Concept of a Resonant-Antennae Observatory for Gravitational Wave Bursts

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We show how, combining their outputs, at least six resonant bar antennae give with isotropic sensitivity the amplitude, polarization, and direction of propagation of a burst and test for two distinctive properties of the Riemann tensor: transversality and tracelessness. If not located at the same site, to exert the two vetoes, the burst arrival times on three antennae must be known; we show how to determine them. Then in addition the propagation of the burst at light velocity is tested.

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The current strategy of assigning to gravitational wave bursts the signals detected by few antennae consists in making coincidences within the optimal postdetection bandwidth [1]. However, such a strategy has a few limitations, which, in the expectation of few events per year, may impair the chances of confident detections. For instance, in order to maximize the probability of such coincidences, the antennae must be oriented parallel and thus the network, due to the bar antenna pattern, comes out to be strongly directional and also blind to signals with the wrong polarization. Also, no fingerprints are provided to assign confidently candidate events (one may notice in this respect that, even in the ideal case of pure thermal noise, when thresholds are set at $\approx 3h_{\min}$, where $h_{\min}$ is the minimum pulse amplitude detectable at unity signal to noise ratio, three antennae will go in coincidence with 0.95 probability at least once per year, a figure comparable with expected event rates); thus, to assign a gravitational wave burst, one has to rely on other signals, such as the light emission in the visible and in x and γ rays and the neutrino flash, but these may be either undetectable or show up on such different time scales that they may be correlated only with difficulty to the few msec duration of the gravitational wave burst.

These considerations have motivated us [2] to explore the possibilities of a strategy of detection such that the network, on one hand, may result in equal sensitivity to any direction and polarization, and reconstruct, besides the amplitude of the wave, its polarization and the direction of propagation (as, for instance, the recently proposed spherical antenna [3]). Additionally we may test the distinctive properties of the Riemann tensor of the gravitational wave to produce vetoes. We have in mind ultracryogenic bar antennae, because those under construction, NAUTILUS at INFN Frascati, AURIGA at INFN Legnaro and the one at Stanford, are expected to be sensitive enough to detect supernova burst signals emitted in the local group of galaxies and we are interested in discussing which would be the minimal number of antennae needed and where they should be located.

Let us first see how the above requests could be, in principle, satisfied with a minimal network of six identical antennae (labeled by $\alpha=1,\ldots,6$), all located at the same site.

The Riemann tensor, which describes the gravitational wave, must be of the kind $R^0{}_{i0j}$, so that it has only six components. Furthermore it must be traceless and transverse. It follows that $R^0{}_{i0j}$ carries only four independent numbers, namely, the amplitude, two angles for the direction of propagation, and the polarization angle in the wave front. In fact the Riemann tensor can be expressed as $R^0{}_{i0j}=\ddot{h}(t)W_{ij}(t)/2c^2$, where $W_{ij}(t)$ is a 3×3 symmetric, traceless, and transverse matrix. The "square amplitude" of the wave $\ddot{h}(t)^2$ can then be calculated as $\ddot{h}(t)^2=2c^2R^0{}_{i0j}(t)R^0{}_{i0j}$, because $\mathrm{Tr}(W^2)$, an invariant under the rotation group, is, for a traceless and transverse matrix, $\mathrm{Tr}(W^2)=2$. Consider now a set of N identical resonant antennae (labeled by $\alpha=1,\ldots,N$). If $\mathbf{n}^\alpha$ is a unit vector parallel to the axis of the αth bar we can write the response of each antenna as

$$X^\alpha(t)=(n^in^j)^\alpha\int_{-\infty}^{+\infty}d\tau\,H(t-\tau)R^0{}_{i0j}(\tau)\equiv\xi(t)W_{ij}(n^in^j)^\alpha\,, \quad (1)$$

where $H(t)$ represents the transfer function of the bar. For $N\geq 4$, if no two antennae are parallel and no three lie in the same plane, Eq. (1) can be, at least in principle, inverted to extract, at a given time, the four independent parameters of $R^0{}_{i0j}(t)$.

However, in order that the sensitivity of such an observatory is independent of the direction of propagation and of the polarization of the wave, one must find an estimate of the square amplitude which has to be a totally symmetric function of the X^α's. To fulfill this condition the detectors should be oriented along a set of N equivalent axes. The most natural choice would be the axes going from the center to the vertices of a regular polyhedron so that they divide the celestial sphere into identical regular polygons. The minimal choices are then (i) $N=4$, the four axes of the cube and (ii) $N=6$, the six axes of the

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regular icosahedron. We rule out (i) because there exists at least one choice for the direction of propagation and for the polarization for which the configuration is blind to the wave (propagation along the axis of intersection of the two planes containing the two pairs of axes and polarization at 45° from these planes).

It turns out that for the $N=6$ configuration instead the quadratic invariant, $\mathcal{H}\equiv\frac{1}{2}\xi^2(t)\mathrm{Tr}(W^2)$, is simply $\mathcal{H}=\frac{5}{8}\sum_{\alpha=1}^{6}X^{\alpha 2}$ and so it is an estimate of the square amplitude of the burst, which is independent of its polarization and direction of propagation. The expressions for the other two independent invariants, namely, $\mathcal{T}\equiv\xi(t)\mathrm{Tr}(W)$ and $\mathcal{D}\equiv\xi^3(t)\mathrm{Tr}(W^3)$, are also very simple,

$$\mathcal{T}=\frac{1}{2}\sum_{\alpha=1}^{6}X^{\alpha},$$

$$\mathcal{D}=\frac{25}{32}\sum_{\alpha=1}^{6}X^{\alpha 3}+\frac{15\sqrt{5}}{32}\sum_{\alpha\neq\beta\neq\gamma}^{6}\varepsilon_\alpha\varepsilon_\beta\varepsilon_\gamma X^\alpha X^\beta X^\gamma,$$

where $\varepsilon_\alpha=1$ for detectors 1,3,5 and $\varepsilon_\alpha=-1$ for detectors 2,4,6. For gravitational waves they both must be identical to zero so they can be used for vetoing against spurious excitations of the network.

For noisy detectors the linear invariant $\mathcal{T}$ is a Gaussian zero-mean stochastic process with autocorrelation $\langle\mathcal{T}(t)\mathcal{T}(t')\rangle=3R(t-t')$, where $R(t-t')$ is the noise correlation function of each antenna; $\mathcal{T}$ does not change if a true gravitational signal is added to the output of the detectors and therefore it can be used as a veto for large extra-noise disturbances (e.g., seismic noise, cosmic rays, etc.). The quadratic invariant $\mathcal{H}$, being proportional to the square of the responses, is connected to the total gravitational wave energy in the network; moreover it is the sum of squared Gaussian stochastic processes, its statistic is a χ^2 with 6 degrees of freedom and its mean and autocorrelation are respectively $\langle\mathcal{H}\rangle=\frac{15}{8}R(0)$, $\langle\mathcal{H}(t)\mathcal{H}(t')\rangle=\frac{15}{8}[R^2(0)+2R^2(t-t')]$. The cubic invariant $\mathcal{D}$ is a measure of the transversality of the wave; because of its nonlinearity, it may be more difficult to use as a veto at low signal-to-noise ratios.

We simulated the noise described by Eq. (2) below [taking $R(0)=\sigma^2=1$] by a Monte Carlo calculation and found the probability distributions of the invariants. Adding a linearly polarized gravitational wave signal to the noise, we found that we can reconstruct its direction and polarization. In Fig. 1 we report a typical result.

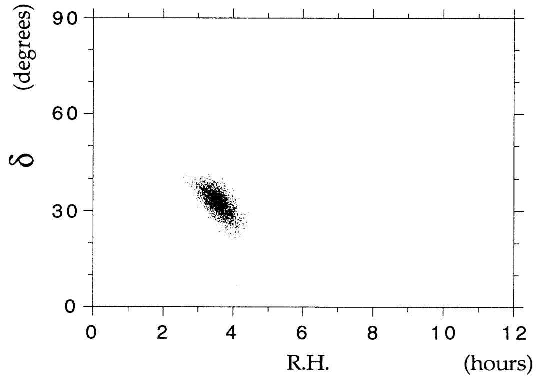

FIG. 1. Scatter plot of reconstructed positions of a source located at right ascension =3 h, 35 min and declination $\delta=32°\ 18'$ over 3000 attempts. The incoming gravitational pulse is assumed linearly polarized with polarization angle $\psi=30°$, the estimate of which has the same accuracy as the other two angles. The R_{SN}'s of the 6 antennae, due to their different figure pattern, are respectively 4.5, 6, 1, 9, 10, and 1.

We see that, in the local case, the $N=6$ configuration, besides allowing us to recover the four independent numbers carried by the $R^0{}_{i0j}(t)$, is minimal in respect to two distinct requirements: the possibility of exerting the linear and cubic veto and the possibility of showing "spherical" sensitivity.

Such a local solution to our problem, although rigorous in its minimal feature, may be highly impractical: Ultracryogenic bars, as they must lie horizontally, cannot be located in the same place [4]; however, they may be suitably distributed on the surface of the Earth in one of the possible $N=6$ configurations, in which their responses do not degenerate and the network keeps its "spherical" sensitivity. To give just one example, with reference at least in part to antennae under construction, they could be put in pairs. The observatory would then need a minimal number of three sites, each one having two detectors lying in the horizontal plane and forming an angle of $\simeq 63°$. One of these sites could, for instance, be located in Italy, one in Stanford, and a third one in Japan (allowing a maximum tilt of 8° from the horizontal plane, which would be feasible with slight modifications of the apparatus under construction). Now one can still use the quadratic invariant to get the total energy carried by the first, but if one wants to also exert the two vetoes, one must in addition determine the arrival time of the burst in order to reconstruct the wave front on which the $X^\alpha(t)$ must be recombined. The preceding analysis then can be carried out just the same provided the responses are shifted by the appropriate delay times. We see that this can be done off line with optimal filtering methods, as briefly outlined below, at the price that in at least three detectors $h\gtrsim 8h_{\min}$. While this may appear only an inconvenience, it gives the further advantage of testing that the signal travels with the velocity of light, an additional independent test. Of course, as we have added the time delay information, the $N=6$ configuration may no more be minimal in principle as far as the vetoing requirements are concerned, but of course it is still minimal under the request of "spherical" sensitivity.

We thus need to discuss how cryogenic bar antenna, whose output is currently analyzed so as to show time resolutions of fractions of 1 s at best, may actually be used to resolve arrival times of rare pulses with the resolution of a fraction of a ms, the resolution needed because of light velocity travel times on Earth surface distances.

The procedure we discuss below will need a digital data analysis, by which the output of the antennae is sampled at least at 5 kHz, in order to obey the sampling theorem. This is possible with current digital methods and instrumentation.

The estimation of the arrival time of a signal in the presence of noise is a well established problem in signal analysis [5]. The ability to measure the arrival time of a pulse signal on a resonant antenna then depends on its signal-to-noise ratio and its postdetection bandwidth. We have considered a simplified model of the antenna but we expect the main results to remain valid for an actual bar detector, if the operating conditions discussed in Ref. [6] are met, in particular the resonant conditions among bar, transducer, and electrical modes. In this model the antenna is considered as a simple harmonic oscillator with mass M, resonant angular frequency ω_0, and decay time τ, excited by a short pulse of force $f(t)$. The oscillator is driven in random motion both by the sum $f_n(t)$ of the Brownian force and by the backaction force noise of the position transducer, this one also contributing an additive position noise $\xi_n(t)$. Both the total force noise $f_n(t)$ and the added noise $\xi_n(t)$ are assumed to have white spectra with values S_f and S_ξ, respectively. In the framework of the standard Wiener filtering theory we can estimate for an impulsive force signal $A(t)=A_0 g(t-t_a)$ both the unknown amplitude A_0 and the unknown time of arrival t_a, defined as the time of maximum output of the given filter. An important case is that in which $g(t)$ is a wave packet with a center frequency close to ω_0 and a duration $\tau_p \approx 1/\omega_0$. This is probably the closest approximation to the expected gravitational wave burst signals. In order to evaluate the uncertainty of the estimation of t_a, let us notice first that, in the absence of any signal, the filter output reduces to a zero-mean Gaussian stochastic process η_w with a correlation function

$$R(t-t')\equiv\langle\eta_w(t)\eta_w(t')\rangle=\sigma^2\int_{-\infty}^{+\infty}\tilde{g}(t-t'')\exp(-|t''-t'|/\tau_{\rm pd})\left\{\cos[\omega_*(t''-t')]+\frac{1}{\omega_*\tau_{\rm pd}}\sin[\omega_*(t''-t')]\right\}dt'', \tag{2}$$

where $\omega_*=\omega_0(1+S_f/M^2S_\xi\omega_0^4)^{1/4}$, $\tau_{\rm pd}^{-2}=\tau^{-2}+2(\omega_*^2-\omega_0^2)$, and $\tilde{g}(t)$ is the convolution of the signal with itself. For actual antennae, optimal sensitivity calculations allow one to expect [6] $\omega_*\approx\omega_0$ and $\tau_{\rm pd}\approx\sqrt{M^2S_\xi\omega_0^2/S_f}\approx 20$ ms, that is an optimal postdetection bandwidth of 50 Hz. On the other hand the response of the Wiener filter to the signal is proportional to the autocorrelation of the output noise. The estimate of A_0 and t_a reduces to the search for the maximum of the absolute value of the stochastic process $\hat{A}(t)=A(t)+\eta_w(t)$ and, by definition, the value $\hat{t}$ for which $|\hat{A}(t)|$ reaches its maximum is taken as an estimate of t_a. Let us call t_r the random part of the estimate of the arrival time. An analytic evaluation of the statistics of the zero-mean random variable t_r in a few situations was obtained long ago [7]. To discuss the main conclusions that apply to our case let us first separate the "phase" part δt of t_r, writing $t_r=\delta t+nT_*/2$, with $T_*=2\pi/\omega_*$, $\delta t\le T_*/4$, and n an integer. The first result is that the standard deviation of δt, $\sigma_{\delta t}$, provided that $g(\omega)$ varies slowly in the range $\omega\approx\omega_*\pm 1/\tau_*$, is given by

$$\sigma_{\delta t}=\frac{T_*}{2\pi R_{\rm SN}}, \tag{3}$$

where the signal-to-noise ratio $R_{\rm SN}$ is defined by $R_{\rm SN}=A_0/\sigma$, which turns out to be actually the pulse sensitivity $h/h_{\rm min}$.

The second important result is that for $R_{\rm SN}\gg 1$ and pulse duration $\tau_p\lesssim 1/\omega_*$ the standard deviation of n is given by

$$\sigma_n=\frac{\tau_{\rm pd}}{T_*R_{\rm SN}^2}. \tag{4}$$

In this limit $A(t)$ shows a cusp for $t=t_a$, which makes detection of the arrival time easier. Notice that as soon as $\sigma_n\le 1$, for instance, when $R_{\rm SN}\ge 8$ for $\tau_{\rm pd}=20$ ms, the total uncertainty σ_t of the arrival time reduces abruptly only to the phase contribution $\sigma_{\delta t}$ as in Eq. (3). For a 1 kHz antenna this gives $\sigma_t\approx 160/R_{\rm SN}$ μs, a figure that easily allows time delay measurement for antennae at different places on the Earth.

To show the confidence in the measurement of the arrival time at such a resolution, we give in Fig. 2 the results of a numerical simulation, using $g(t)=\delta(t)$: The zero-delay channels within $\sim$0.03 ms contain more than 95% of the data, in about 300 attempts. Only in less than 5% of the attempts did the procedure give arrival times wrong by $\pm nT/2$, with n never larger than $n=2$. We have also investigated numerically the effect of the dura-

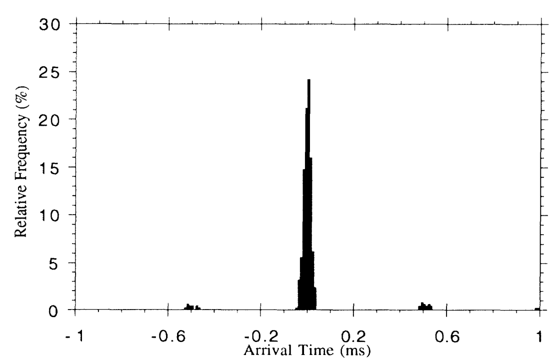

FIG. 2. The relative frequency, over 300 attempts, with which the arrival time is determined by the procedure in the text. The "true" arrival time is $t=0$. The signal has $R_{\rm SN}=10$ and $\omega_*\tau_{\rm pd}=20$.

tion τ_p of the pulse: As long as $\tau_p < 1/\omega_*$, the results are quite insensitive both to τ_p and to the shape of the pulse [8].

An actual ultracryogenic antenna is not a single mechanical oscillator, but rather a system of three coupled oscillators: two mechanical, the bar and the transducer, and one electrical, the circuit coupling the transducer to the amplifier. So one may wonder if all this is still applicable. Fortunately the answer is in the affirmative and particularly transparent just as in the case of optimal sensitivity, when one tunes the three resonators to a tight coupling: We find that the output of the Wiener filter behaves like that of a single resonator. Thus we are confident that the present results remain valid.

In summary, if we have $R_{SN} \geq 8$ in at least three antennae, we can determine the pulse arrival times therein and thus velocity and direction of propagation of the burst wave front. Recombining the responses as in the local case, after appropriately shifting them in time, we can exert the linear and cubic vetoes to reject spuria and determine the polarization and (again) the direction of propagation; the agreement between the two determinations of the direction effectively tests for speed of light propagation.

The observatory will then fully and autonomously reconstruct a signal, i.e., assign it unequivocally to a gravitational wave event, without, for instance, the help of neutrino coincidences. This last feature may be crucial for observations of gravitational collapses beyond the Andromeda galaxy because the range of even the largest neutrino detector under development for supernova events should be limited to the distance of that galaxy, as one anticipates taking as reference the neutrino emission of supernova 1987A.

If for all antennae $R_{SN} < 8$, the capability will still be preserved to recover, with isotropic sensitivity, the total energy of the burst, if the six squared amplitudes are integrated over times longer than the light-times delays.

All these results can be easily adapted to a network of interferometric antennae, provided their location on Earth is properly chosen. Our concept as given here differs from the solution of the inverse problem worked out in Ref. [9], where a complementary point of view is taken. In Ref. [9] the distinctive tensorial properties of the wave are rather an untested assumption, which enables one to reconstruct the signal with the minimum number of antennae. It should be noticed that also in that case it is requested to deal with signals of amplitude $h \gtrsim 10 h_{\text{min}}$.

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[4] This statement should be qualified: It is valid for the cryogenic and ultracryogenic antennae either in operation or under advanced construction, but there is at least one proposal to set up an antenna along a vertical axis [M. Bocko (private communication)] and that suspension methods are being developed which would allow in principle any angle with the horizontal [P. Michelson (private communication)].

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[8] In the opposite situation, when $\tau_p \gtrsim 1/\omega_*$ Eq. (4) becomes $\sigma_n = \tau_p / T_* R_{SN}$. Of course a train of pulses, coming too close to each other, would reduce to such a case.

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