Flow Properties of the Axiplanar Phase of ^{3}He-A

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(Received 9 August 1993)

Because of the recently raised suspicion about the correctness of the conventional (axial) identification of ^{3}He-A, we have studied the flow properties of the axiplanar state, the only other possible candidate for the A phase. We show that for small "planar" mixture, both phases have very similar flow properties. However, the superflow of the axiplanar phase is always more stable than that of the axial phase, in qualitative (though not yet quantitative) agreement with current experiments. We also propose a method to distinguish these two states clearly using flow experiments.

PACS numbers: 67.57.De, 43.35.+d, 47.20.−k

Since its discovery about twenty years ago, ^{3}He-A has been identified with the so-called Anderson-Morel or axial state. In a recent paper [1], summarizing his recent works and a reanalysis of previous thermodynamic measurements [2], Gould has questioned the correctness of this identification. He pointed out that the current thermodynamic measurements cannot rule out a much less well-known candidate, namely, the axiplanar state.

The axiplanar state was first suggested by Mermin and Stare [3] in the early days of ^{3}He-A as a possible candidate besides the axial state. Stare [4] also showed that, like the axial state, the axiplanar state exhibits longitudinal and transverse NMR frequency shifts. Despite its compatibility with early experiments, the axiplanar phase has never been studied in detail because of the popularity of the much simpler axial state, which was thought to be consistent with all experiments until now. In addition to the thermodynamic measurements [2], Gould and coworkers have also found discrepancies in the axial interpretation in both zero sound [5] and flow measurements [6]. For the latter, a series of experiments was performed to map out the phase boundary for the transition from the uniform to helical texture in parallel superflow and magnetic field. It was found that the transition takes place at velocities unaccountably higher than those predicted for the axial state [7]. A recent measurement by Hall and Hook [8] on the ratio of longitudinal and transverse superfluid density $\rho_\parallel/\rho_\perp$ of ^{3}He-A near T_c also shows that it lies above the axial prediction (i.e., $\frac{1}{2}$) by about 10%. While it is conceivable that this discrepancy can be caused by textural effects despite efforts to eliminate them, it has a simple explanation in the axiplanar picture. In view of the rising suspicion of the axial interpretation, it will be useful to have a better understanding of the properties of the axiplanar state, especially those that can distinguish it from the axial.

The purpose of this paper is to study the flow properties of the axiplanar state. As we shall see, the axiplanar state can be regarded as a mixture of the axial and the so-called "planar" state. The planar admixture (however small) changes the symmetry group of the axial state completely. As a result, unlike the axial state, it is possible to define a global phase for the axiplanar order parameter. This means that it has the usual type of superfluid velocity like that of ^{4}He and ^{3}He-B, and not the peculiar kind of the axial state. However, in the limit of a small planar mixture, the flow properties of axial and axiplanar states are very similar. Despite the fundamental difference in superfluid velocity, essentially all the key features of the axial state are intact in this limit, i.e., the existence of a "textural" axis, nonsingular vortices, and the helical distortion of the texture in the presence of a strong superflow. There are, however, qualitative differences. Unlike the axial state, the axiplanar superfluid density is not axisymmetric. Near T_c, the ratio between the longitudinal and transverse superfluid density is generally greater than $\frac{1}{2}$ and is *pressure dependent.* A similar but more subtle feature occurs in the helical distortion of the texture induced by a parallel superflow u and magnetic field H. When both u and H are expressed in appropriate dimensionless units (defined later as w and h), at temperatures close to T_c, the boundary in the w-h plane separating the uniform and the helical texture is pressure dependent (independent) for the axiplanar (axial) state. These pressure dependences provide a simple and direct way of distinguishing these two phases.

(I) The axial and axiplanar order parameter.—The order parameters of p-wave superfluids are 3×3 complex matrices $A_{\mu i}$, where μ and i are spin and orbit indices, respectively. The axiplanar order parameter is

$$A_{\mu i}=\Delta[\hat{\mathbf{f}}_3(\xi\hat{\mathbf{e}}_1+i\zeta\hat{\mathbf{e}}_2)-\eta\hat{\mathbf{f}}_1\hat{\mathbf{e}}_3]_{\mu i}e^{i\theta}, \tag{1}$$

where Δ is the overall magnitude, ξ, ζ, and η are all positive, $\zeta^2+\xi^2+\eta^2=1$. The phase θ displays the gauge symmetry. The axial state corresponds to $\xi=\zeta=1/\sqrt{2}$, $\eta=0$. In this case, it is well known that the phase factor can be absorbed into the orientation of the vector $\hat{\mathbf{e}}_1+i\hat{\mathbf{e}}_2$. Minimizing the bulk free energy using the axiplanar order parameter, it is straightforward to show that [4]

$$\xi^2-\zeta^2=\frac{-\beta_1\beta_{45}}{2\beta_{13}\beta_{345}-\beta_3^2},\quad \eta^2=\frac{\beta_{13}\beta_{45}}{2\beta_{13}\beta_{345}-\beta_3^2}, \tag{2}$$

where the β's are the coefficients of the quartic terms of

0031-9007/93/71(24)/3999(4)$06.00

the bulk free energy. We have used the standard notation $\beta_{ijk\ldots} \equiv (\beta_i + \beta_j + \beta_k + \cdots)$. Note that in the limit of small β_{45}, Eq. (1) can be written as a sum of an axial state $\mathbf{f}_3(\hat{\mathbf{e}}_1 + i\hat{\mathbf{e}}_2)$ with a small planar admixture $(\xi/\zeta - 1)\mathbf{f}_3\hat{\mathbf{e}}_1 - (\eta/\zeta)\mathbf{f}_1\hat{\mathbf{e}}_3$.

Figure 1 shows the relative stability between different p-wave states in β space that are consistent with the observed A-phase properties (i.e., nonreduced susceptibility and no spontaneous magnetization) [3,4]. The results of Tang *et al.* in Ref. [2] have constrained the β's near T_c to lie on a curve at a given pressure. With these constraints, further determination of either β_3 or β_{45} at a given pressure would have identified the order parameter of the A phase. As we shall see, the stability of laminar flow can provide precisely what is needed for this identification.

(II) Superfluid density, laminar flow, and its stability.— By laminar flows, we mean order parameters of the form $A_{\mu i} = e^{i2muz/\hbar}A^0_{\mu i}$, where A^0 is constant in space and the phase factor describes a uniform superfluid velocity $\mathbf{v}_s = u\hat{\mathbf{z}}$. In the following, we shall also consider a magnetic field $\mathbf{H}$ parallel to $\hat{\mathbf{z}}$, which is the experimental configuration [6]. The orientation of A^0 is determined by minimizing the total orientational energy $F_{\rm or} = F_G + F_D + F_H$, where

$$F_G = \tfrac{1}{2}\left[K_1\partial_i A_{\mu i}\partial_j A^*_{\mu j} + K_2\partial_i A_{\mu j}\partial_i A^*_{\mu j} + K_3\partial_i A_{\mu j}\partial_j A^*_{\mu i}\right],$$

$$F_D = g_D\left[|\mathrm{Tr}A|^2 + \mathrm{Tr}AA^* - \tfrac{2}{3}\mathrm{Tr}AA^\dagger\right],$$

$$F_H = g_H H_\mu (AA^\dagger)_{\mu\nu} H_\nu .$$

With A^0 given by the axiplanar form, Eq. (1), these energies become

$$F_G = \frac{1}{2}\left(\frac{2m}{\hbar}\right)^2 \Delta^2 u^2 \{K_2 + K_{13}[\xi^2(\hat{\mathbf{e}}_1\cdot\hat{\mathbf{z}})^2 + \zeta^2(\hat{\mathbf{e}}_2\cdot\hat{\mathbf{z}})^2 + \eta^2(\hat{\mathbf{e}}_3\cdot\hat{\mathbf{z}})^2]\}, \tag{3}$$

$$F_D = g_D\Delta^2[(\xi\hat{\mathbf{f}}_3\cdot\hat{\mathbf{e}}_1 - \eta\hat{\mathbf{f}}_1\cdot\hat{\mathbf{e}}_3)^2 + \xi^2(\hat{\mathbf{f}}_3\cdot\hat{\mathbf{e}}_1)^2 + \eta^2(\hat{\mathbf{f}}_1\cdot\hat{\mathbf{e}}_3)^2 + 2\zeta^2(\hat{\mathbf{f}}_3\cdot\hat{\mathbf{e}}_2)^2 - 2\eta\xi(\hat{\mathbf{f}}_1\cdot\hat{\mathbf{e}}_1)(\hat{\mathbf{f}}_3\cdot\hat{\mathbf{e}}_3) - \tfrac{2}{3}], \tag{4}$$

$$F_H = g_H\Delta^2[(\xi^2 + \zeta^2)(\hat{\mathbf{f}}_3\cdot\mathbf{H})^2 + \eta^2(\hat{\mathbf{f}}_1\cdot\mathbf{H})^2]. \tag{5}$$

In the limit of a small planar mixture, $\eta \ll 1$, $\zeta,\xi \sim 1/\sqrt{2}$, and $\hat{\mathbf{e}}_3$ is the direction of minimum superfluid density, which we identify as the "texture" axis. From Eq. (3), it is clear that the superfluid density is not axisymmetric about $\hat{\mathbf{e}}_3$. Near T_c, where $K_1 = K_2 = K_3$, we have $\rho_s^{zz}/\rho_s^{xx} = (1+2\eta^2)/(1+2\xi^2)$, $\rho_s^{zz}/\rho_s^{yy} = (1+2\eta^2)/(1+2\zeta^2)$. For the axial state, these ratios are exactly $\frac{1}{2}$, whereas they are strictly greater than $\frac{1}{2}$ and are pressure dependent for axiplanar states.

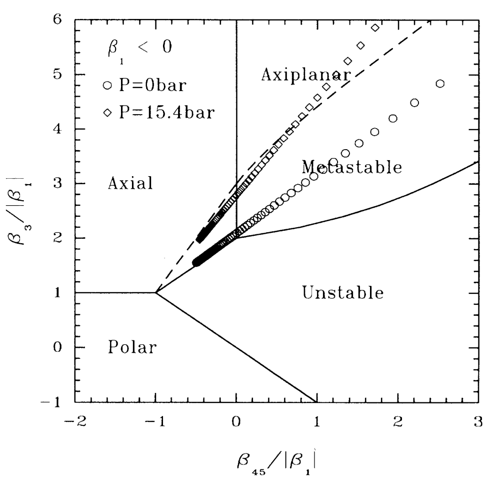

FIG. 1. Relative stability of all possible candidates for ^{3}He-A [3,4]. The curves labeled by 0 and 15.4 bars are constraints on the β coefficients at T_c and at the respective pressures, derived from the results of Tang *et al.* in Ref. [2]. In the "metastable" region (bounded above by the broken line), both axiplanar and axial phases are locally stable but have higher energy than the isotropic Balian-Werthamer phase. None of the A-phase candidates is stable in the "unstable" region.

It is clear that F_G tends to align $\hat{\mathbf{e}}_3$ with $\mathbf{v}_s$. F_D is minimized when the orbital triad $\{\hat{\mathbf{e}}_i\}$ is completely aligned with the spin triad $\{\hat{\mathbf{f}}_i\}$. F_H is minimized when $\hat{\mathbf{f}}_2$ is aligned with $\mathbf{H}$. It is straightforward (though tedious) to show that the laminar configuration $(\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3) = (\hat{\mathbf{f}}_1,\hat{\mathbf{f}}_2,\hat{\mathbf{f}}_3) = (\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})$ is a stationary point of $F_{\rm or}$, and is locally stable against any uniform change in the orientations of the spin and orbital triads in a substantial portion of the u-H plane (the region in Fig. 2 above the inclined straight line) [9]. Within this region, we have

$$A^0_{\mu i} = \Delta[\hat{\mathbf{z}}(\xi\hat{\mathbf{x}} + i\zeta\hat{\mathbf{y}}) - \eta\hat{\mathbf{x}}\hat{\mathbf{z}}]_{\mu i}. \tag{6}$$

Next, we consider the stability of A^0 against nonuniform deformations in orientations that only vary along z. The general form of such a deformation with a single wave vector can be shown to be [10]

$$A_{\mu i}(z) = e^{i2muz/\hbar}[S(z)A^0 T^{-1}(z)]_{\mu i}, \tag{7}$$

where S and T are spin and orbital rotations of the form

$$T(z) = R(\hat{\mathbf{z}},qz)R(\hat{\mathbf{y}},\epsilon)R(\hat{\mathbf{z}},-qz),$$
$$S(z) = R(\hat{\mathbf{z}},qz)R(\hat{\mathbf{y}},\sigma)R(\hat{\mathbf{z}},-qz), \tag{8}$$

$R(\hat{\mathbf{n}},\theta)$ denotes a rotation about axis $\hat{\mathbf{n}}$ by an angle θ. The angles ϵ and σ are small angles. The rotation T generates a precession of the orbital triad along z such that the texture $\hat{\mathbf{e}}_3$ precesses about the $\hat{\mathbf{z}}$ axis with wavelength $2\pi/q$ and an opening angle ϵ. S describes a similar precession for the spin triad.

Since A^0 is a local minimum, the (spatially averaged) difference $\delta F = F_{\rm or}(A) - F_{\rm or}(A^0)$ is quadratic in ϵ and σ. After a straightforward but lengthy calculation, δF can be written in dimensionless units as

$$\delta f = [A(\bar{q})\epsilon^2 + B(\bar{q})\epsilon\sigma + C(\bar{q})\sigma^2] + D(\epsilon-\sigma)^2 + Eh^2\sigma^2 , \quad (9)$$

$$A(\bar{q}) = \tfrac{1}{4}w^2\bar{K}_{13}(1-3\eta^2) + \bar{q}w\bar{K}_{123}\zeta\xi + \tfrac{1}{4}\bar{q}^2\{\bar{K}_{123} + (\bar{K}_2 - \bar{K}_{13})\eta^2\} , \quad (10)$$

$$B(\bar{q}) = -\bar{q}w\bar{K}_{13}\eta\zeta - \tfrac{1}{2}\bar{q}^2(\bar{K}_{13} + 2\bar{K}_2)\eta\xi , \quad (11)$$

$$C(\bar{q}) = \tfrac{1}{4}\bar{q}^2\{\bar{K}_{13}\eta^2 + \bar{K}_2(2-\eta^2)\} , \quad (12)$$

$$D = 1 + \tfrac{5}{2}\eta\xi, \quad E = -(1 - \tfrac{3}{2}\eta^2) , \quad (13)$$

where we have expressed the energies and lengths in units of the dipole energy $(g_D\Delta^2)$ and the dipole length $[L_D \equiv (K_{123}/3g_D)^{1/2}]$:

$$\delta f = \delta F/g_D\Delta^2 , \quad h^2 = g_H H^2/g_D ,$$

$$w = 2muL_D/\hbar, \bar{q} = qL_D , \quad \bar{K}_i \equiv 3K_i/K_{123} .$$

Optimizing with respect to σ, one finds

$$\delta f = Y(\bar{q};w,h)\epsilon^2 = \left[\frac{4[C(\bar{q}) + D + Eh^2][A(\bar{q}) + D] - [B(\bar{q}) - 2D]^2}{4[C(\bar{q}) + D + Eh^2]} \right] \epsilon^2 . \quad (14)$$

Instability towards a helical distortion will result if δf is negative. To determine the boundary between the uniform and the helical texture (referred to as the laminar boundary), we note that the numerator of Y is quartic in $\bar{q}$, and is non-negative for all $\bar{q}$ if all roots of the equation $Y(\bar{q}) = 0$ are complex, while the denominator is non-negative for all $\bar{q}$. Instability is signaled when a pair of (conjugated) complex roots become real as one varies w and h starting from the laminar side.

Figure 2 shows the axiplanar boundary for a given β_3 and a series of β_{45}. The latter illustrates the effect of planar admixture on the "stability loop." The case $\beta_{45} = 0$ corresponds to the axial state. We have found numerically that the laminar boundary is rather insensitive to β_3. On the other hand, as β_{45} (i.e., the planar mixture) increases from zero, the size of the stability loop expands rapidly. It reaches a maximum size (substantially larger than that of the axial phase) and then shrinks back, implying that the axiplanar laminar flow is more stable than that of the axial phase for at least a range of the planar mixture. This feature is qualitatively consistent with the experimental observation of Bates *et al.* [6], which shows the laminar flow is stable at velocities and magnetic fields higher than those predicted by the axial interpretation. However, as we try to fit the data of Ref. [6], we find that the axiplanar stability loop is still not big enough to match the data. Rather than speculating on the reasons for the mismatch, we would like to point out a special feature of the axiplanar laminar boundary which clearly distinguishes it from the axial phase. Near T_c, $K_1 = K_2 = K_3$ and hence $\bar{K}_i = 1$; the functions A,B,C,D,E in Eq. (9) depend only on ξ, ζ, and η. Since these coefficients are pressure dependent, the axiplanar laminar boundary will vary with pressure. This feature is absent in the axial state for ζ, ξ, and η are constants. Unfortunately, the measurements in Ref. [6] were done at different temperatures (fixed pressure) instead of at different pressures. We believe repeating these experiments at different pressures will not only produce a clearer distinction between these two states, but will also enable one to determine the ratio β_{45}/β_1, in the event that ^{3}He-A is axiplanar.

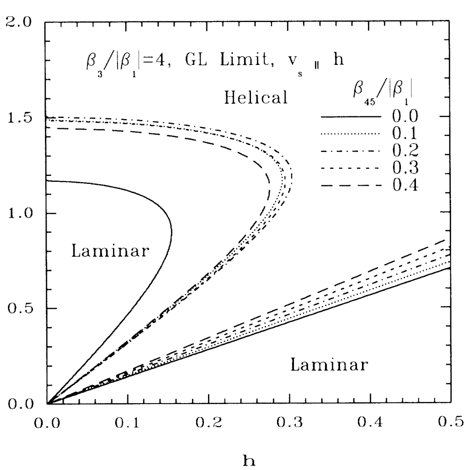

FIG. 2. Region of stability of laminar flow in the w-h plane: Above the inclined straight line, the laminar flow $(\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3) = (\hat{\mathbf{f}}_1,\hat{\mathbf{f}}_2,\hat{\mathbf{f}}_3) = (\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})$ is a stationary point, which is stable (unstable) against helical distortion in the region labeled laminar (helical). Below the straight line, the stationary configuration is $\{\hat{\mathbf{e}}_i = \hat{\mathbf{f}}_i\}, \hat{\mathbf{f}}_2 = \hat{\mathbf{z}}, \hat{\mathbf{f}}_3 = \hat{\mathbf{x}}, \hat{\mathbf{f}}_1 = \hat{\mathbf{y}}$ (see Narasimhan and Ho in Ref. [9]).

(III) Vortices and vortex nucleation in the axiplanar state.—The axial state is known to have nonsingular vortices [11]. This is a consequence of its orbital symmetry group, SO(3). This symmetry also allows a superflow to be collapsed completely down to zero by textural motions [12]. In fact, the remarkable periodic textural motions in ^{3}He-A in the presence of a heat flow [13] can be interpreted in terms of this collapsing process, with nonsingular vortices (i.e., textural variations) being nucleated continuously to reduce the increasing superfluid velocity generated by the heat flow [14].

The orbital symmetry group of the axiplanar state is SO(3)×U(1). The U(1) gauge symmetry only allows the formation of the ordinary "phase" vortices. It also imposes a topological stability (of the usual kind) on the superflow. Because of this stability, even though the axiplanar textures can undergo helical distortions to reduce the flow energy as we have shown in the previous section, it can never collapse the superflow completely as in the axial case. At first sight, it appears that the axiplanar state would have serious difficulty in explaining the heat flow driven periodic textural motions, which is essentially a flow collapsing process in repetition. There is, however, a way that the axiplanar state can escape this topological stability. In the limit of small planar mixture, it will cost little energy for the axiplanar state to turn axial in the regions where the flow energy is high, or near the surface of the container where the order parameter is generally suppressed. Once turned axial, the system can get rid of any amount of superflow within the transformed region. In this way, essentially all major flow properties of the axial state are preserved. Nonsingular vortices can exist in the axiplanar state provided that they have an axial core. These vortices can be nucleated near the surface in the axial mode [14], evolving into the full axiplanar form as they emerge into the bulk.

The similarities between the axial and the axiplanar flow properties certainly make it difficult to distinguish these states. However, as discussed in the previous section, clear distinctions can still be made from the symmetry of the superfluid density as well as from the pressure dependence of the laminar phase boundary.

T.L.H. would like to thank Chris Gould for many informative discussions This work is supported by NSF Grant No. DMR87-17574.

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