms for $D < 3$:

$$G(\mathbf{k}, \omega + i\delta) = \frac{1}{\lambda_1 |\omega|^{D/3} \mathrm{sgn}\omega - \epsilon_\mathbf{k} + i\lambda_2 |\omega|^{D/3} \mathrm{sgn}\delta}, \quad (12)$$

$$\Lambda_\mu(\bar{k}, \bar{k} + \bar{q}, \bar{q}) = \Lambda k_F^\mu, \quad (13)$$

where λ_1, λ_2, and Λ are all constants. Note that in order to assume the scaling form (12), we need $\Sigma(\mathbf{k}, \omega + i\delta)$ for general $\mathbf{k}$ and ω which has been calculated in [4]. Since $\Sigma(\mathbf{k}, \omega + i\delta)$ depends on $\mathbf{k}$ only when $\omega < (k - k_F)^3$, the $\mathbf{k}$ dependence of $\Sigma(\mathbf{k}, \omega + i\delta)$ is irrelevant under scaling, justifying (12).

The three irreducible objects, (12), (13), and the gauge propagator given by (3), have to satisfy the Dyson equations (Fig. 2). At $T = 0$, they are

$$\Sigma''(\mathbf{k}, \omega) = -\frac{g^2 \Lambda}{m^2} \sum_\mathbf{q} [k^2 - (\mathbf{k} \cdot \hat{q})^2] \int_{-\omega}^0 \frac{d\nu}{\pi} G''(\mathbf{k} + \mathbf{q}, \omega + \nu) D''(q, \nu), \quad \omega > 0, \quad (14)$$

$$\Pi''(\mathbf{q}, \nu) = \frac{g^2 \Lambda}{m^2} \sum_\mathbf{k} [k^2 - (\mathbf{k} \cdot \hat{q})^2] \int_{-\nu}^0 \frac{d\omega}{\pi} G''(\mathbf{k}, \omega) G''(\mathbf{k} + \mathbf{q}, \omega + \nu), \quad \nu > 0, \quad (15)$$

$$\Lambda = 1 - \frac{g^2 \Lambda^3}{m^2} \sum_\mathbf{q} [k_F^2 - (\mathbf{k}_F \cdot \hat{q})^2] \int_{-\infty}^0 \frac{d\nu}{\pi} \{ 2G'(\mathbf{k}_F + \mathbf{q}, \nu) G''(\mathbf{k}_F + \mathbf{q}, \nu) D'(\mathbf{q}, \nu)$$

$$+ [G'(\mathbf{k}_F + \mathbf{q}, \nu)^2 - G''(\mathbf{k}_F + \mathbf{q}, \nu)^2] D''(\mathbf{q}, \nu) \} + \cdots, \quad (16)$$

where Π'' is the imaginary part of the photon self-energy. Note that the vertex equation (16) contains a series of infinite skeleton diagrams and only the expressions for the first two have been written out. The integrations can be carried out in $D < 3$ keeping only the leading powers in frequency and momentum. We find

$$\Sigma''(k_F, \omega) = -\frac{\Lambda g^2 v_F}{2(2\pi)^{D-1}} \gamma^{D/3-1} \omega^{D/3}$$
$$\times \int_0^\infty dx \, x^{D/3-1} \ln\left(1 + \frac{1}{x^2}\right), \quad (17)$$

$$\Pi''(\mathbf{q}, \nu) = \frac{\pi \Lambda g^2 k_F^{D-1}}{2(2\pi)^{D-1}} \frac{\nu}{q}, \quad (18)$$

$$\Lambda = 1 + \frac{\Lambda^3 g^2 v_F}{(2\pi)^{D-1}} \frac{\gamma^{D/3-1}}{\lambda_1} \int_0^\infty dx \frac{1}{(1 + x^2) x^{D/3}} + \cdots. \quad (19)$$

The important point is that (19) is well behaved and no higher singular terms are generated in (17) and (18). Strictly speaking, we have to verify that no infrared divergence will be generated in every skeleton diagram of (16). Nevertheless, as explained above, we do not expect divergence in higher order skeleton diagrams because g is exactly marginal; they may, however, contribute to determining the constants λ_1, λ_2, and Λ. Thus, we conclude that (12), (13), and the gauge propagator given by (3) are indeed the asymptotic low energy solution.

From (12), we see $G''(k_F, \omega) \sim \omega^{-D/3}$. As $D \to 3$, the exponent is discontinuous from the $D = 3$ value given by (11). It is then instructive to study the dimensional crossover as we lower the pertinent energy scale, treating $D = 3 - \varepsilon$ as a continuous parameter and $|\varepsilon| \ll 1$. To analyze (8), we define a small energy scale: $T_\varepsilon = \Omega e^{-3/|\varepsilon|}$. At $\omega > T_\varepsilon$, we find $(\gamma\omega)^{-\varepsilon/3} \simeq 1$ and
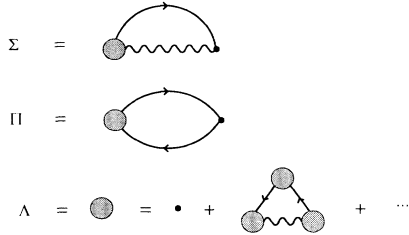
FIG. 2. Dyson equations. Σ, Π, and Λ_μ are the irreducible electron, photon self-energies, and the irreducible gauge interaction vertex, respectively. All lines are full Green's functions.
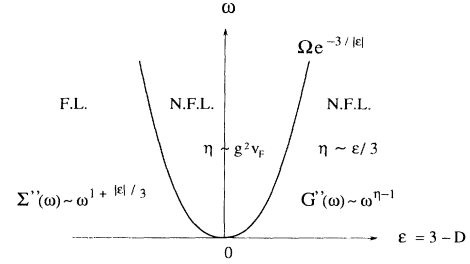

FIG. 3. Illustration of the dimensional crossover. For thermodynamic quantities, the temperature T corresponds to ω.

$\Sigma'(\omega)/\omega \sim g^2 v_F \ln(\Omega/\omega)$. Comparing with (9), we see the same behavior as in 3D at $\omega > T_\varepsilon$ for all ε.

At $\omega < T_\varepsilon$, we see from (8) that the situation is different for $D > 3$ and $D < 3$. In $D > 3$, $\ln(\Omega/\omega)$ appearing at $\omega > T_\varepsilon$ is now cut off by $3/|\varepsilon|$. The frequency dependence of $\Sigma'(\omega)/\omega$ dies as $\omega^{|\varepsilon|/3}$. The system eventually flows to a Fermi-liquid-like fixed point with a quasiparticle scattering rate given by (7). In $D < 3$, we have $\Sigma'(\omega)/\omega \sim (\gamma\omega)^{-\varepsilon/3}/\varepsilon$ from direct evaluation. The effect of $(\gamma\omega)^{-\varepsilon/3}$ starts to become important at $\omega < T_\varepsilon$. As we have argued, there are no other singular terms. In the numerical prefactor of $(\gamma\omega)^{-\varepsilon/3}$, each $\ln(\Omega/\omega)$ appearing at $\omega > T_\varepsilon$ is again replaced by $3/\varepsilon$ and the series in $3/\varepsilon$ can be summed up to give a constant for finite ε. We thus recover the $D < 3$ behavior (12). An illustration of the dimensional crossover is sketched in Fig. 3.

Although the electron Green's function is gauge dependent, physical results derived from it are not. As an example, we calculate the resistivity from the Kubo formula. Since (13) has no singularity in $D < 3$, the vertex correction in the resistivity does not change the temperature dependence. So, we find $\rho \sim T^{D/3}$.

The similar occurrence of the electron critical scattering at the quantum phase transition in the presence of a Fermi surface probably provides the easiest experimental realization. The role of the gauge field is then played by the soft mode of the critical fluctuations. In the pressure driven $T_c \to 0$ itinerant ferromagnetic transition, the critical mode is the magnon excitation. The power law divergence in the electron spectral density is directly related to physical observables in the Fermi surface measurements. It is interesting to note that many heavy fermion systems show low temperature critical behavior, markedly different from the Fermi-liquid expectation [17,18]. The critical scattering has been seen at the heavy fermion metamagnetic transition [19].

[1] P. W. Anderson, Science **256**, 1526 (1992).

[2] C. M. Varma, Int. J. Mod. Phys. B **3**, 2083 (1989); C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. **63**, 1996 (1989).

[3] T. Holstein, R. E. Norton, and P. Pincus, Phys. Rev. B **8**, 2649 (1973).

[4] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[5] P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989).

[6] N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990); Phys. Rev. B **43**, 1233 (1991).

[7] L. B. Ioffe and A. I. Larkin, Phys. Rev. B **39**, 8988 (1989).

[8] A. J. Schofield and J. M.Wheatley, Phys. Rev. B **47**, 11 607 (1993).

[9] M. Yu. Reizer, Phys. Rev. B **39**, 1602 (1989); **40**, 11 571 (1989).

[10] B. Blok and H. Monien, Phys. Rev. B **47**, 3454 (1993).

[11] J. Polchinski, Institute of Theoretical Physics Report No. ITP-93-33 (to be published); D. V. Khveshchenko and P. C. Stamp, Phys. Rev. Lett. **71**, 2118 (1993).

[12] J. A. Hertz, Phys. Rev. B **14**, 1165 (1976); A. J. Millis, Phys. Rev. B **48**, 7183 (1993).

[13] H. Fukuyama, H. Ebisawa, and Y. Wada, Prog. Theor. Phys. **42**, 494 (1969).

[14] J. Gan, N. Andrei, and P. Coleman, Phys. Rev. Lett. **70**, 686 (1993).

[15] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967); P. Nozières and C. T. De Dominicis, Phys. Rev. **178**, 1097 (1969).

[16] L. B. Ioffe, D. Lidsky, and B. L. Altshuler (to be published).

[17] C. L. Seaman et al., Phys. Rev. Lett. **67**, 2882 (1991).

[18] B. Andraka and A.M. Tsvelik, Phys. Rev. Lett. **67**, 2886 (1991); A. M. Tsvelik and M. Yu. Reizer, Phys. Rev. B **48**, 9887 (1993).

[19] P. Haen, J. Flouquet, F. Lapierre, P. Lejay, and G. Remenyi, J. Low Temp. Phys. **67**, 391 (1987).