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Experimental Signature for Statistical Multifragmentation

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Multifragment production was measured for the 60 MeV/nucleon ^{197}Au+^{27}Al, ^{51}V, and ^{nat}Cu reactions. The branching ratios for binary, ternary, quaternary, and quinary decays were determined as a function of the excitation energy E and are independent of the target. The logarithms of these branching ratios when plotted vs $E^{-1/2}$ show a linear dependence that strongly suggests a statistical competition between the various multifragmentation channels. This behavior seems to relegate the role of dynamics to the formation of the sources, which then proceed to decay in an apparently statistical manner.

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Multifragment production [1–12], a most interesting feature of intermediate-energy heavy-ion reactions, still remains elusive in its interpretation. For instance, it is still unclear whether there exists a homogeneous mechanism that may be labeled multifragmentation, or whether the process is simply a collage of weakly correlated features occurring at various stages of the reaction.

Recently, some experimental progress has been made, by isolating and characterizing what appear to be true multifragmentation sources formed in reverse kinematics reactions [2,11]. These sources are formed in a process akin to incomplete fusion, whereby one partner of the collision picks up, and fuses with, a variable portion of the other partner. Kinematically, it is possible to determine how much mass has been picked up and the excitation energy associated with the fused object [13]. Surprisingly, these sources, once characterized as described above, undergo multifragment decay in a way that is singularly independent of the formation process. The observed branching ratios for binary, ternary, quaternary, and quinary decays seem to depend almost exclusively upon the excitation energy E of the fused object, and remarkably little upon the target-projectile combination or even the bombarding energy [2]. Similar features of target independence suggesting the formation of a source decaying independently of the formation process have been observed by other groups [4,5,7].

The obvious question that we want to address is the following: What is the multifragmentation mechanism of these sources? In particular, is this decay controlled by dynamics [14–18] or by statistics [19–31]?

The role of statistics in these reactions has been expounded in a variety of modes such as chemical equilibrium models [21,22], the liquid-gas phase transition [24–27], or hybrid approaches such as evaporation occurring simultaneously with dynamical expansion [28], dynamics followed by statistical decay [29–31], etc. While these models, or approaches, may be well justified *a priori*, inevitable limitations may make their application to actual data somewhat problematic [32]. In other words, while the models may be sound in their essence, they may be too schematic and thus unable to fit the data satisfactorily.

An alternative way of searching for statistical effects would be to examine the data themselves in order to see whether they contain signatures that may be brought forth without the help, or impediment, of any given model. As an example very much to the point, in Ref. [33] the rise of the fission probability P with excitation energy in electron or bremsstrahlung induced fission was shown to be statistical in origin by demonstrating the presence of a characteristic energy dependence [$\ln(P) \propto E^{-1/2}$]. This dependence is a generic attribute of statistical decay that has been verified with well-understood fission reactions (see Fig. 1). In this Letter we apply a similar approach to intermediate-energy heavy-ion reactions in order to demonstrate the statistical nature of the multifragmentation branching ratios.

Let us suppose that the hot nuclear system formed in

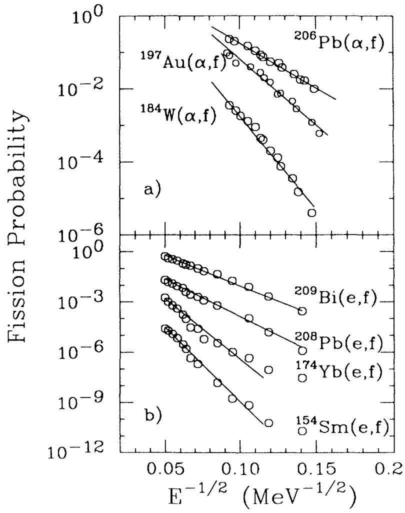

FIG. 1. (a) The fission probability plotted as a function of $E^{-1/2}$ for the α induced reactions ^{206}Pb(α,f), ^{197}Au(α,f), and ^{184}W(α,f) and (b) for the electron induced reactions ^{209}Bi(e,f), ^{208}Pb(e,f), ^{174}Yb(e,f), and ^{154}Sm(e,f). (The data are taken from Ref. [33].)

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the heavy-ion reaction decays statistically, and that a barrier of some sort governs this decay. Alternatively, in the framework of the chemical equilibrium picture, one can consider the potential energy of each configuration as a barrier. It is conceivable that, in these pictures, there might arise a hierarchy of "barriers" such that all the binary configurations would have barriers closer to each other than to those of the ternary configurations, and so on. Thus, let us assume that $B_2, B_3, \ldots, B_n$ are the average "barriers" associated with binary, ternary, and n-body decays. The decay probability for each channel should be proportional to the level density of the system $\rho(E)$ (dominated by the internal degrees of freedom) at an excitation energy equal to the available energy minus the barrier:

$$P_n(E) \propto \rho(E - B_n) . \tag{1}$$

For a Fermi gas level density, we have

$$P_n(E) \propto e^{2\sqrt{a(E-B_n)}} , \tag{2}$$

where a is the level density parameter. For $E \gg B_n$ one obtains

$$P_n(E) \propto e^{2\sqrt{aE}} e^{-B_n\sqrt{a/E}} \propto e^{-B_n/T} . \tag{3}$$

For convenience, we want the ratio of the n-fold events to the binary events:

$$\ln[P_n/P_2] \propto -\sqrt{a/E}\,(B_n - B_2) . \tag{4}$$

Thus, a plot of $\ln(P_n/P_2)$ vs $E^{-1/2}$ should give a straight line.

As mentioned above, this simple theoretical prediction has been empirically tested in Ref. [33] for the overall fission probabilities in the Pb region, and used to prove that the rapid rise of the fission cross section in e^- induced fission of similar nuclei is due to statistics. In Fig. 1(a) the total fission probability is plotted vs $E^{-1/2}$ for three α induced reactions in an energy regime where compound nucleus formation is well established. The expected linear dependence is observed, and the slopes correlate quantitatively with the known fission barriers. It is important to notice that the linear dependence extends even to regions of excitation energy where multiple-chance fission contributes substantially. Thus, one should consider this linear dependence as "empirical" evidence for statistical decay.

In Fig. 1(b) a similar plot is shown for four e^- induced fission reactions. The energy dependence of the fission probability was extracted by unfolding the e^- induced fission cross sections from the virtual photon spectrum. The observed linear dependences and the correlation of the slopes with the fission barriers proved that the rise of the fission cross section with increasing e^- energy is a statistical effect arising from the phase spaces associated with competing decay channels [33].

To see whether a similar dependence exists in the multifragmentation branching ratios, we have performed an experiment with the specific purpose of determining the multifragment branching ratios as a function of the excitation energy of the decaying source. The experiment [34] was performed at the Lawrence Berkeley Laboratory Bevalac. Beams of ^{197}Au ions impinged on targets of ^{27}Al, ^{51}V, and $^{\mathrm{nat}}$Cu at 60 MeV/nucleon. The reaction products with $Z > 5$ were detected on an array of twenty Si(0.3 mm)-Si(5 mm) telescopes [35]. The twenty telescopes were arranged in a 5×5 configuration closely packed around the beam with the central and corner array elements missing. The maximum angular coverage was approximately ±17° in the horizontal and vertical planes. Since in reverse kinematics the fragments have high kinetic energies and are emitted within a narrow cone around the beam direction, a detection efficiency of about 60% was obtained for inclusive events. The detection efficiency was calculated via Monte Carlo simulations that included the geometry of our detector setup.

The decay of the hot nuclear systems formed in these reactions was studied, following closely the approach of Ref. [2], by determining the ratio of the n-fold events ($n=3$, 4, and 5) with respect to the twofold events as a function of the excitation energy E. In the incomplete fusion model [36], the excitation energy is approximately related to the parallel component $V_s^{\parallel}$ of the source velocity V_s by $E = E_b(1 - V_s^{\parallel}/V_b)$, where E_b is the bombarding energy and V_b is the beam velocity. This formula does not take into account preequilibrium emission; thus the calculated value of the excitation energy should be regarded as an upper limit [32].

The parallel source velocity was calculated from the source velocity V_s of the multifold events which was determined by $V_s = \sum_i m_i V_i / \sum_i m_i$, where m_i and V_i are, respectively, the mass and velocity in the laboratory frame of the ith fragment and the summation is performed over all the detected fragments. The resulting velocity distributions are very similar to those observed in Ref. [2] for a ^{139}La projectile. Typically, they consist of a broad peak whose width increases with increasing target mass. It has been shown [2,11] that most of this width is due to the actual range of source velocities, and only a fraction is due to the perturbation introduced by light particle emission prior and subsequent to heavy fragment emission.

The number of binary and multibody events was determined for different bins of the source velocity and thus of the excitation energy of the source. By this procedure, we obtained the probabilities for ternary, quarternary, and quinary decays, as a function of the calculated excitation energy, shown in Fig. 2. The measured probabilities were then corrected for the detection efficiency. Sets of twofold, threefold, fourfold, and fivefold events were generated by simulating the reactions following the procedure described in Ref. [31], where the dynamics is given by a Landau-Vlasov calculation [31] and the subsequent statistical decay of the primary fragments is described by

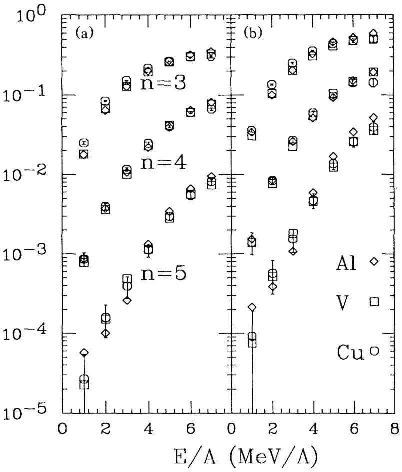

FIG. 2. (a) Uncorrected relative probabilities for the ternary, quaternary, and quinary decays as a function of the source excitation energy for the 60 MeV/nucleon ^{197}Au+^{27}Al, ^{51}V, and $^{\rm nat}$Cu reactions. (b) Same as in (a) after efficiency corrections (see text). Statistical errors are shown for the Cu target when they exceed the size of the symbols.

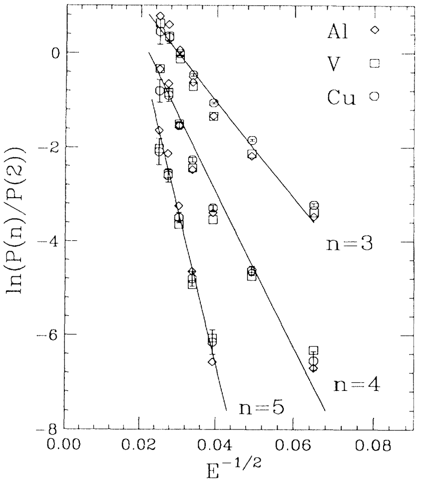

FIG. 3. The natural logarithm of the ratio of the corrected threefold, fourfold, and fivefold probabilities to the twofold probability (symbols) as a function of $E^{-1/2}$ for the 60 MeV/nucleon ^{197}Au+^{27}Al, ^{51}V, and $^{\rm nat}$Cu reactions. The lines are the best fits to the data. Statistical errors are shown for the Cu target when they exceed the size of the symbols.

the statistical code GEMINI [22]. The simulated events were then filtered through a software replica of our detector to estimate the efficiency and the spillover of higher folds into lower fold coincidences.

Let e_j^i be the efficiency for a j-fold event to be detected as an i-fold event. Then the observed number of i-fold events n_i is related to the true number of i-fold events N_i by the set of equations:

$$n_5 = N_5 e_5^5 \,, \tag{5}$$

$$n_4 = N_4 e_4^4 + N_5 e_5^4 \,, \tag{6}$$

$$n_3 = N_3 e_3^3 + N_4 e_4^3 + N_5 e_5^3 \,, \tag{7}$$

$$n_2 = N_2 e_2^2 + N_3 e_3^2 + N_4 e_4^2 + N_5 e_5^2 \,. \tag{8}$$

Using the values of N_i from the simulations and the corresponding values of n_i from the output of the detector filter, one can solve for the efficiencies e_j^i. These efficiencies were then used to correct the experimental data shown in Fig. 2(b).

The first striking observation is that the data from all the targets fall on the same curves. This is a strong confirmation of the results obtained for the La induced reactions [2,11]. More specifically, once the multifragmentation source is characterized in terms of the kinematically determined excitation energy, the branching ratios for the various multifragment channels seem to be fixed and independent of the specific reaction that has produced the source. This decoupling between entrance and exit channel suggests a "statistical" kind of decay.

This statistical feature is brought forth by the $E^{-1/2}$ plot shown in Fig. 3, that indeed generates straight lines. Similar straight lines are obtained from the La data [32]. We believe that the observed linear dependence for both the Au and La induced reactions is a strong signature for processes controlled by phase space. Since this dependence demonstrates statistical equilibrium between "different" channels, it may be deemed more significant evidence for deep equilibration than the thermalization of the kinetic spectrum within a given channel.

Can this signature differentiate between the various statistical models? Equation (4) has been derived for a statistical multifragmentation process. It is immaterial whether we refer to a transition-state model [19] or a "freeze-out" equilibrium model [20,21]. In the former case, B_n is the barrier to be crossed in order to reach an n-body decay configuration. In the latter case, B_n is the "potential energy" of the n-body system at the freeze-out configuration. However, the same dependence can be obtained for sequential decay. Let us suppose that the system undergoes sequential decay with probabilities $P(E) \ll 1$ and with barriers $b_1, b_2, b_3, \ldots, b_n$ for the successive binary decays. The probability to obtain n fragments is

$$P_n(E) \propto K(n) e^{-b_1/T_1} e^{-b_2/T_2} \cdots \propto K(n) e^{-(b_1+b_2\cdots)/T}$$
$$\propto K(n) e^{-B_n/T} \propto K(n) e^{-B_n\sqrt{a/E}} \,, \tag{9}$$

where $K(n)$ is a combinatorial factor and $B_n = b_1 + b_2 + \cdots$. Thus, even for multiple sequential binary decay we expect a linear dependence of $\ln P_n$ with $E^{-1/2}$. Therefore, the observed linear dependence *per se* does not discriminate between a prompt or sequential multifragmentation mechanism.

In principle, however, one can obtain more specific information from the slope of the straight line [see Eq. (4)]. Since B_n could be very different for simultaneous or sequential decay, a greater experience with both the data and the models might lead to a discrimination between the two possibilities. Still, we already have a strong message, that the role of dynamics may be limited to the process of source formation (incomplete fusion, for instance), while phase space seems to control the ultimate fate of the source.

In conclusion, the evidence presented above strongly suggests the following picture for multifragmentation:

(1) The dynamics of the reaction seems to be limited to the formation of a source of a given mass, energy, and angular momentum through a mechanism similar to incomplete fusion.

(2) Once this source is formed, its decay is apparently independent of its mode of formation.

(3) The branching ratios between the various multifragmentation channels are dictated by the available phase space as shown by the excitation functions.

(4) The qualitative features of the excitation functions do not permit distinguishing between a sequential or simultaneous decay mechanism, but the quantitative features may contain relevant information in this regard.

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Note added.—After submission of our paper, we received a preprint [37] that, following our approach, demonstrates the same linear dependence in the breakup of ^{16}O in the reaction of ^{197}Au+^{16}O at 50 MeV/nucleon.

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