Optical Stochastic Cooling

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In this paper, we consider the utilization of optical amplifiers with bandwidth $\Delta f \cong 10^{14}$ Hz for use in stochastic cooling. It is shown that quadrupole and dipole wigglers can be used as pickups and kickers, respectively. The proposed method increases the application of the stochastic cooling method beyond the traditional area of proton-antiproton cooling. For example, the method has application to electron-positron cooling as well as potential use in muon cooling. The proposed method makes possible the independent choice of damping time and source of excitation of emittance, thereby providing a new direction for the design of low-emittance damping rings.

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Synchrotron radiation, electron cooling, and stochastic cooling are methods used to damp the emittance of charged particle beams. Heretofore, synchrotron radiation has been the only method used for damping the emittances of electron and positron beams. The damping time τ for stochastic cooling of N particles is determined by the bandwidth Δf (see, for example, Ref. [1]). The damping time τ can be estimated by $\tau/T \cong N/\Delta f(c/l_b) \simeq 2N_u$, where N_u is the number of particles in the bandwidth, T is the period of revolution, and l_b is the bunch length. For a given number of particles, the limitations are determined by the bandwidth Δf of the amplifiers, pickups, and kickers. While successful for protons, if applied to electrons with the typical bandwidths used ($\simeq 10^9$ Hz), this method gives longer damping times than those associated with synchrotron radiation. Therefore, this method has not been used for cooling electron and positron beams. However, if the bandwidth is extended to the optical amplifier range (10^{14} Hz), stochastic cooling can be successfully applied to cool electron and positron beams. For practical utilization, of course, all hardware, such as pickups, amplifiers, and kickers, must function at typical optical amplifiers bandwidths.

A sequence of the quadrupole lenses with changing polarities, a quadrupole wiggler (QW), can be used as a pickup. Radiation in a quadrupole field is defined by the fluctuation of current density in a cross section. This function of the QW is similar to that of pickup electrodes in conventional schemes of stochastic cooling. The number of periods of the QW [the number of the periods of the focusing-defocusing (FODO) structure], defines the bandwidth of the radiation corresponding to the bandwidth of the optical amplifier. The betatron phase shift must be small to prevent enlarging the bandwidth by betatron oscillations. Polarization of the radiation makes it possible to distinguish fluctuations in both transverse directions independently.

Optical amplifiers exist with relative bandwidths $\Delta f/f$ in the range of 10% to 20% [2]. These amplifiers normally operate at a central wavelength λ of $\lambda \cong 0.3$ to 1 μm.

After amplification, the optical signal from the QW goes to the longitudinal kicker, which is placed downstream in the beam trajectory, where if the phase, amplitude, and dispersion function are chosen correctly, the fluctuation dampens to the necessary level. Transverse kick can be feedback, for example, through an energy jump at a place with nonzero dispersion function in the lattice [3]. For different cooling schemes both transverse and longitudinal kickers can be used, as described in [3]. In optical cooling, transverse kickers are not effective, due to cancellation of the transverse force from the electric and magnetic field $\cong 1/2\gamma^2$ in the electromagnetic wave, which propagates in the same direction as the particle. So we consider only schemes with longitudinal kickers. In this way the transverse emittance can be damped efficiently. A dipole wiggler with corresponding period can be used as a longitudinal kicker, if it is installed at a point of nonzero dispersion with the correct betatron phase shift relative to the quadrupole wiggler.

The general layout of the proposed method is represented in Fig. 1. The bend between the quadrupole and dipole wigglers is made in such a manner so that fluctuations which produced the radiation in the QW are not eliminated.

The quadrupole wiggler is not the only choice to act as a pickup electrode in optical cooling. The dipole wiggler can also be used as a broadband beam position monitor as well, but it requires additional optical devices.

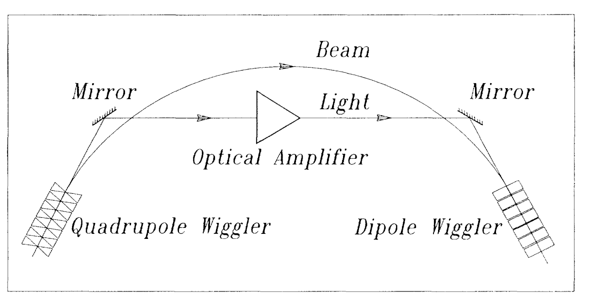

FIG. 1. The general layout of the system.

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I. Radiation from the quadrupole wiggler.—The radiation from the particles passing through the wiggler is related to the current density distribution. For a random distribution of particles in the beam, the field amplitude is known to depend on $\cong\sqrt{N}$ and the radiated intensity is known to depend on N, where N is the total number of particles.

A quadrupole wiggler consists of a sequence of focusing and defocusing lenses, i.e., FODO structure. For a steady current of particles passing through a quadrupole field wiggler, for each particle with a given transverse position x, there exists another particle at $-x$ position, which is accelerated in the opposite direction. This process yields destructive radiation interference in the forward direction. Hence, radiation in a quadrupole field is defined by the fluctuation of current density in a cross section. The period of the FODO structure is chosen to obtain the desired radiation wavelength. If the particle causes only a small part of betatron oscillation in the whole QW structure, then the central wavelength is defined by the period $2L$ of the FODO structure, $\lambda_u \cong L/\gamma^2$, where L is the distance between the F and D lenses. This wavelength must fall within the frequency range of the optical amplifier, i.e., $\lambda_u \cong 0.3$ to 1 μm [2]. If we take $\gamma \cong 10^3$, $\lambda_u \cong 1$ μm, then $L \cong 100$ cm, or a period of about 200 cm.

The undulatority factor K is defined as

$$K=\frac{eH_\perp 2L}{2\pi mc^2},$$

where m is the mass of the electron, c is the speed of light, and e is the charge of the electron.

For an optimal undulatority factor $K\cong(1/2)^{1/2}$ [4], we need the field strength $H_\perp \cong GA_0 \cong 4\times10^{-3}$ T for electrons and 8 T for protons, where G is the gradient of the lens and A_0 is the amplitude of the betatron oscillation. For μ mesons with the same γ factor $H_\perp \cong 0.8$ T.

The number of photons ΔN_γ radiated by one particle on the first harmonic in the energy interval calculated from $E_{\max}$ to $E_{\max}(1-1/M)$, where M is the number of periods, can be estimated for $K\leq 1$ according to [4]

$$\Delta N_\gamma \cong 4\alpha\frac{K^2}{1+K^2},$$

where $\alpha=-e^2/\hbar c$, and the energy of the quanta E_γ is defined by the formula

$$E_\gamma \cong \frac{2\pi\hbar c\gamma^2}{L(1+K^2+\gamma^2\theta^2)},$$

where θ is the angle of observation. We will be interested in the range $K\leq 0.7$. The number of photons radiated by the particle with amplitude A in the relative bandwidth $\Delta f/f \simeq 1/M$ becomes

$$\Delta N_\gamma \cong \alpha(A/A_0)^2,$$

where A_0 is the initial amplitude at the beginning of the cooling, when $K\cong 0.7$ and A is the amplitude after cooling.

For a bunch with N particles and length l_b, the number of particles in the bandwidth is equal to

$$N_u \cong MN(\lambda_u/l_b) \cong MN(L/l_b)/\gamma^2.$$

The number of photons ΔN_γ^u in the bandwidth is defined by fluctuations of the centroid of charge from these N_u particles and is given by

$$\Delta N_\gamma^u \cong \Delta N_\gamma N_u \cong \frac{1}{3}\alpha N_u \frac{\varepsilon}{\varepsilon_0},$$

where ε_0 is the initial emittance at the beginning of the cooling, when $K\cong 0.7$, and ε is the emittance after cooling. The total number of radiated photons in the bandwidth in each pass is

$$\Delta N' \cong \frac{1}{3}\alpha N\frac{\varepsilon}{\varepsilon_0}.$$

During damping, the factor K also decreases which introduces a wavelength shift $\Delta\lambda \simeq 1/(1+K^2)$. There are techniques available for overcoming this problem; for example, the beam envelope function in the wiggler can be dynamically changed to restore the K factor.

II. Necessary energy change and amplification.—The energy $\Delta\mathcal{E}_\gamma$ carried by N' photons produced by a beam whose center is off in position by $\Delta x \cong A/(N_u)^{1/2}$ is given approximately by

$$\Delta\mathcal{E}_\gamma \cong E_\gamma N' \cong 2\left(\frac{e^2}{L}\right)N\gamma^2(A/A_0)^2.$$

Following the general idea of stochastic cooling, we must damp this amplitude Δx with a kicker. The well-known longitudinal kicker method can be used for optical cooling; a transverse displacement is provided by an energy change $\Delta E/E$ at a lattice position with nonzero dispersion function (see [3]).

To damp the amplitude $\Delta x \cong A/(N_u)^{1/2}$, we must provide an energy change $\Delta E/E$ which corresponds to

$$\Delta x \cong A/(N_u)^{1/2} \cong \eta(\Delta E/E),$$

where η is the dispersion function of the lattice of the dipole wiggler. If P is the undulatority factor of the dipole wiggler and the number of periods M is the same as in the quadrupole wiggler, then

$$\eta\frac{eE_\perp PL}{E\gamma}M = A/(N_u)^{1/2},$$

where $E_\perp$ is the electric field strength.

The energy $\mathcal{E}$ that must be contained in the photon radiation is given by

$$\mathcal{E}=\frac{c}{4\pi}E_\perp^2\mathcal{L}\frac{l_b}{c} \cong \frac{c}{4\pi}\frac{A^2}{N_u}\left(\frac{E\gamma}{ePL\eta M}\right)^2\mathcal{L}\frac{l_b}{c},$$

where $\mathcal{L}$ is the cross section of the emitted radiation light spot at the interaction point. If we compare with the energy radiated by the particles in the quadrupole wiggler, the coefficient of amplification κ is obtained,

$$\kappa \cong (\mathcal{E}/\Delta\mathcal{E}_\gamma)^{1/2} \cong \frac{1}{4}\frac{\varepsilon_\parallel}{r_0}\frac{1}{N}\frac{\Delta f}{f},$$

where $r_0 = e^2/mc^2$, $\varepsilon_\parallel = \gamma l_b \Delta E/E$ is invariant longitudinal emittance, and $\mathcal{L}$ is to be estimated as $\mathcal{L} \cong \pi A_0^2 \cong 2\pi\lambda LM$. In the last expression we used the optimal beam size as the transverse size of coherence and assumed $P \cong 1$. The dispersion function was also estimated $\eta \cong A_0(\Delta E/E)$, where $\Delta E/E$ is the energy spread in the beam and $M \cong f/\Delta f$.

If we substitute $N \cong 1\times 10^{10}$, $l_b \cong 15$ cm, $M = 5$, $\Delta E/E \cong 10^{-3}$, and $\gamma \cong 10^3$ (500 MeV), we obtain $\kappa \cong 3\times 10^2$ for an electron damping ring which is well below $\kappa \cong 10^5$, the typical amplification factor [2]. For the above parameters, the required pulsed power is $\cong 5$ kW at an average power of $\cong 25$ W, assuming a repetition rate of $\cong 10$ MHz. Thus, there is no apparent limitation for the design of such an amplifier to effectively cool positron and electron beams by this method. The amplification coefficient κ increases for protons and muons with the same γ factor as $r_{p,\mu}/r_0$, where r_p, r_μ are the classical radius of a proton and muon, respectively. The power approaches the technical limits of present day amplifiers. If, however, this relatively fast damping time (see below) is not necessary, the amplification coefficient can be decreased to an acceptable level.

III. Damping time and temperature.— At each pass the individual particle gets a kick which is correlated with its coordinate in the pickup $\Delta x \cong A/N_u$ and an averaged uncorrelated kick $\Delta x \simeq A/(N_u)^{1/2}$ which heats the particle. Following the standard technique described [1,3] one can obtain that the resulting action under optimal conditions gives a change of emittance per turn as

$$\frac{d\varepsilon}{dn} + \frac{\varepsilon}{N_u} = 0.$$

So the number of revolutions is approximately N_u. The process of cooling will stop when the number of radiated quanta is of the order unity. The theoretical limit for the noise of the amplifier is of the same order, i.e., one photon in the coherence volume. This corresponds to one photon per sample which has N_u electrons in it. This gives a decrease in emittance equal to

$$\alpha N_u \varepsilon_f/\varepsilon_0 \cong 1 \quad \text{or} \quad \varepsilon_f/\varepsilon_0 \cong 1/(\alpha N_u).$$

For our example, the reduction factor is 10^3.

The damping time τ in the exponent $\varepsilon \cong \varepsilon_0 \exp(-t/\tau)$ is approximately equal to

$$\tau \cong N_u T = \frac{N}{(\Delta f/f)}(\lambda_u/l_b)T.$$

For $1/T = 10$ MHz, this implies that $\tau \cong 10^5/10^7 = 10$ ms for a single cooling system, installed in a damping ring.

For cooling of the energy spread, the QW must be installed in a place where the beam size is mostly defined by the energy spread. For both transverse emittances and energy spread cooling in the same time it is necessary to arrange the bend between the pickup and the kicker so that the betatron phase shift must be more than $\pi/2$ [3]. This makes the bend not isochronous, and yields the existence of a threshold emittance.

IV. Transverse dynamics.— The equations which define the equilibrium emittance in the damping ring are as follows [5]:

$$\frac{d}{dt}\varepsilon_{x,y} \cong \left\langle (H_{x,y} + \beta_{x,y}/\gamma^2)\frac{d}{dt}(\Delta E/E)_i^2 \right\rangle - \varepsilon_{x,y}(1/\tau_s + 1/\tau),$$

where τ_s, τ is the time of cooling by synchrotron radiation and optical stochastic cooling, respectively, and

$$H_{x,y} = [\eta_{x,y}^2 + (\beta_{x,y}\eta'_{x,y} - \tfrac{1}{2}\beta'_{x,y}\eta_{x,y})^2]/\beta_{x,y}.$$

$\beta_{x,y}$ is the envelope function, $\eta_{x,y}$ is the dispersion function, and $\eta'_{x,y}$ its derivative. The averaging ($\langle \cdots \rangle$) is made over the longitudinal distance, while x and y denote two transverse coordinates.

The energy spread is defined by the relation

$$\frac{d}{dt}(\Delta E/E)_i^2 = \frac{d}{dt}(\Delta E/E)_{\rm IBS}^2 + \frac{d}{dt}(\Delta E/E)_{\rm QE}^2.$$

It should be noted that there are two distinct components, denoted by the subscripts QE and IBS, which refer to quantum excitations and intrabeam scattering, respectively. The reduction in the energy spread due to cooling is given by the expression

$$\frac{d}{dt}(\Delta E/E)^2 = \frac{d}{dt}(\Delta E/E)_i^2 - (\Delta E/E)^2(1/\tau_s + 1/\tau).$$

The increase in the energy spread due to IBS can be expressed as

$$\frac{d}{dt}(\Delta E/E)_{\rm IBS}^2 \cong \frac{N r_0^2(\varepsilon_x\beta_x)^{1/2}\ln_c c}{\gamma^3\varepsilon_x(\varepsilon_y\beta_y)^{1/2}\sigma_1\sigma_{sx}},$$

where $\sigma_{sx} = [\varepsilon_x\beta_x + \eta_x^2(\Delta E/E)^2]^{1/2}$, $\ln_c$ is Coulomb's logarithm, and $\sigma_1 \cong l_b/2$. The increase in the energy spread attributable to QE can be approximated by

$$\frac{d}{dt}(\Delta E/E)_Q^2 \cong \frac{55}{48(3)^{1/2}}\frac{r_0^2 c\gamma^5}{\alpha|\rho|^3},$$

where ρ is the bending radius.

The equilibrium emittance is defined by the conditions

$$\frac{d}{dt}(\Delta E/E)^2 = 0, \quad \frac{d}{dt}\varepsilon_{x,y} = 0.$$

Thus, as the energy is lowered, the source of excitation of the transverse emittance from synchrotron radiation is also reduced by $1/\gamma^5$ for fixed radius, and the effective time

$$1/\tau_s + 1/\tau \cong 1/\tau$$

is approximately the same as the time for stochastic cooling.

The primary source of heat for the beam is associated with intrabeam scattering. Present designs for low-emittance damping rings are optimized under the condition that the synchrotron radiation defines the damping time. Because the damping times with optical cooling are practically independent of the envelope functions in the damping ring, the optimization must be done at relatively lower energy in order to minimize IBS. IBS can be decreased by enlarging the vertical beam envelope function and enlarging the bending radius, thereby decreasing H_x, since the damping time is not now defined by synchrotron radiation. By using this optimization procedure the transverse invariant emittances and the energy spread can be decreased to sufficiently low values.

In conclusion, stochastic cooling offers a powerful technique for reducing energy spread and transverse emittances in electron and positron beams when extended to optical bandwidths. All the hardware such as pickups, amplifiers, and kickers have analogs in the optical region, such as quadrupole wigglers, optical amplifiers, and dipole wigglers. The estimated optical amplification of signals, energetics, damping times, and final beam temperatures are all within the realm of practical implementation for obtaining low-emittance (low energy spread) electron and positron beams. This technique may also be useful for developing low-emittance damping rings for future linear colliders and free electron lasers, as well. By considering the use of damping rings for linear collider applications, the emittance can be lowered while maintaining the same luminosity which makes it possible to decrease the number of particles, lowering the damping time. Proton-antiproton rings have distinctly different dimensional and temporal scales. Therefore it is possible to install few optical cooling devices and obtain significant damping by using the techniques proposed in this paper. The proposed method also offers the possibility of cooling muon beams.

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